Journal of Applied Analysis and Computation
Volume 5, Number 2, May 2015, 251–261
Website:http://jaac-online.com/
doi:10.11948/2015023

DESIGN AND CIRCUIT IMPLEMENTATION FOR A NOVEL CHARGE-CONTROLLED CHAOTIC MEMRISTOR SYSTEM*

Zengqiang Chen[1,3†], Hang Tang[1,2] , Zhonglin Wang[2]
Qing Zhang[3] and Jingwei Han[2]

Abstract This paper deals with the simulation and implementation of a new charge-controlled memristor based on the simplest chaotic circuit. The circuit we used has only three basic elements in series. Some period-one and period-doubling chaotic routes are generated by this circuit with changes in its component values. Device-level simulation is conducted by using Multisim to provide the basis for building the real chaotic circuit. The results of numerical simulations are identical to those of circuit simulations, demonstrating that the circuit is feasible.

Keywords Memristor, chaotic circuit, multisim, period-doubling.

MSC(2000) 93c15, 94c30

1. Introduction

In recent years, the memristor has attracted a wide attention in science fields. The concept of memristor was first postulated in 1971 by Leon O. Chua [5]. It is a new two-terminal circuit element which has every right to be as basic as the three classical circuit elements already in existence, namely, the resistor, inductor, and capacitor. However, for over thirty years, the memristor hadn't played any role in circuit theory, until 2008 Stan Williams and colleagues [18] fabricated a solid-state implementation of memristor in HP labs. Using a simple analytical example, that memristance arises naturally in nanoscale systems in which solid-state electronic and ionic transport are coupled under an external bias voltage.

Later, many other types of memristor models have also been introduced [1–3,6,9, 10,13]. But, those memristor-based circuits only display transient chaotic behaviors and their steady trajectories asymptotically tend to limit cycles or diverge [3, 10]. Moreover, some papers have suggested the implementation of a memristor in which a practical implementation of a memtistor-based chaotic circuit is provided, where the memristor characterized by a continuous cubic nonlinearity was realized by using off-the-shelf components such as resistor, capacitor, operational, amplifier and analog multiplier [13]. At the same time, in order to demonstrate the complex dynamical

†The corresponding author. Email address:chenzq@nankai.edu.cn(Z. Chen)
1Department of Automation, Nankai University, 300071, Tianjin, China
2Department of Physics and Electronics, Binzhou University, 256600, Binzhou, China
3College of Science, Civil Aviation University of China, 300300, Tianjin, P. R. China

behaviors [8,15,17,21] and to design memristors based on oscillatory networks [10], memristor oscillators have been investigated as a new kind of nonlinear circuits, such as oscillatory memories and neuromorphic circuits [4,11,14,19,20].

Recently, several implementations based on chaotic memristor circuits have been presented. Ref [13] proposed an implementing flux-controlled memristor based on chaotic circuits and showed how to build a memristor based on chaotic circuits on a breadboard. Ref [9] designed a chaotic circuit for flux-controlled memristor and charge-controlled memristor using PSPICE, and the PSPICE simulation results verified the correctness of the theoretical analysis. Ref [12] presented a breadboard circuit for the implementation of this chaotic circuit, by employing only the four basic circuit elements: the resistor, capacitor, inductor and memristor.

This paper constructs a new charge-controlled memristor circuit which is different from Chua's circuit in its products terms presenting the nonlinearity. Although the circuit has only three elements, chaos can still be generated [16]. And we have implemented this circuit by physical means with integrated chaotic circuit equipment in our lab. This approach is useful for further study of memristor-based oscillators [7].

This paper is organized as follows: we begin the memristor model by discussing the theoretical knowledge and then introduce a new circuit structure for memristor. Next, we show the equilibrium and perform stability analysis, and display several plots of waveforms from the physical circuit and the Lyapunov spectrum which illustrate the period-one route to chaos. At last, we demonstrate the circuit implementation and experimental results.

