THE EFFECT OF MASS TRANSFER ON MHD FREE CONVECTIVE RADIATING FLOW OVER AN IMPULSIVELY STARTED VERTICAL PLATE EMBEDDED IN A POROUS MEDIUM

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Abstract The laminar convective heat and mass transfer flow of an incompressible, viscous, electrically conducting fluid over an impulsively started vertical plate with conduction-radiation embedded in a porous medium in presence of transverse magnetic field has been studied. An exact solution is derived by solving the dimensionless governing coupled partial differential equations. As the equations are nonlinear, so Laplace transform technique is used to solve it. The effects of important physical parameters on the velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number have been analyzed through graphs. The results of the present study agree well with the previous solutions obtained without mass transfer. After the consideration of mass transfer, some different results are noticed. Applications of the present study arise in material processing systems and different industries.

Keywords MHD, Porosity, Thermal radiation, shear stress, Nusselt number, Sherwood number.

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1. Introduction

Nomenclature

C^*	Species concentration (kgm^{-3}),	G_r	Thermal Grashof number,
C_p	Specific heat at constant pressure ($Jkg^{-1}K$),	G_m	Mass Grashof number,
C_∞^*	Species concentration in the free stream (kgm^{-3}),	K_r	Permeability parameter,
C_w^*	Species concentration at the surface (kgm^{-3}),	M	Hartmann number,
D	Chemical molecular diffusivity (m^2s^{-1}),	N_u	Nusselt number,
g	Acceleration due to gravity (ms^{-2}),	P_r	Prandtl number,
q_r	Radiative heat flux,	S_h	Sherwood number,

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T_w^*	Fluid temperature at the surface (K),	S_c	Schmidt number
T_∞^*	Fluid temperature in the free stream (K),	T^*	Temperature (K),
u	Dimensionless velocity component (ms^{-1}),	u_0	plate velocity (ms^{-1}),
Greek symbols			
β	Coefficient of volume expansion for heat transfer(K^{-1}),	β_c	Coefficient of volume expansion for mass transfer(K^{-1}),
θ	Dimensionless fluid temperature (K),	k	Thermal conductivity $(Wm^{-1}K^{-1})$,
ν	Kinematic viscosity (m^2s^{-1}),	ρ	Density (kgm^{-3}),
σ	Electrical conductivity,	τ	Shearing stress (Nm^{-2})
C	Dimensionless species concentration (kgm^{-3}),		
Subscripts			
w	Conditions on the wall,	∞	Free stream condition

Many transport processes exist in industries and technology where the transfer of heat and mass occurs simultaneously as a result of thermal diffusion and diffusion of chemical species. Natural convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest in nature and in many industrial applications such as meteorology, chemical industry, cooling of nuclear reactors, cosmic fluid dynamics, magneto hydrodynamics power generators and in the earths core. Many studies that considered combined heat and mass transfer in natural convection boundary layer flows over heated surfaces with various geometries can be found in the monograph by Gebhart et.al [10].

The subject of magneto hydrodynamics has attracted the attention of a large number of scholars due to its diverse applications in several problems of technological importance. Heat transfer by thermal radiation is becoming of great importance when we are concerned with space applications, higher operating temperatures and also power engineering. In astrophysics and geophysics, it is mainly applied to study the stellar and solar structures, radio propagation through the ionosphere etc. Recently it is of great interest to study the effects of magnetic field and other participating parameters on heat and mass transfer in porous medium when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing thermal radiation. Ganeswara Reddy M and Bhaskar Reddy N [13] studied mass transfer and heat generation effects on MHD free convection flow in a porous medium. B Shankar et.al [15] investigated radiation and mass transfer effects on unsteady MHD free convective fluid flow in a porous medium with heat generation. Ahmed et.al [3] studied the finite difference approach in porous media transport modelling for Magneto hydrodynamic unsteady flow over a vertical plate. Ahmed et.al [4] gave an exact solution to the Hartmann newtonian radiating MHD flow for a rotating vertical porous channel immersed in a Darcian porous regime.

