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THE DYNAMICAL BEHAVIOR AND APPLICATION OF ONE ALPINE MEADOW MODEL*

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Abstract In recent years, the alpine meadow has degraded seriously, before restoring the degraded alpine meadow, it is necessary to know the cause of degrading and to find the effective restoration measure. One forage grass-livestock-rodent-raptor dynamical model is formulated and analyzed. Based on this model, the cause of alpine meadow degrading is analyzed, the efficiency of restoration measures is evaluated. Overgrazing and uncontrolled hunting are the causes of alpine meadow degradation, and may lead to more serious disaster. Long time supplementary feeding aggravates alpine meadow degrading further. Climate change is not the unique cause of alpine meadow degradation at least. Both determining livestock number by grass yield together with controlling rodent with natural enemy and ecological resettlement together with controlling rodent with natural enemy are effective strategies of restoring degraded alpine meadow. Human beings should fully understand the consequence of their behavior, regulate their behavior, make human beings and nature develop harmoniously.

Keywords Dynamical model, alpine meadow, cause of degradation, restoration measure.

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1. Introduction

In China, the 63.72 million hectares alpine meadow is important base of animal husbandry, is carrier of national culture [16], is habitat for an abundance of unique wildlife, and provides great ecosystem function and services [2]. The alpine meadow is the source region of Yangtze, Yellow and Mekong rivers, and has significant effect on the economic development and ecological environment in their middle and lower reaches.

Because of its special geographical and climatic environment, alpine meadow

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ecosystem is extremely fragile and sensitive to climate change and human activities [4, 17, 21]. In recent years, the alpine meadow has degraded seriously. The height, coverage and biomass of vegetation decreased, the proportion of forage grass declined, the economic potential and service function reduced. The number of livestock increased. The number of pests (include rodent such as *Ochotona curzoniae*, *Myospalax baileyi* and insect such as *Gynaephora*) increased. The biodiversity decreased, the species and quantity of plants, birds and beasts reduced. The water-holding capacity decreased, soil and water loss seriously. On the Qinghai Tibet Plateau, there are 16.2 million hectares degraded alpine meadows and many of them are even barren.

Alpine meadow degradation has caused a great threat to the survival of both human and livestock, to the conservation of biodiversity, to the ecological security, and to economic development. The protection of alpine meadow has always been the research focus of grassland management. The main content is analyzing the causes of degradation and evaluating the efficiency of restoration measures.

Researches show that the possible causes of alpine meadow degrading are as follows [3, 8, 18, 19, 23, 24, 27]. 1) Climate change: Including temperature rising, uneven precipitation and others. 2) Overgrazing: The number of livestock gradually increases and the forage grass is overused. 3) Rodent damage: The degraded alpine meadow is suitable for the survival of plateau pika, as a result, the number of plateau pika increases, the vegetation is further damaged, and the alpine meadow degrades more seriously. 4) Uncontrolled hunting: This refers to the killing of birds and carnivores beasts, which leads to the reduction of predators of rodents, insects and livestock, so that the food web is out of balance. 5) Improper grassland utilization: This refers to that grassland reclamation, abandonment, road construction, mining, collecting medicinal plant, tourism and so on destroy vegetation and reduce forage grass yield.

The ordinary measures used to restore degraded alpine meadow are as follows [5, 9, 22, 25]. 1) Meliorate vegetation: Establish artificial grassland or promote vegetation growth by means of loosening soil, replanting, fertilizing, irrigating and so on in order to increase forage grass yield and proportion. 2) New grazing strategy: Implement new grazing strategies, such as determining livestock number by grass yield, grazing prohibition and seasonal grazing, to depress overfeeding of vegetation. 3) Rodent control: Reduce the amount of rodent through lethal control, contraception control, natural enemy control, destroying habitat. 4) Ecological resettlement: A large number of people and livestock moving out of the degraded alpine meadow. 5) Supplementary feeding: Provide additional forage for livestock in the hope of reducing forage grass intake. 6) Greenhouse construction: The greenhouse can reduce the death rate of livestock in winter and increase the survival rate of young.

So far, the cause of degradation has not been fully determined and long-term effective restoration measures have not been found. The object of this paper is to investigate the dynamics of alpine meadow, analyze the cause of degradation, and seek effective restoration measure by means of dynamical model.