2. Memristor model and circuit structure

Figure 1 shows the HP memristor that is a passive two-terminal electronic device, described by a nonlinear relation:

$$v = M(q)i, \quad i = W(\varphi)v. \tag{2.1}$$

Here, the functions $M(q)$ and $W(\varphi)$ mean the memristance and memductance, v is the device terminal voltage and i is the terminal current:

$$M(q) = \frac{d\varphi(q)}{dq} \tag{2.2}$$

and

$$W(\varphi) = \frac{dq(\varphi)}{d\varphi}. \tag{2.3}$$

The slope of a scalar function $\varphi = \varphi(q)$ and $q = q(\varphi)$ means the memristor constitutive relation. A memristor characterized by a differentiable $q - \varphi$ (resp. $\varphi - q$) characteristic curve is passive, if and only if its small-signal memristance $M(q)$ (resp. small-signal memductance $W(\varphi)$) is non-negative; i.e.

$$W(q) = \frac{d\varphi(q)}{dq} \geq 0, \quad W(\varphi) = \frac{dq(\varphi)}{d\varphi} \geq 0. \tag{2.4}$$

In other papers [1,3], the memristor is considered as a two-terminal element in which the magnetic flux (φ) between the terminals is a function of the electric

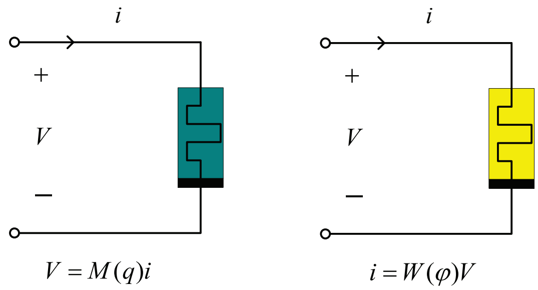


Figure 1. Charge-controlled memristor (left) and flux-controlled memristor (right).

charge (q) that passes through the device [5]. A flux-controlled memristor which is characterized by its incremental memductance function $W(\varphi)$ is frequently used [18]. And function $W(\varphi)$ describes the flux-dependent rate of change of charge.

More specifically, the memristor used in this work is a charge-controlled memristor, thereby possessing a simple structure and a practicability for the memristor applications. And that is characterized by its incremental memristance [18] function $M(q)$, which describes the charge-dependent rate of charge [16]:

$$\begin{cases} V_M = \beta(r^2 - 1)i_M, \\ \dot{r} = i_M - \alpha r - i_M r. \end{cases} \tag{2.5}$$

Here, V_M is voltage across the ends of memristor and i_M indicates the corresponding current through it, and r is an interal variable of this memristive model [19]. The simplest chaotic circuit which only contains three elements in series, i.e., the inductor, capacitor and a memristor, is shown in Figure 2.

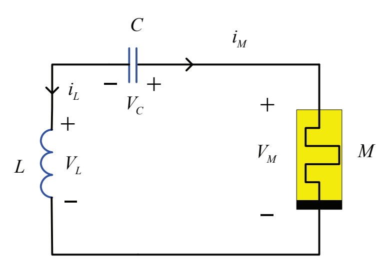


Figure 2. The circuit of memristor.

From the constitutive relation of a linear capacitor [11], we get the following equation from Figure 2:

$$C\frac{dV_C}{dt} = i_L. \tag{2.6}$$

Applying Kirchhoffs Voltage Law in the loop in Figure 2 and simplifying the constitutive relations of the inductor, capacitor and memristor, we get:

$$V_L + V_C = V_M. \tag{2.7}$$

For this memristor, $R(M) \triangleq \beta(r^2 - 1)$. So, we define the differential equation governing the internal state of our memristor as:

$$\frac{dr}{dt} = i_M - \alpha r - i_M r. \tag{2.8}$$

And we know that:

$$V_M = R(M)i_M; \quad V_L = L\frac{di_L}{dt}; i_M = -i_L. \tag{2.9}$$

Substituting V_M, V_L, from equation (2.7) and simplifying the equation above, we get:

$$\frac{di_L}{dt} = -\frac{1}{L}\left[\beta(r^2-1)i_M + V_C\right]. \tag{2.10}$$

Thus, simplifying equation (2.6), (2.8), and (2.10), we get:

$$\begin{cases} \dfrac{dV_C}{dt} = \dfrac{1}{C}i_L, \\ \dfrac{di_L}{dt} = -\dfrac{1}{L}\left[\beta(r^2-1)i_L + V_C\right], \\ \dfrac{dr}{dt} = (r-1)i_L - \alpha r. \end{cases} \tag{2.11}$$

Let $V_C = x, i_L = y, r = z, 1/C = a$ and $1/L = b$. Thus, equation (2.11) can be transformed into a dimensionless form:

$$\begin{cases} \dot{x} = ay, \\ \dot{y} = -b\left[\beta(z^2-1)y + x\right], \\ \dot{z} = (z-1)y - \alpha z. \end{cases} \tag{2.12}$$

3. Equilibrium and stability analysis

Consider the new chaotic memristor circuit which takes the form of equation (2.12). The divergence of the vector field is:

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = y - \alpha. \tag{3.1}$$

If $y < \alpha$, the divergence is negative.

Here the paramemters a, b, α and β are positive. By solving equation (2.12) we get the only equilibrium point $O = [\beta, 0, 0]$ which represents many infinitely equilibrium points and they fill the entire x axis, so it is an equilibrium point set.

Moreover, by considering the Jacobian matrix for one of these equilibriums and calculating their eigenvalues, we can investigate the stability of each equilibrium based on the roots of the characteristic equation. The corresponding Jacobian matrix is

$$J = \begin{bmatrix} 0 & a & 0 \\ -b & 0 & -2b\beta yz \\ 0 & -1 & y-\alpha \end{bmatrix}. \tag{3.2}$$

Its eigenvalues given by the roots of the characteristic equation

$$\lambda^3 - C_1\lambda^2 + C_2\lambda - C_3 = 0, \tag{3.3}$$

where

$$\begin{cases} C_1 = y - \alpha, \\ C_2 = 2b\beta yz + ab, \\ C_3 = ab(y-\alpha). \end{cases} \tag{3.4}$$

According to the Routh-Hurwitz theorem of stability criterion, in order to stabilize an equilibrium point, the necessary and sufficient conditions are:

$$\begin{cases} \Delta_1 = C_1 > 0, \\ \Delta_2 = C_1 C_2 - C_0 C_3 > 0. \end{cases} \tag{3.5}$$

In equation (3.3), $C_0 = 1$. Thus, we get the stable conditions for this system:

$$\beta < 0. \tag{3.6}$$

Apparently, when $\beta > 0$, this system is unstable and chaos will be generated. These simulation results are combined into Figures 3-6 (shown in the next section).

