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EXISTENCE RESULTS FOR A NEW CLASS OF FRACTIONAL IMPULSIVE PARTIAL NEUTRAL STOCHASTIC INTEGRO-DIFFERENTIAL EQUATIONS WITH INFINITE DELAY*

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Abstract In this paper, a new class of fractional impulsive partial neutral stochastic integro-differential equations with infinite delay is introduced. Under some dissipative conditions, we obtain the existence, uniqueness and continuous dependence of mild solutions for these equations. An application involving a fractional stochastic parabolic system with not instantaneous impulses is considered.

Keywords Fractional neutral integro-differential equations, impulsive partial neutral stochastic integro-differential equations, infinite delay, solution operator, fixed point.

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1. Introduction

Fractional differential equations have attracted the attention of many researchers in the last decades, because of their applications in numerous fields of science, engineering, physics, economy and so on (see [20,26,28]). In particular, the existence of solutions for fractional semilinear differential or integro-differential equations is one of the theoretical fields that investigated by many authors [11,12,14,35]. On the other hand, the deterministic models often fluctuate due to noise, which is random or at least appears to be so. Therefore, we must move from deterministic problems to stochastic ones. In fact, stochastic partial differential equations arise naturally in the mathematical modeling of various phenomena in the natural and social sciences; see [33]. The existence, uniqueness, and qualitative analysis of solutions of stochastic differential equations have been considered in abstract spaces. For some of these applications, one can see [3,25,34] and the references therein. Furthermore, fractional stochastic partial differential equations involving the Riemann-Liouville fractional derivative or the Caputo fractional derivative have been paid more and more attentions (see, e.g. [8,15,29]).

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In addition, the theory of impulsive differential equations has been an object of interest because of its wide applications in physics, biology, engineering, medical fields, industry, and technology (see [21]). The reason for this applicability arises from the fact that impulsive differential problems are an appropriate model for describing process which at certain moments change their state rapidly and which cannot be described using the classical differential problems. Therefore, it seems interesting to study the fractional impulsive differential equations. For some recent work on the existence and uniqueness of mild solutions for these equations in an abstract space; see [2,4,9,32] and the references therein. However, besides impulsive effects, stochastic effects likewise exist in real systems. So, impulsive stochastic differential equations describing these dynamical systems subject to both impulse and stochastic changes have attracted considerable attention [1, 22, 36]. Among them, Sakthivel et al. [30] studied the existence of mild solutions for the fractional impulsive stochastic differential equation with infinite delay in Hilbert spaces. Also, the existence of solutions for fractional impulsive stochastic semilinear differential equations with nonlocal conditions has been discussed.

Recently, Hernández and O'Regan [18] introduced a new class of first order abstract impulsive differential equations for which the impulses are not instantaneous. In the model, the impulses start abruptly at the points t_i and their action continue on a finite time interval $[t_i, s_i]$. This situation as an impulsive action which starts abruptly and stays active on a finite time interval. Further, Pierri et al. [27] studied the existence of solutions for a class of first order semi-linear abstract impulsive differential equations with not instantaneous impulses by using the theory of analytic semigroup and fractional power of closed operators.

Motivated by the researches mentioned previously, we will study the following fractional impulsive partial neutral stochastic integro-differential equations with infinite delay of the form

$$dD(t, x_t) = \int_0^t \frac{(t-s)^{\alpha-2}}{\Gamma(\alpha-1)} AD(s, x_s)dsdt + \sigma(t, x_t)dt + f(t, x_t)dw(t), \quad t \in (s_i, t_{i+1}], i = 0, 1, \ldots, N, \tag{1.1}$$

$$x(t) = g_i(t, x_t), \; t \in (t_i, s_i], i = 1, \ldots, N, \tag{1.2}$$

$$x_0 = \varphi \in \mathcal{B}, \tag{1.3}$$

where the state $x(\cdot)$ takes values in a separable real Hilbert space H with inner product $\langle \cdot, \cdot \rangle_H$ and norm $\| \cdot \|_H$. $1 < \alpha < 2$, $A : D(A) \subset H \to H$ is a linear densely defined operator of sectorial type on H. Let K be another separable Hilbert space with inner product $\langle \cdot, \cdot \rangle_K$ and norm $\| \cdot \|_K$. Suppose $\{w(t) : t \geq 0\}$ is a given K-valued Wiener process with a covariance operator $Q > 0$ defined on a complete probability space $(\Omega, \mathcal{F}, P)$ equipped with a normal filtration $\{\mathcal{F}_t\}_{t\geq 0}$, which is generated by the Wiener process w. The time history $x_t : (-\infty, 0] \to H$ given by $x_t(\theta) = x(t + \theta)$ belongs to some abstract phase space $\mathcal{B}$ defined axiomatically; let $0 = t_0 = s_0 < t_1 \leq s_1 \leq t_2 < ... < t_{N-1} \leq s_N \leq t_N \leq t_{N+1} = b$, are prefixed numbers, and $f, \sigma, q, D(t, \varphi) = \varphi(0) + q(t, \varphi), \varphi \in \mathcal{B}$, $g_i (i = 1, ..., N)$, are given functions to be specified later. The initial data $\{\varphi(t) : -\infty < t \leq 0\}$ is an $\mathcal{F}_0$-adapted, $\mathcal{B}$-valued random variable independent of the Wiener process w with finite second moment.