There are many general population dynamic models [7, 10, 20], but few of them are related to alpine meadow. Li etc [11] established a optimal control model of grazing management for alpine meadow, and put forward a strategy for its sustainable utilization. Chang etc [1] formulated a eagle-pika-grass dynamical model and found that increasing eagle and grass can restore degraded alpine meadow. By means of the Lotka-Volterra interspecies competition model, Wu etc [26] predicted that it would take at least 20 years for the artificial grassland on the "black beach" to obtain a more stable plant community. Based on the relationship between plateau pika and vegetation, Liu etc [15] and Liu etc [12] established two prey-predator models in which the carrying capacity of predators depends on the amount of their prey. They show that increasing grazing rate or decreasing plateau pika mortality may cause alpine meadow degradation and reducing grazing rate and increasing plateau pika mortality may recover the degraded alpine meadow effectively. From the analysis of a vegetation-pika model with Holling II functional response, Liu etc [14] found that global warming and decreasing of raptor or other natural enemy of rodent may cause alpine meadow degrading, and controlling rodent or protecting its natural enemy can restore the degraded alpine meadow. Liu etc [13] formulated and analyzed the dynamical property of a rodent-forage grass-raptor-livestock model. Their analysis shows that the increasing of raptor mortality and the decreasing of livestock mortality (or the increasing of the rate at which livestock increases by consuming forage grass) are the major causes of alpine meadow degradation. Accordingly, controlling the amount of livestock according to grass yield or ecological resettlement, together with protecting raptor, is an effective strategy to restore degraded alpine meadow.

However, the model in Liu etc [13] did not involve the functional response of predator to its prey. In the present paper, we formulate a forage grass-livestock-rodent-raptor model in which the predator rate of livestock to forage grass is Holling II type. We discuss why alpine meadow degrading and how restoring the degraded alpine meadow based on the theoretical analysis of this model.

2. Model formulation and analysis

Forage grass, livestock (Domestic herbivores, e g: *Bos grunniens*, *Ovis aries*, *Capra hircus*), rodent (Small herbivores, e g: *Ochotona curzoniae*, *Myospalax baileyi*) and raptor (Predator of rodent, e g: *Buteo hemilasius*, *Falco cherrug*, *Vulpes ferrilata*, *Mustela altaica*) are important components of alpine meadow ecosystem, and they would be usually mentioned when discussing alpine meadow degradation. Livestock and rodent feed on forage grass, raptor preys on rodent. One can formulate model (2.1).

$$\begin{cases} x' = rx(1-\dfrac{x}{K}) - \dfrac{\alpha xy}{\delta + x} - \mu xz, \\ y' = -d_2 y + \dfrac{\beta xy}{\delta + x}, \\ z' = -d_3 z + \eta xz - pzu, \\ u' = -d_4 u + qzu. \end{cases} \tag{2.1}$$

Here, $x(t)$, $y(t)$, $z(t)$, $u(t)$ are the amount of forage grass, livestock, rodent, raptor at time t respectively. The parameters $r > 0$, $K > 0$ are the intrinsic growth rate and carrying capacity of forage grass, $d_2 > 0$, $d_3 > 0$, $d_4 > 0$ are the mortality rates of livestock, rodent and raptor when there is no food. In comparison, grazing has the greatest impact on alpine meadow. Assume that the predator rate of livestock to forage grass is Holling II type, that is, $\frac{\alpha x}{\delta + x}$ [6], where $\alpha > 0$ is the maximum feeding rate, and feeding rate is half maximal at $x = \delta > 0$. Parameter $\beta > 0$ is the maximum birth rate of new born livestock, $\mu > 0$ is the feeding rate of rodent on forage grass, $\eta > 0$ is the birth rate of new born rodent, $p > 0$ is the predation rate of raptor on rodent, and $q > 0$ is the birth rate of new born raptor.