4. Bifurcation analysis and Lyapunov spectrum

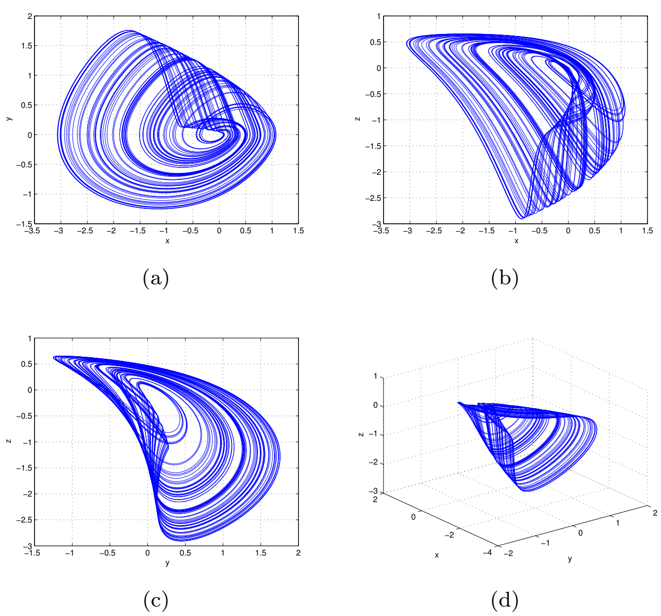


Figure 3. The plots of chaotic attractor of charge-controlled memristor based on the simplest chaotic circuit. By calculating the equation (2.12), the phase diagram is obtained from each plane above: (a) Chaotic trajectory ($i_L(t)$ versus $V_C(t)$); (b) Chaotic trajectory (r versus $V_C(t)$); (c) Chaotic trajectory (r versus $i_L(t)$); (d) The three-dimensional chaotic trajectory (r versus $i_L(t)$ and $V_C(t)$). The scales along the axes are: 0.5V/division on each axis in (a), (b), (c) and 1V/division on each axis in (d). $\beta = 1.5$. With initial conditions $(V_C(t), i_L(t), r) = (0.1, 0, 0.1)$, circuit parameters: $C = 1, L = 3.3, \alpha = 0.6$.

In this paper, we take into account the following typical values for the simplest chaotic circuit: $a = 1, b = 1/3.3, \beta = 1.5, \alpha = 0.6$. This leads to $C = 1F, L = 3.3H$. Moreover, the initial conditions are $(x(0), y(0), z(0)) = (0.1, 0, 0.1)$, the rescaled system in equation (2.12) is simulated with MATLAB. The circuit exhibits a chaotic attractor shown in Figure 3. It is easy to understand why we resort to the more general memristive system. From basic circuit theory, it is not possible to have a single-loop circuit with three independent state variables if we use the ideal charge-controlled memristor. This can be easily seen if we recall the definition of a charge-controlled memristor [5] as:

$$\begin{cases} V_M = M(q)i_M, \\ \dot{q} = i_M. \end{cases} \tag{4.1}$$

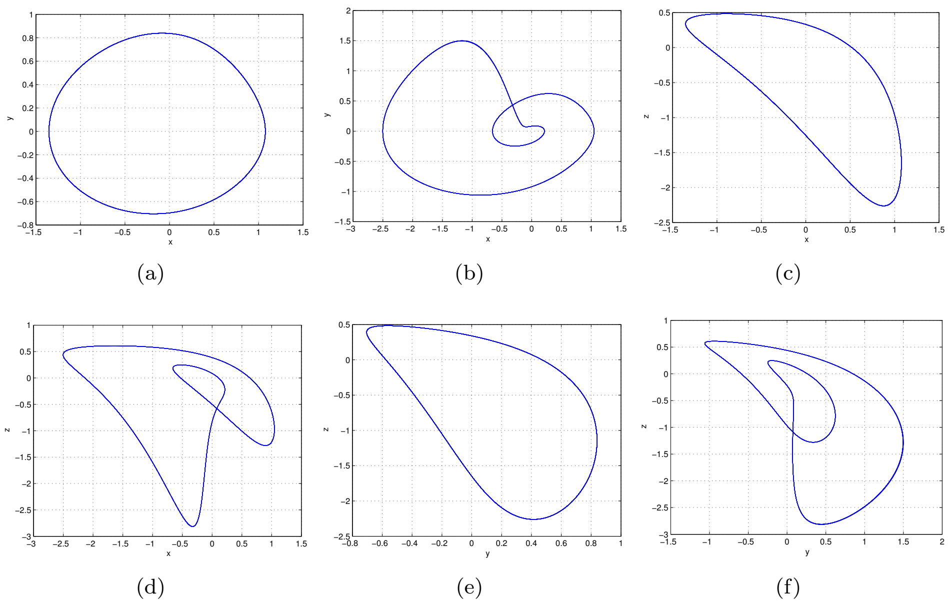


Figure 4. The plots of period-one route and period-doubling route. Figures (a) and (b) are phase plots of ($i_L(t)$ versus $V_C(t)$); Figures (c) and (d) are plots of (r versus $V_C(t)$); Figures (e) and (f) are plots of (r versus $i_L(t)$); The axes scale are: 0.5V/division on each axis. When $\beta = 0.3$ the phase diagrams is in the left and when $\beta = 1.2$ its in the right. The initial conditions and circuit parameters are consistent with Figure 3.