Extensive research work has been published on an impulsively started vertical plate with of course under different boundary conditions depending upon the physics of the problem. An exact solution to the Navier-Stokes equation of the flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate moving in its own plane was first examined by Stokes [1] which is being often referred as Rayleigh's problem in the literature and has inspired several investigators. For the first time, the exact solution of the Stokes problem for the case of infinite vertical

plate was obtained by Soundalgekar [17]. Jaiswal and Soundalgekar [12] obtained an approximate solution to the problem of an unsteady flow past an infinite vertical plate with constant suction and embedded in a porous medium with oscillating plate temperature. The effects of radiation and viscous dissipation on the transient natural convection-radiation flow of viscous dissipation fluid along an infinite surface embedded in a porous medium by means of network simulation method, was investigated by Zueco [19]. Hossain and Takhar [11] studied the radiation effects on mixed convection along a vertical plate with uniform surface temperature using the Rosseland flux model. Senapati and Dhal [14] studied magnetic effect on mass and heat transfer of hydrodynamic flow past a vertical oscillating plate in presence chemical reaction. Chamka [7] presented thermal radiation and buoyancy effects on hydro magnetic flow over an accelerating permeable surface with heat source or sink. Chen [8] studied heat and mass transfer with variable wall temperature and concentration. Cookey et al [9] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Recently Ahmed et.al [2] studied the convective laminar radiating flow over an accelerated vertical plate embedded in a porous medium with an external magnetic field. Ahmed et.al [6] analyzed the Non-linear Magneto hydrodynamic Radiating flow over an impulsively started vertical plate in a saturated porous regime. Ahmed et.al [1,5] investigated the magneto hydrodynamic chemically reacting and radiating fluid past an impulsively started vertical plate using numerical technique.

Here it is proposed to study the effect of mass transfer on MHD convective laminar radiating flow over an impulsively started vertical plate embedded in a porous medium. The magnetic field is imposed transversely to the plate. The whole system is solved by Laplace transform method to obtain an exact solution.

2. Mathematical Formulation

The laminar convective heat and mass transfer flow of an incompressible, viscous, electrically conducting fluid over an impulsively started vertical plate with conduction-radiation embedded in a porous medium in presence of transverse magnetic field has been studied. The x^* axis is taken along the plate in the vertical upward direction and the y^* axis is taken normal to the plate. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. It is also assumed that the thermal radiation along the plate is negligible as compared to that in the normal direction. The induced magnetic field and viscous dissipation is assumed to be negligible. Initially it is assumed that the plate and fluid are at the same temperature T_∞^* in the stationary condition with concentration level C_∞^* at all the points. At time, $t^* > 0$ the plate is given an impulsive motion in its own plane with velocity u_0. The temperature of the plate and the concentration level are also raised to T_w^* and C_w^* . They are maintained at the same level for all time $t^* > 0$. Then under the above assumption the unsteady flow with usual Boussinesq's approximation is governed by the following equations

$$\frac{\partial u^*}{\partial t^*} = g\beta\Big(T^* - T_\infty^*\Big) + g\beta_c\Big(C^* - C_\infty^*\Big) + \nu\frac{\partial^2 u^*}{\partial y^{*2}} - \Big(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{K^*}\Big)u^*, \qquad (2.1)$$

$$\rho C_p \frac{\partial T^*}{\partial t^*} = k\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{\partial q_r}{\partial y^*}, \qquad\qquad\qquad\qquad\qquad (2.2)$$

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}}. \tag{2.3}$$

The initial and boundary conditions are

$$
\begin{aligned}
&t^* \le 0: u^* = 0, T^* = T_\infty^*, C^* = C_\infty^* \text{ for every } y, \\
&t^* > 0: u^* = 0, T^* = T_W^*, C^* = C_W^* \text{ at } y = 0, \\
&t^* > 0: u^* \to 0, T^* \to T_\infty^*, C^* \to C_\infty^* \text{ at } y \to \infty.
\end{aligned} \tag{2.4}
$$