For convenience, let $\frac{dt}{\delta+x} = d\tau$, that is, $dt = (\delta + x)d\tau$, then model (2.1) can be transformed to model (2.2), and these two models have same equilibriums and same dynamical behavior.

$$\begin{cases} \dfrac{dx}{d\tau} = rx(1 - \dfrac{x}{K})(\delta + x) - \alpha xy - \mu xz(\delta + x), \\ \dfrac{dy}{d\tau} = -d_2 y(\delta + x) + \beta xy, \\ \dfrac{dz}{d\tau} = z(\delta + x)(-d_3 + \eta x - pu), \\ \dfrac{du}{d\tau} = u(\delta + x)(-d_4 + qz). \end{cases} \tag{2.2}$$

For simplicity of notation, introduce the following quantities,

$$A = \frac{\mu}{r} > 0, B = \frac{\delta d_2}{\beta - d_2}, C = \frac{d_3}{\eta} > 0, D = \frac{d_4}{q} > 0.$$

Theorem 2.1. *The following statements on equilibriums of model* (2.2) *are true.*
(i) There are always equilibriums $O : x = y = z = u = 0$ *and* $E_1 : x = K, y = z = u = 0$.
(ii) When $C < K$, *there is equilibrium* $E_2 : x = x_2 \triangleq C$, $z = z_2 \triangleq \frac{1}{A}(1 - \frac{C}{K})$, $y = u = 0$.
(iii) When $K - C - AKD > 0$, *there is equilibrium* $E_3 : x = x_3 \triangleq K(1 - AD)$, $y = 0$, $z = z_3 \triangleq D$, $u = u_3 \triangleq \frac{\eta}{p}(K - C - ADK)$.
(iv) When $K > B > 0$, *there is equilibrium* $E_4 : x = x_4 \triangleq B$, $y = y_4 \triangleq \frac{\beta r B}{\alpha K d_2}(K-B)$, $z = u = 0$.
(v) When $B > C > 0, K - B - AKD > 0$ *(Implies* $1 > AD$*), there is equilibrium* $E_5 : x = x_5 \triangleq B$, $y = y_5 \triangleq \frac{\beta r B}{\alpha K d_2}(K - B - ADK)$, $z = z_5 \triangleq D$, $u = u_5 \triangleq \frac{\eta}{p}(B - C)$.

The proof is simple and is omitted.

Next, study the stability of equilibriums.

Theorem 2.2. *For model* (2.2)*, the following statements hold.*
(i) The equilibrium O *is unstable.*
(ii) The equilibrium E_1 *is globally asymptotically stable if* $C > K, \frac{1}{B} < \frac{1}{K}$.
(iii) The equilibrium E_2 *is globally asymptotically stable if* $K > C > K(1-AD), \frac{1}{B} < \frac{1}{C}$.
(iv) The equilibrium E_3 *is globally asymptotically stable if* $K(1 - AD) > C, \frac{1}{B} < \frac{1}{K(1-AD)}$.
(v) The equilibrium E_4 *is globally asymptotically stable if* $K > B > 0, C > B, K - \delta < B$.
(vi) The equilibrium E_5 *is globally asymptotically stable if* $B > C, K - B - ADK > 0, \delta + B - K + ADK > 0$ *(Implies* $1 > AD$*).*

Proof. Only the conclusion (vi) is proved, and the proof of other conclusions is similar.

The Jacobian matrix of model (2.2) at equilibrium E_5 is

$$J_{E_5}=\begin{pmatrix} \frac{rx_5}{K}(K-2x_5-\delta)-\mu x_5 z_5 & -\alpha x_5 & -\mu x_5(\delta+x_5) & 0 \\ (-d_2+\beta)y_5 & 0 & 0 & 0 \\ \eta z_5(\delta+x_5) & 0 & 0 & -pz_5(\delta+x_5) \\ 0 & 0 & qu_5(\delta+x_5) & 0 \end{pmatrix}.$$

The characteristic equation is $a_0\lambda^4+a_1\lambda^3+a_2\lambda^2+a_3\lambda+a_4=0$, where $a_0=1$, $a_1=\frac{rx_5}{K}(\delta+x_5)-rx_5(1-\frac{x_5}{K})+\mu x_5 z_5$, $a_2=\alpha(\beta-d_2)x_5y_5+\eta\mu x_5 z_5(\delta+x_5)^2+pqu_5z_5(\delta+x_5)^2$, $a_3=pqz_5u_5(\delta+x_5)^2 a_1$, $a_4=\alpha\beta\delta pqx_5y_5z_5u_5(\delta+x_5)$.