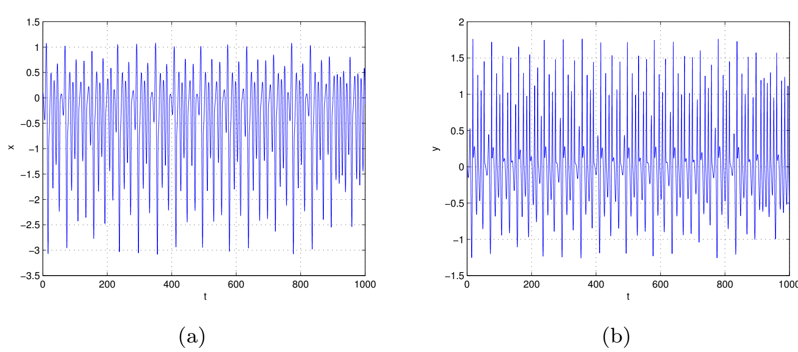
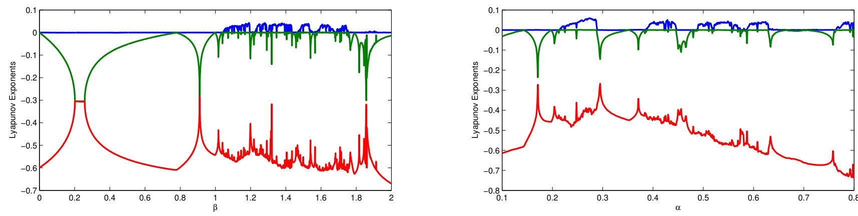


Figure 5. Time domain waveforms of the variable x and y. (a) The vertical coordinate is $V_C(t)$ and the horizontal coordinate is time (s); (b) The vertical coordinate is $i_L(t)$ and the horizontal coordinate is time (s). The scales along the axes are: 0.5V/division on vertical axis and $100s$ on horizontal axis. The initial conditions and circuit parameters are consistent with Figure 3.



(a) (b)

Figure 6. The plots of the Lyapunov exponents spectrum when the parameter β and α changed. (a) The vertical coordinate is exponents and the horizontal coordinate is parameter β. The first Lyapunov exponent is marked in blue, the second Lyapunov exponent is marked in green, the third is marked in red. (b) The vertical coordinate is exponents and the horizontal coordinate is parameter α. The first Lyapunov exponent is marked in blue, the second Lyapunov exponent is marked in green, the third is marked in red. Hence, The scales along the axes are: (a) 0.1V/division on vertical axis and 0.2 on horizontal axis; (b) 0.1V/division on vertical axis and 0.1 on horizontal axis. The initial conditions and circuit parameters are consistents as in Figure 3.

Figure 4 shows the results from the physical circuit. We have plotted state variables x, y, z (here, $x = V_C$ is the voltage across the capacitor, $y = i_L$ is the current through the inductor and $z = r$ is the internal variable of the charge-memristor) which illustrates period-one route and period-doubling route to chaos. The bifurcation parameter from equation (2.12) is β.

Figure 5 indicates the time-domain waveforms of x and y. Thus, Lyapunov exponents provide empirical evidence of chaotic behavior. They characterize the rate of separation of infinitesimally close trajectories in state space [8]. The rates of separation are different for different orientations of the initial separation vector, hence the number of Lyapunov exponents is equal to the number of dimensions in phase space. So, for a three-dimensional autonomous continuous-time system, we have three Lyapunov exponents. Notice that one of the exponents is positive and the sum of the exponents is negative indicating the presence of chaos [14]. Figure 6 shows the Lyapunov spectrum via β and α changed.