The radiative heat flux term is simplified by making use of the Rosseland approximation [16] as

$$q_r = \frac{-4}{3} \frac{\sigma^* \partial T^{*4}}{a^* \partial y^*} \tag{2.5}$$

where σ^* and a^* are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. It should be noted that by using the Rosseland approximation, we limit our analysis to optically thik fluids. If temperature differences within the flow are sufficiently small, such that T^{*4} may be expressed as a linear function of the temperature, then the Taylor series for T^{*4} about T_∞^* , after neglecting higher order terms, is given by

$$T^{*4} \cong 4T^* T_\infty^{*\,3} - 3T_\infty^{*\,4}. \tag{2.6}$$

Substituting (2.5) and (2.6) in (2.2) we have

$$\rho C_p \frac{\partial T^*}{\partial t^*} = \left[k + \frac{16}{3} \frac{\sigma^*}{a^*} T_\infty^* \right] \frac{\partial^2 T^*}{\partial y^{*2}}. \tag{2.7}$$

Let us introduce the following non dimensional terms in (2.1), (2.7) and (2.3).

$$
\begin{aligned}
&y = \frac{u_0 y^*}{\nu}, u = \frac{u^*}{u_0}, P_r = \frac{\rho \nu C_p}{k}, S_c = \frac{\nu}{D}, t = \frac{u_0^2 t^*}{\nu}, K_r = \frac{u_0^2 K^*}{\nu^2}, \\
&\theta = \frac{T^* - T_\infty^*}{T_W^* - T_\infty^*}, C = \frac{C^* - C_\infty^*}{C_W^* - C_\infty^*}, M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, N_a = \frac{ka^*}{4\sigma^* T_\infty^{*\,3}}, \\
&G_r = \frac{\nu g \beta \left(T_W^* - T_\infty^* \right)}{u_0^3}, G_m = \frac{\nu g \beta_c \left(C_W^* - C_\infty^* \right)}{u_0^3}.
\end{aligned} \tag{2.8}
$$

Hence the non dimensional form of (2.1), (2.7) and (2.3) are

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \left(M + K_r^{-1} \right) u + G_r \theta + G_m C, \tag{2.9}$$

$$3 N_a P_r \frac{\partial \theta}{\partial t} = \left(3 N_a + 4 \right) \frac{\partial^2 \theta}{\partial y^2}, \tag{2.10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2}. \tag{2.11}$$

The transformed initial and boundary conditions are

$$
\begin{aligned}
&t \le 0: u = 0, \theta = 0, C = 0 \text{ for every } y, \\
&t > 0: u = 1, \theta = 1, C = 1 \text{ at } y = 0, \\
&t > 0: u \to 0, \theta = \to 0, C \to 0 \text{ as } y \to \infty.
\end{aligned} \tag{2.12}
$$

3. Method of Solution

The equations (2.9) to (2.11) are nonlinear, coupled partial differential equations.
So we want to solve them by using Laplace transform technique. Taking Laplace
Transform, the equations (2.9), (2.10) and (2.11) reduce to

$$\frac{\partial^2 \bar{u}}{\partial y^2} - \left(N + s\right)\bar{u} + G_r\bar{\theta} + G_m\bar{C} = 0, \tag{3.1}$$

$$\frac{\partial^2 \bar{\theta}}{\partial y^2} - As\bar{\theta} = 0, \tag{3.2}$$

$$\frac{\partial^2 \bar{C}}{\partial y^2} - sS_c\bar{C} = 0, \tag{3.3}$$

where s is Laplace transform parameter.

The boundary conditions (2.12) reduce to

$$\bar{u} = \frac{1}{s}, \bar{\theta} = \frac{1}{s}, \bar{C} = \frac{1}{s} \text{ when } y = 0,$$
$$\bar{u} = 0, \bar{\theta} = 0, \bar{C} = 0 \text{ when } y \to \infty. \tag{3.4}$$

Solving (3.1), (3.2) and (3.3) with boundary conditions (3.4) we get

$$\bar{\theta} = \frac{1}{s}e^{-\sqrt{As}y}, \tag{3.5}$$

$$\bar{C} = \frac{1}{s}e^{-\sqrt{S_c s}y}, \tag{3.6}$$

$$\bar{u} = \left[\frac{1}{s} + \frac{1}{s}\left(\frac{G_r}{As - (N+s)} + \frac{G_m}{S_c s - (N+s)}\right)\right]e^{-\sqrt{s+N}y}$$
$$- \frac{1}{s}\left[\frac{G_r e^{-\sqrt{As}y}}{As - (N+s)} + \frac{G_m e^{-\sqrt{S_c s}y}}{S_c s - (N+s)}\right]. \tag{3.7}$$