To use Routh-Hurwits criterion, one can calculate

$$\begin{aligned}
&\Delta_1=a_1=\frac{\mu B}{AK}(\delta+2B-K+ADK),\\
&\Delta_2=\begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}=[\alpha\delta d_2 y_5+\eta\mu x_5 z_5(\delta+x_5)^2]a_1,\\
&\Delta_3=\begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ 0 & a_4 & a_3 \end{vmatrix}=\eta\mu pqx_5 z_5^2 u_5(\delta+x_5)^4 a_1^2>0,\\
&\Delta_4=\Delta_3 a_4>0.
\end{aligned}$$

So, when $B>C, K-B-ADK>0, \delta+2B-K+ADK>0$ hold, E_5 is local stable.

Define a Lyapunov function

$$\begin{aligned}
V=&x-x_5-x_5ln\frac{x}{x_5}+\frac{\alpha}{\beta-d_2}(y-y_5-y_5ln\frac{y}{y_5})\\
&+\frac{\mu}{\eta}(z-z_5-z_5ln\frac{z}{z_5})+\frac{\mu p}{\eta q}(u-u_5-u_5ln\frac{u}{u_5}).
\end{aligned}$$

The total derivative of V along solutions of model (2.2) is

$$\frac{dV}{d\tau}\Big|_{(2.2)}=\frac{-r}{K}(x-x_5)^2(x+\delta+B-K+ADK).$$

Thus $\frac{dV}{d\tau}\Big|_{(2.2)}\le 0$, and $E=\{\frac{dV}{d\tau}\Big|_{(2.2)}=0\}=\{x=x_5\}$. On set E, model (2.2) reduces to

$$\begin{cases}
x=B, & (2.3)\\
0=r(1-\frac{B}{K})(B+\delta)-\alpha y-\mu z(B+\delta), & (2.4)\\
y'=0, & (2.5)\\
z'=z(B+\delta)(-pu-d_3+\eta B), & (2.6)\\
u'=u(B+\delta)(qz-d_4). & (2.7)
\end{cases}$$

From equations (2.5) and (2.4), one gets that $y = constant$ and $z = constant$, then equation (2.6) gives $z = 0$ or $u = u_5$. If $z = 0$, then equations (2.4) and (2.7) imply $y = y_4$ and $u = \gamma exp(\frac{-\beta\delta d_4}{\beta - d_2}\tau)$, where γ is an arbitrary constant, thus $u \to 0(\tau \to +\infty)$, so E_4 belongs to the invariant set of E. If $u = u_5$, then equation (2.7) implies $z = z_5$, equation (2.4) implies $y = y_5$, so E_5 belongs to the invariant set of E. Therefore, the largest invariant set of E is $\{E_4, E_5\}$. Notice that E_4 is unstable under the condition in (vi), so E_5 is globally asymptotically stable. This completes the proof. □

According to Theoren 2.2, the upper half plane of plane $B - C$ can be divided into domains D1, D2, D3, D4, D5 and D (Figure 1). When point (B, C) is located in domain D1, D2, D3, D4, D5, equilibrium E_1, E_2, E_3, E_4, E_5 is global asymptotically stable respectively. When point (B, C) is located in domain D, all equilibriums are unstable, and there may exist limit cycle (Figure 2). Figure 1 shows the case of $AD < 1$, and when $AD \geq 1$, domains D2, D3, D5 are combined as D2.

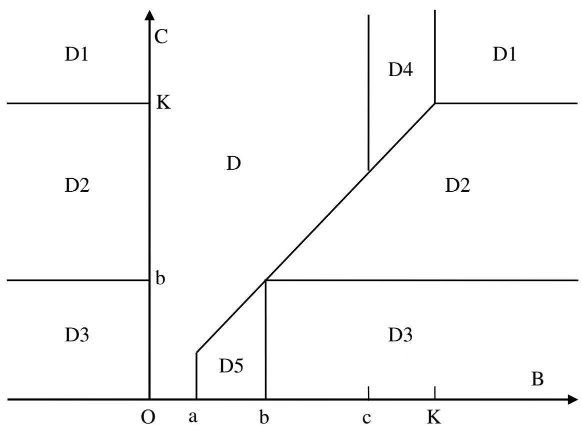

Figure 1. The stability domains of equilibriums in $B - C$ plane. Where $a = K(1 - AD) - \delta, b = K(1 - AD), c = K - \delta$ (c may be less than b, and a may be less than 0).