We have obtained that Lyapunov exponents are 0.029, 0 and -0.47, and the Kaplan-Yorke dimension is 2.0671. Fortunately, this system is immune to most of the difficulties encountered with the chaotic systems except that both the maximum Lyapunov exponent and the maximum Kaplan-Yorke dimension apparently occur just before the orbits become unbounded, although such orbits occur without very long duration transients and are easy to detect relatively.

5. Circuit implementation and experimental results

The schematic of three-element memristor-based chaotic circuit which is used to realize equation (2.12) is displayed in Figure 7. The schematic consists of nine Op-Amps $LF347BN$, two analog multiplier $AD633$, nineteen resistors and three capacitors. Moreover, we set $R1 = R16 = 100K\Omega, R8 = 330K\Omega, R9 = 200K\Omega, R10 = 5K\Omega, R15 = 160K\Omega$, and the other resistors are $10K\Omega$ and $C_1 = C_2 = C_3 = 5nF$. In the physical circuit, we used two potentiometers in series for $R9(300K)$ and $R15(200K)$. The circuit values corresponding to the memductance parameters α and β are shown in Figure 1, and the formulaes relating α and β to the circuit parameters are:

$$\begin{cases} \alpha = 10\dfrac{R18}{R15}, \\ \beta = \dfrac{3}{10} \times \dfrac{R11}{R9}. \end{cases} \tag{5.1}$$

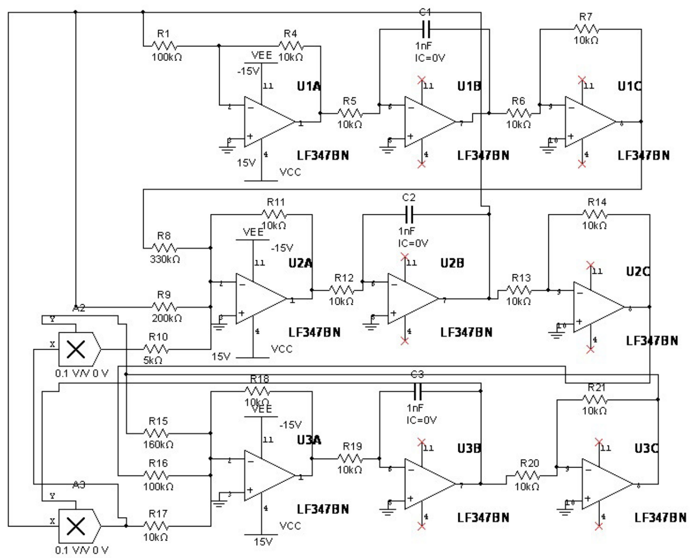


Figure 7. The schematic of the memristor based on the simplest chaotic circuit.

The power is supplied by the ICs which are $VCC = +15V$ and $VEE = -15V$. The chaotic attractor obtained from the physical implementation is also plotted.