Inverting the equations (3.5), (3.6) and (3.7) we get

$$\theta = erfc(\eta\sqrt{A}), \tag{3.8}$$

$$C = erfc(\eta\sqrt{S_c}), \tag{3.9}$$

$$u = f_1\left[e^{-2\eta\sqrt{Nt}}erfc(\eta - \sqrt{Nt}) + e^{2\eta\sqrt{Nt}}erfc(\eta + \sqrt{Nt})\right]$$
$$+ f_2\left[e^{-2\eta\sqrt{Abt}}erfc(\eta - \sqrt{Abt}) + e^{2\eta\sqrt{Abt}}erfc(\eta + \sqrt{Abt})\right]$$
$$+ f_3\left[e^{-2\eta\sqrt{S_cbt}}erfc(\eta - \sqrt{S_cbt}) + e^{2\eta\sqrt{S_cbt}}erfc(\eta + \sqrt{S_cbt})\right]$$
$$- f_2\left[e^{-2\eta\sqrt{Abt}}erfc(\eta\sqrt{A} - \sqrt{bt}) + e^{2\eta\sqrt{Abt}}erfc(\eta\sqrt{A} + \sqrt{bt})\right]$$
$$- f_3\left[e^{-2\eta\sqrt{S_cbt}}erfc(\eta\sqrt{S_c} - \sqrt{bt}) + e^{2\eta\sqrt{S_cbt}}erfc(\eta\sqrt{S_c} + \sqrt{bt})\right]$$
$$+ f_4erfc(\eta\sqrt{A}) + f_5erfc(\eta\sqrt{S_c}), \tag{3.10}$$

where $b = \frac{N}{A-1}$, $N = M + \frac{1}{K_r}$, $A = \frac{3N_aP_r}{3N_a+4}$, $\eta = \frac{y}{2\sqrt{t}}$, $f_1 = \frac{1}{2} - \frac{1}{2}\left(\frac{G_r+G_m}{N}\right)$,
$f_2 = \frac{1}{2}\left(\frac{G_r}{N}\right)e^{bt}$, $f_3 = \frac{1}{2}\left(\frac{G_m}{N}\right)e^{bt}$, $f_4 = \frac{G_r}{N}$, $f_5 = \frac{G_m}{N}$.

The Skinfriction at the surface of the plate is given by

$$
\tau = -\left[\frac{\partial u(y,t)}{\partial y}\right]_{y=0} = -\frac{1}{2\sqrt{t}}\left[\frac{\partial u(y,t)}{\partial \eta}\right]_{\eta=0}
$$

$$
= \frac{1}{2\sqrt{t}}\left[f_1\left(4\sqrt{Nt}erf(\sqrt{Nt}) + \frac{4}{\sqrt{\pi}}e^{-Nt}\right) + f_2\left(4\sqrt{Abt}erf(\sqrt{Abt}) + \frac{4}{\sqrt{\pi}}e^{-Abt}\right)\right.
$$

$$
+ f_3\left(4\sqrt{S_cbt}erf(\sqrt{S_cbt}) + \frac{4}{\sqrt{\pi}}e^{-S_ct}\right) - f_2\left(4\sqrt{Abt}erf(\sqrt{bt}) + \frac{4}{\sqrt{\pi}}\sqrt{A}e^{-bt}\right)
$$

$$
\left. - f_3\left(4\sqrt{S_cbt}erf(\sqrt{bt}) + \frac{4}{\sqrt{\pi}}\sqrt{S_c}e^{-bt}\right) + f_4\left(2\sqrt{\frac{A}{\pi}}\right) + f_5\left(2\sqrt{\frac{S_c}{\pi}}\right)\right]. \qquad (3.11)
$$

The Nusselt number and the Sherwood number at the plate are respectively

$$
N_u = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \sqrt{\frac{A}{\pi t}},
$$

$$
\text{and} \quad S_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = \sqrt{\frac{S_c}{\pi t}}. \qquad (3.12)
$$

4. Results and Discussion

To discuss the physical significance of various parameters involved in the results (3.8) to (3.12), the numerical calculations have been carried out. Our results are found in agreement with the results of Ahmed and Batin [14] in the absence of the mass transfer parameter. The effects of the various parameters entering in the governing equations on the velocity, temperature, skin friction, Nusselt number and Sherwood number are shown through graphs.