3. The cause of alpine meadow degradation

Alpine meadow degrading is a slow and continuous process. During this process, no forage grass, livestock, rodent or raptor was extinct, and their numbers did not fluctuate significantly. So it is reasonable to think that equilibrium E_5 is always stable and point (B, C) is always located in domain D5.

The parameters in model (2.2) are formed during the long-term evolution of organisms, and do not change in general. However, due to the climate change and the intensification of human activities, the values of some parameters in model (2.2) alter, the coordinates of E_5 and some important thresholds alter, as a result, alpine meadow degraded. The possible causes of alpine meadow degradation are climate change, overgrazing, improper grassland utilization, rodent damage, uncontrolled hunting, supplementary feeding, greenhouse construction, etc.

Overgrazing is a phenomenon of more livestock and less forage grass, it is caused by herdsmen's behavior mainly. Herdsmen provide their livestock better medical condition, take good care of their newborn livestock, build greenhouses for their

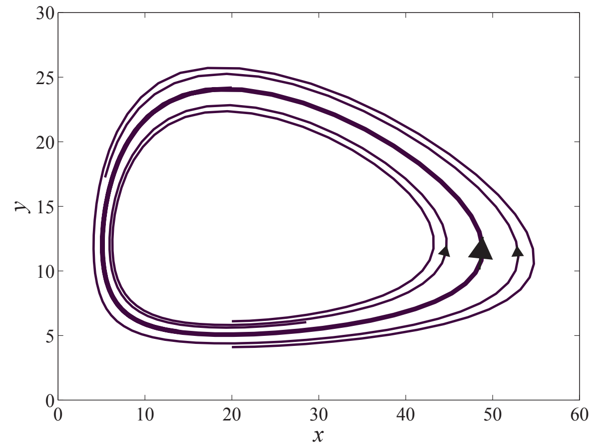

Figure 2. The limit cycle and trajectories near it in plane $x-y$. Where the values of parameters are $r = 2.3year^{-1}, K = 100Kg, \delta = 50Kg, \alpha = 10Kg \cdot head^{-1} \cdot year^{-1}, \beta = 4.3year^{-1}, \mu = 0.01head^{-1} \cdot year^{-1}, \eta = 0.01Kg^{-1} \cdot year^{-1}, p = 0.02head^{-1} \cdot year^{-1}, q = 0.01head^{-1} \cdot year^{-1}, d_2 = 1.2year^{-1}, d_3 = 0.3year^{-1}, d_4 = 0.5year^{-1}$.

livestock to survive the cold winter, supplement their livestock additional forage when food is scarce, drive away the predator for their livestock, and sell less livestock. All of these reduce the removal rate of livestock and increase the survival rate, as a result, the parameter d_2 in model (2.2) decreases.

The effect of climate change on alpine meadow is complex. Temperature rising with abundant precipitation is beneficial to the growth of vegetation, but these two cannot be realized at the same time in general. Up to now, there is no definite conclusion about how climate change affects alpine meadow, so how parameters r and K in model (2.2) vary is uncertain.

Improper grassland utilization restrain forage grass growing and (or) reduce growth area, so reduce the values of parameters r and K in model (2.2).

Rodent damage derives from degraded alpine meadow being suitable for rodent survival, this time, parameter d_3 in model (2.2) decreases.

Uncontrolled hunting increases the value of parameter d_4 in model (2.2).

If supplement livestock additional forage for long time, then livestock feeds on less forage grass but can produce more offspring, this is equivalent to the decrease of parameter α in model (2.2) and the increase of parameter β in model (2.2).

If the aforementioned parameters vary, the coordinates of E_5 and some thresholds would change. When equilibrium E_5 is globally asymptotically stable, that is, the condition of $\frac{d_3}{\eta} < \frac{\delta d_2}{\beta - d_2} < K(1 - \frac{\mu d_4}{qr}) < \frac{\beta\delta}{\beta - d_2}$ holds, one can calculate the partial derivatives of variables $x_5, y_5, z_5, u_5, C = \frac{d_3}{\eta}, B = \frac{\delta d_2}{\beta - d_2}, a = K(1 - AD) - \delta = K(1 - \frac{\mu d_4}{rq}) - \delta, b = K(1 - AD) = K(1 - \frac{\mu d_4}{rq}), AD = \frac{\mu d_4}{rq}$ with respect to parameters $r, K, \alpha, \beta, d_2, d_3, d_4$ respectively and determine the influence of parameters change on these variables from the sign of these partial derivatives (Table 1).