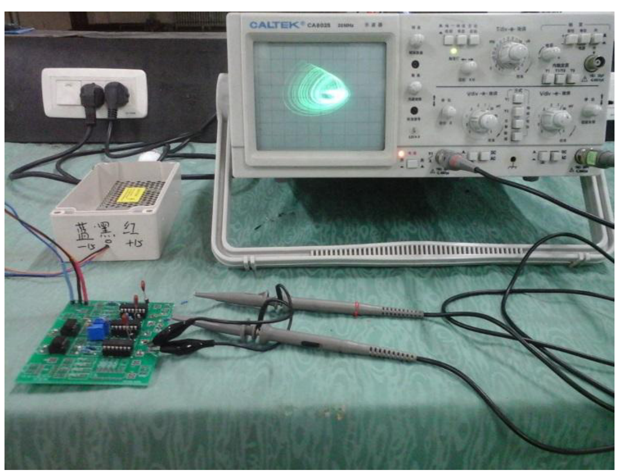
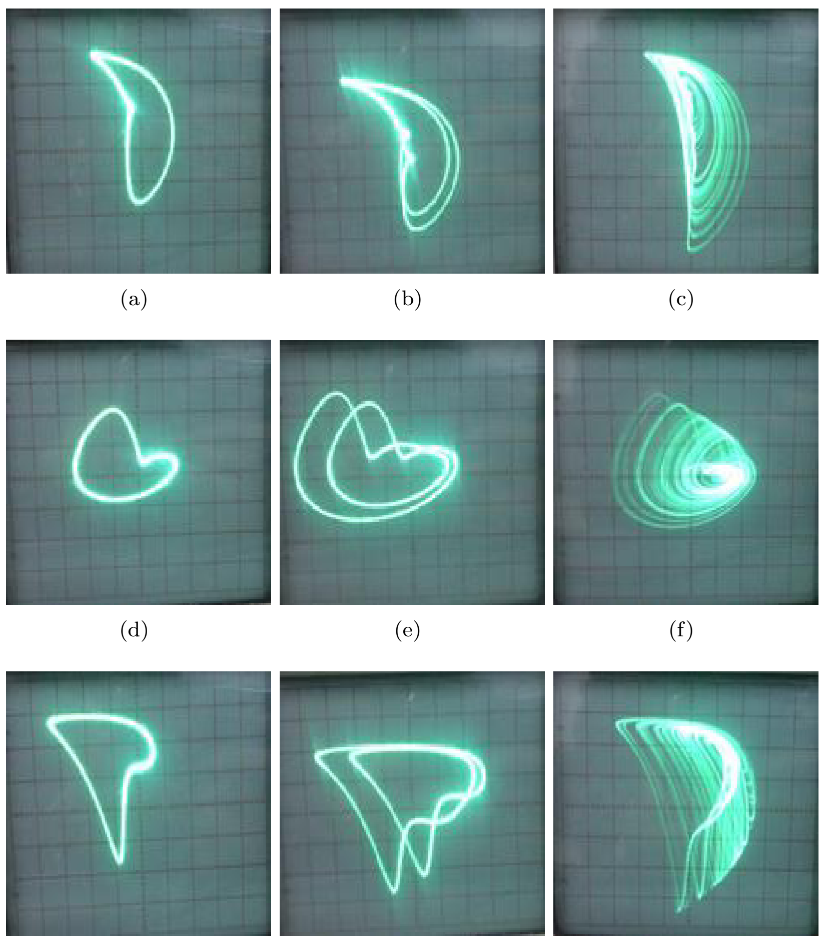


Figure 8. Integrated chaotic circuit equipment in our lab.

Figure 8 indicates the integrated circuit. Figure 9 shows the results observed on an analog oscilloscope which illustrates the process of period-one route to chaos, when we increase the variable β values. It's a great consistence with the theoretical result. While the corresponding circuit simulation with MATLAB simulation software has a significant difference, mainly due to the simulator integration drift.

There are two debugging hints about the physical circuit. The first one is after soldering circuit boards, one should check if limit-cycle behavior occurs when applying power and regulate potentionmeters. If one instead observes DC signals on all variables, it means a wiring error. Another one is that one should connect a resistor in the output terminal to offset the current distortion.



(a) (b) (c)
(d) (e) (f)
(g) (h) (i)

Figure 9. The phase diagram observed in the Oscilloscope corresponding to simulation in Multisim. Figures (a), (b) and (c) are (r versus $V_C(t)$); Figures (d), (e) and (f) are ($i_L(t)$ versus $V_C(t)$); Figures (g), (h) and (i) are (r versus $i_L(t)$); Figures (g) and (h) show some distortion when the current values is lower. The scales along the axis are: 500mV/division on vertical axis and 1V/division on horizontal axis in the whole figures.