Figure 1: In this figure, the effects of M, G_r and G_m on the velocity are noticed, while other parameters are kept constant. The Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force. It is noticed that when thermal buoyancy force dominates over viscous force then velocity increases. This result agrees with the result of Ahmed and Batin [14]. Again Modified Grashof number Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. The fluid velocity increases due to increase in the species buoyancy force relative to viscous hydrodynamic force. Also it is observed that as the magnetic parameter M increases velocity decreases.

Figure 2: The velocity profiles for different values of P_r, t and K_r are plotted in this figure. It is found that the velocity increases with increasing values of K_r. This signifies that the increasing values of K_r reduce the drag force which assists the fluid considerably to move fast. Figure shows that the increasing values of Pr result in a decreasing velocity. Again for increasing values of time parameter t, velocity increases.

Figure 3: Here it is found that velocity decreases with increase in radiation parameter N_a. This is also in accordance with the previous result of Ahmed and Batin [14] (i.e. flow without mass transfer). Again the effect of Sc on velocity is same as N_a.

Figure 4: Here it is noticed that both of S_c and t have the same effect on the concentration. As these parameters increase, concentration increases.

Figure 5: This figure illustrates that the temperature rises for decreasing values of P_r. On the other hand the parameters t and K_r are directly proportional to the temperature.

Figure 6:It reveals that the effect of parameters N_a and S_c on skin friction show quite opposite to that on the velocity of the fluid. But parameter K_r has same effect on velocity and skin friction.

Figure 7: This figure shows that the skin friction increases as parameters G_r and Gm are increased. But parameter M has opposite effect on skin friction.

Figure 8:The Nusselt number is found to increase with the increasing value of the Prandtl number. The same trend is also noticed for the parameter N_a.

Figure 9: Interestingly this figure shows that the Schmidt number has same effect on Sherwood number as it shows for concentration i.e. the Sherwood number increases with the increasing values of S_c.
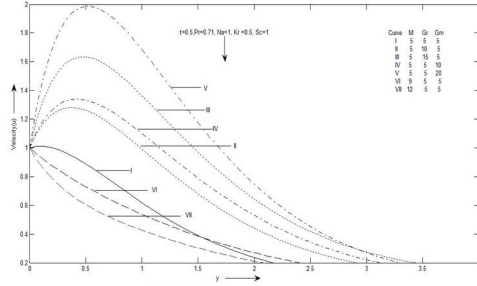
Figure 1. Velocity profiles for M, G_r and G_m.
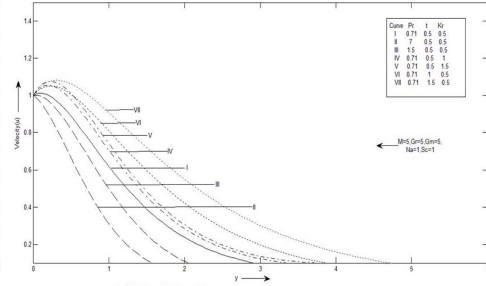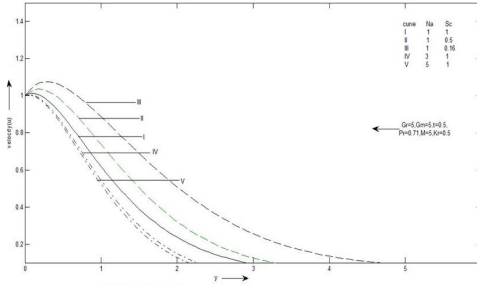
Figure 2. Velocity profiles for P_r, t and K_r.