With alpine meadow degrading, x_5 decreases, y_5 increases, z_5 increases, and u_5 decreases. From Table 1, one knows that the change of z_5 caused by uncontrolled hunting is consistent with the degradation of alpine meadow, the change of x_5, y_5, z_5 caused by overgrazing or long time supplementary feeding is consistent with the degradation of alpine meadow. Notice that herdsmen began to supplement their livestock with additional feeding only after alpine meadow degraded. Therefore,

Table 1. The influence of possible causes of degradation.

Possible cause of degradation	Parameter varying	x_5	y_5	z_5	u_5	B	C	b	a	AD
Overgrazing	$d_2 \searrow$	$\searrow$	$\nearrow$	–	$\searrow$	$\searrow$	–	–	–	–
Climate change	$r?$	–	?	–	–	–	–	?	?	?
	$K?$	–	?	–	–	–	–	?	?	–
Improper grassland utilization	$r \searrow$	–	$\searrow$	–	–	–	–	$\searrow$	$\searrow$	$\nearrow$
	$K \searrow$	–	$\searrow$	–	–	–	–	$\searrow$	$\searrow$	–
Rodent damage	$d_3 \searrow$	–	–	–	$\nearrow$	–	$\searrow$	–	–	–
Uncontrolled hunting	$d_4 \nearrow$	–	$\searrow$	$\nearrow$	–	–	–	$\searrow$	$\searrow$	$\nearrow$
Long time supplementary feeding	$\alpha \searrow$	–	$\nearrow$	–	–	–	–	–	–	–
	$\beta \nearrow$	$\searrow$	$\nearrow$	–	$\searrow$	$\searrow$	–	–	–	–

overgrazing and uncontrolled hunting are the causes of alpine meadow degradation. The effect of overgrazing and uncontrolled hunting on y_5 is opposite, in reality, the increase of y_5 can be considered as the comprehensive effect of both, and the effect of overgrazing is greater. Although long time supplementary feeding is not the cause of alpine meadow degradation, it undoubtedly aggravates alpine meadow degrading further.

Table 1 shows that improper grassland utilization has no effect on x_5, z_5, u_5 and the effect on y_5 is opposite to alpine meadow degradation, so improper grassland utilization is not the cause of alpine meadow degradation. Similarly, rodent damage has no effect on x_5, y_5, z_5 and the effect on u_5 is opposite to alpine meadow degradation (Table 1), so rodent damage is not the cause of alpine meadow degradation too.

Whether climate change increases or decreases r and K, it can change y_5 only and can not change x_5, z_5, u_5 (Table 1), therefore, climate change is not the unique cause of alpine meadow degradation at least, more detailed research is expected.

In conclusion, overgrazing and uncontrolled hunting are the causes of alpine meadow degradation. They reduce the parameter d_2 and increase the parameter d_4 in model (2.2), so they reduce thresholds $B, K(1-AD), K(1-AD)-\delta$ and increase threshold AD. These can result in 4 results: 1) Point (B, C) is still in domain D5, E_5 is stable. 2) AD is greater than 1, D5 merges into D2, E_2 is stable. 3) The left and right boundaries of domain D5 move leftward, the point (B, C) move into domain D3, E_3 is stable. 4) The point (B, C) moves into domain D, limit cycle appears.

If result 2) occurs, livestock and raptor will be extinct, if result 3) occurs, livestock will be extinct, both result 2) and 3) will make alpine meadow ecosystem incomplete and make the animal husbandry collapse. If result 4) occurs, the amounts of forage grass, livestock, rodent and raptor will fluctuate periodically. This is a kind of interference to alpine meadow, if it exceeds the tolerance of alpine meadow, then alpine meadow system will collapse and cause ecological disaster. Therefore, the degradation of alpine meadow may lead to more serious consequence.

4. The efficiency of restoration measure

Have known that overgrazing (d_2 decreasing) and uncontrolled hunting (d_4 increasing) are the causes of alpine meadow degradation, so accordingly any measure that can increase d_2 and decrease d_4 can restore the degraded alpine meadow. The supplementary feeding and greenhouse construction can cause alpine meadow degradation, so they have no help to restore degraded alpine meadow. Next, evaluate the efficiency of other ordinary restoration measures.