6. Conclusion and Future work

In this paper, we showed how to build a simplest chaotic circuit on the integrated circuit board. The novelty of this paper is the physical circuit realization of the memristor circuit via operational amplifiers and analog multipliers. And we illustrated a series of characteristics about it. Furthermore, we can foresee a lot of opportunities based on these results (in the applications of biology, medicine, cytology, information electronics), which show rich chaotic dynamical behavior expanding the HP memristor in chaotic circuits.

7. Acknowledgement

This work is partially supported by the Natural Science Foundation of China Grants No. 61174094, Tianjin Nature Science Foundation Grant No. 14JCYBJC18700, and Shandong Provincial Natural Science Foundation Grant No. ZR2012FM034.

References

[1] A. Buscarino and L. Fortuna, *A gallery of chaotic oscillators based on HP memristor*, Int. J. Bifur. Chaos, 23(2013),1-13.

[2] R. Barboza and L. O. Chua, *The four-element Chuas circuit*, Int. J. Bifur. Chaos, 18(2008), 943-955.

[3] B. Bao, Z. Liu and J. Xu, *Dynamical analysis of memristor chaotic oscillator*, Acta Physica sinica 59(2010), 3785-3793. (in Chinese)

[4] A. Buscarino, L. Fortuna, M. Frasca and L. Gambuzza, *A chaotic circuit based on Hewlett-Packard memristor*, Chaos, 22(2012), 1-9.

[5] L. O. Chua, *Memristor-The missing circuit element*, IEEE Trans. Circuit Theory, 18(1971), 507-519.

[6] L. O. Chua and S. M. Kang, *Memristive devices and systems*, Proceedings of the IEEE, 64(1976), 209-223.

[7] G. Chen, *Leon Chuas memristor*, Circuits and Systems Magazine, IEEE Second Quarter, 8(2008), 55.

[8] J. P. Eckman and D. Ruelle, *Ergodic theory of chaos and strange attractors*, Reviews of Modern Physics, 57(1985), 617-656.

[9] Q. Hong and Y. Zeng, *Design and simulation of chaotic circuit for flux-controlled memristor and charge-controlled memristor*, Acta Physica Sinica 62(2013), 1-7. (in Chinese)

[10] M. Itoh and L. O. Chua, *Memristor oscillators*, Int. J. Bifur. Chaos, 18(2008), 3183-3206.

[11] M. Itoh and L. O. Chua, *Duality of memristor circuits*, Int. J. Bifur. Chaos, 23(2012), 1-50.

[12] Y. Li and L. Zhao, *Implementation of a new memristor based chaotic system*, Fifth International Workshop on Chaos-Fractals Theories and Applications, 2012.

[13] B. Muthuswamy, *Implementing memristor based chaotic circuit*, Int. J. Bifur. Chaos, 20(2010), 1335-1350.

[14] B. Muthuswamy and P. P. Kokate, *Memristor based chaotic circuits*, IETE Technical Review, 26(2009), 417-429.

[15] T. Matsumoto, *A chaotic attractor from Chuas circuit*, IEEE Transactions on Circuits and Systems, 31(1984), 1055-1058.

[16] B. Muthuswamy and L. O. Chua, *Simplest chaotic circuit*, Int. J. Bifur. Chaos, 20(2010), 1567-1580.

[17] I. Petras, *A note on the fractional-order Chuas system*, Chaos, Solitons and Fractals, 38(2008), 140-147.

[18] D. B. Strukov, G. S. Snider, G. R. Stewart and R. S. Williams, *The missing memristor found*, Nature 453(2008), 80-83.

[19] Y. Zhang and L. Sun, *The simulations of chaotic based on memritor*, Journal of EEE, 35(2013), 54-57.

[20] Y. Zhang, X. Zhang and J. Yu, *Approximate SPICE model for memristor*, International Conference on Communications Circuits and Systems, 2009.

[21] G. Zhong, *Implementation of Chuas circuit with a cubic nonlinearity*, IEEE Transactions on Circuits and Systems, 41(1994), 934-941.