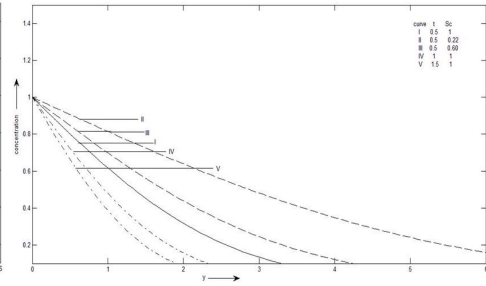**Figure 3.** Velocity profiles for N_a and S_c.

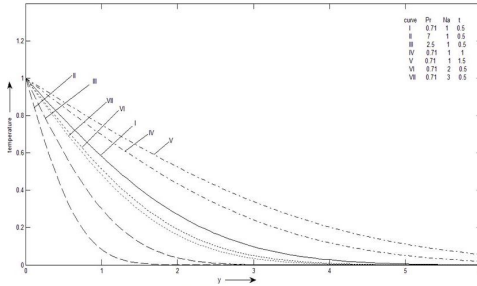**Figure 4.** Concentration profiles for S_c and t.

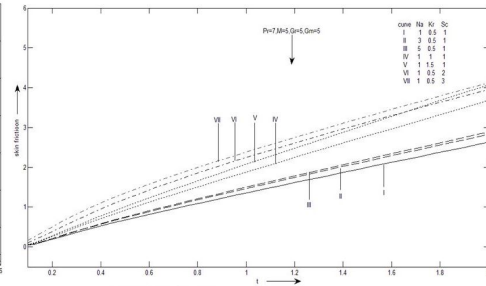**Figure 5.** Temperature profiles for P_r, N_a and t.

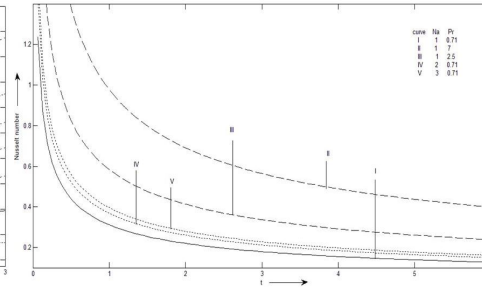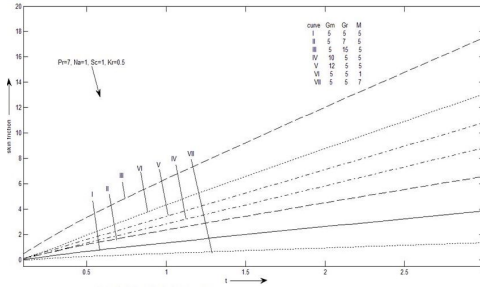**Figure 6.** Skin friction profiles for N_a, K_r and S_c.

Figure 7. Skin friction profiles for G_m, M and G_r. **Figure 8.** Nusselt number profiles for N_a and S_c.
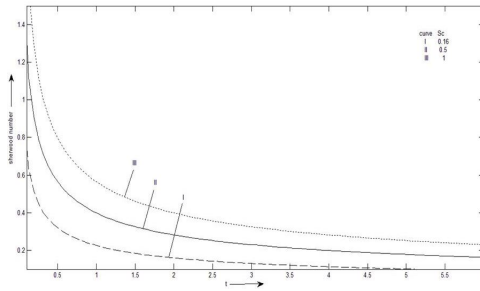

Figure 9. Sherwood number profiles for S_c.

5. Conclusion

In this paper, the problem of convective laminar radiating flow with mass transfer over an impulsively started vertical plate embedded in a porous medium in the presence of magnetic field was analyzed. The governing system of equations was solved analytically with the help of Laplace transform method. The effects of the parameters on the velocity, concentration, temperature skin friction, Sherwood number were studied in details.

Some important conclusions are summarized as follows.

- The flow is generally accelerated with the increase of porosity parameter (K_r).
- Velocity and temperature are decreased with increasing values of Prandtl number (P_r) and radiation parameter Na.
- Schmidt number and time parameter have same effects on concentration. When these parameters increase, concentration decreases.
- With increasing values of convection parameter G_r and modified Grashof number (G_m), the flow velocity is accelerated. But M shows opposite effects.
- Increasing radiation parameter and Schmidt number serves to depress shear stress.

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