In usual, the restoration measure would alter the parameters in model (2.2). Meliorating vegetation increases parameters r and K, killing rodent increases parameter d_3, controlling rodent with sterilant decreases parameter η, controlling rodent through protecting, attracting, releasing its natural enemy decreases parameter d_4. Determining livestock number by grass yield can balance grass and livestock and avoid overgrazing. In the degraded alpine meadow, there are more livestock and less forage grass, so it is necessary to reduce the number of livestock, that is, to increase the removal rate of livestock d_2. After ecological resettlement, the left herdsmen have enough livestock, and the left livestock have enough forage grass, so herdsmen do not need to take additional care of their livestock, so the removal rate of livestock returns to the natural value, that is, parameter d_2 increases.

When equilibrium E_5 is globally asymptotically stable, one can calculate the partial derivatives of variables x_5, y_5, z_5, u_5 with respect to parameters $r, K, \eta, d_2, d_3, d_4$ respectively and determine the influence of parameters change on these variables from the sign of partial derivatives (Table 2).

Table 2. The influence of restoration measures.

Restoration measure	Parameter varying	Influence on variable			
		x_5	y_5	z_5	u_5
Meliorating vegetation	$r \nearrow$	–	$\nearrow$	–	–
	$K \nearrow$	–	$\nearrow$	–	–
Determining livestock number by grass yield	$d_2 \nearrow$	$\nearrow$	$\searrow$	–	$\nearrow$
Killing rodent	$d_3 \nearrow$	–	–	–	$\searrow$
Controlling rodent with sterilant	$\eta \searrow$	–	–	–	$\searrow$
Controlling rodent with natural enemy	$d_4 \searrow$	–	$\nearrow$	$\searrow$	–
Ecological resettlement	$d_2 \nearrow$	$\nearrow$	$\searrow$	–	$\nearrow$

Contrary to alpine meadow degrading, the restoration of degraded alpine meadow is to increase x_5, decrease y_5, decrease z_5 and increase u_5.

Table 2 shows that meliorating vegetation increases y_5, killing rodent and controlling rodent with sterilant decrease u_5, all of these are opposite to the restoration of degraded alpine meadow, so these measures can not restore degraded alpine meadow. Both determining livestock number by grass yield and ecological resettlement can increase d_2 and controlling rodent with natural enemy can decrease d_4, therefore, both determining livestock number by grass yield together with controlling rodent with natural enemy and ecological resettlement together with controlling rodent with natural enemy are effective strategies of restoring degraded alpine meadow. Note that after implementing each of these two effective strategies, y_5 may increase and may decrease, the change of y_5 is determined by the comprehensive effect.

5. Discussion

Based on a forage grass-livestock-rodent-raptor dynamical model, analyzed the cause of alpine meadow degrading, and evaluated the efficiency of ordinary restoration measures.

Overgrazing and uncontrolled hunting are the causes of alpine meadow degradation, and may lead to more serious consequence. Long time supplementary feeding aggravates alpine meadow degradation further. Improper grassland utilization, rodent damage are not the cause of alpine meadow degradation. Climate change is not the unique cause of alpine meadow degradation at least.

Supplementary feeding and greenhouse construction can cause alpine meadow degradation, so they have no help to restore degraded alpine meadow. Meliorating vegetation, killing rodent and controlling rodent with sterilant cannot restore degraded alpine meadow. Determining livestock number by grass yield together with controlling rodent with natural enemy and ecological resettlement together with controlling rodent with natural enemy are effective strategies of restoring degraded alpine meadow.

Overgrazing is the result of improper human behavior, uncontrolled hunting is an improper human behavior, both of them are related to improper human behavior. The development of society, science, technology and so on provides necessary conditions for overgrazing and uncontrolled hunting, while human beings do not realize or ignore the serious consequences of their improper behavior. Therefore, the contradiction between the rapid development of society and the backwardness of human ecological consciousness is the basic reason of alpine meadow degrading. Nowadays, human beings can alter the nature more greatly. Human beings should fully understand the consequence of their behavior, regulate their behavior, and make human beings and nature develop harmoniously.

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