We notice that the convolution integral in (1.1) is known as the Riemann-Liouville fractional integral (see [5–7,13] and the references therein). On this subject, the authors [6] established the existence of S-asymptotically ω-periodic solutions for fractional order functional integro-differential equations with infinite delay. dos Santos and Cuevas [13] considered the existence and uniqueness of an asymptotically almost automorphic mild solution to the abstract fractional partial integro-differential neutral equation with unbounded delay. To the best of our knowledge, there is no work reported on the existence , uniqueness and continuous dependence of mild solutions of fractional partial neutral stochastic integro-differential equations with infinite delay and not instantaneous impulses, which is expressed in the form (1.1)-(1.3). To close the gap in this paper, we study this interesting problem, which are natural generalizations of the concept of mild solution for impulsive evolution equations well known in the theory of infinite dimensional deterministic systems.

The rest of this paper is organized as follows. In Section 2, we introduce some notations and necessary preliminaries. In Section 3, we give the existence and uniqueness results of mild solutions for system (1.1)-(1.3). In Section 4, continuous dependence of mild solutions is discussed. In Section 5, an example is given to illustrate our results.

2. Preliminaries

Let H, K be two real separable Hilbert spaces and we denote by $\langle\cdot,\cdot\rangle_H, \langle\cdot,\cdot\rangle_K$ their inner products and by $\|\cdot\|_H, \|\cdot\|_K$ their vector norms, respectively. $L(K,H)$ be the space of bounded linear operators mapping K into H equipped with the usual norm $\|\cdot\|_H$ and $L(H)$ denotes the Hilbert space of bounded linear operators from H to H. Let $\{w(t) : t \geq 0\}$ denote an K-valued Wiener process defined on the probability space $(\Omega, \mathcal{F}, P)$ with covariance operator Q, that is $E\langle w(t), x\rangle_K \langle w(s), y\rangle_K = (t \wedge s)\langle Qx, y\rangle_K$, for all $x, y \in K$, where Q is a positive, self-adjoint, trace class operator on K. In particular, we denote $w(t)$ an K-valued Q-Wiener process with respect to $\{\mathcal{F}_t\}_{t\geq 0}$.

In order to define stochastic integrals with respect to the Q -Wiener process $w(t)$, we introduce the subspace $K_0 = Q^{1/2}(K)$ of K which is endowed with the inner product $\langle\tilde{u}, \tilde{v}\rangle_{K_0} = \langle Q^{-1/2}\tilde{u}, Q^{-1/2}\tilde{v}\rangle_K$ is a Hilbert space. We assume that there exists a complete orthonormal system $\{e_n\}_{n=1}^{\infty}$ in K, a bounded sequence of nonnegative real numbers $\{\lambda_n\}_{n=1}^{\infty}$ such that $Qe_n = \lambda_n e_n$, and a sequence β_n of independent Brownian motions such that

$$\langle w(t), e\rangle = \sum_{n=1}^{\infty} \sqrt{\lambda_n}\langle e_n, e\rangle \beta_n(t),\ e \in K, t \in [0, b],$$

and $\mathcal{F}_t = \mathcal{F}_t^w$, where $\mathcal{F}_t^w$ is the σ-algebra generated by $\{w(s) : 0 \leq s \leq t\}$. Let $L_2^0 = L_2(K_0, H)$ be the space of all Hilbert-Schmidt operators from K_0 to H with the norm $\| \psi \|_{L_2^0}^2 = \mathrm{Tr}((\psi Q^{1/2})(\psi Q^{1/2})^*)$ for any $\psi \in L_2^0$. Clearly for any bounded operators $\psi \in L(K,H)$ this norm reduces to $\| \psi \|_{L_2^0}^2 = \mathrm{Tr}(\psi Q \psi^*)$. Let $L^p(\mathcal{F}_b, H)$ be the Banach space of all $\mathcal{F}_b$-measurable pth power integrable random variables with values in the Hilbert space H. Let $C([0,b]; L^p(\mathcal{F}, H))$ be the Banach space of continuous maps from $[0,b]$ into $L^p(\mathcal{F}, H)$ satisfying the condition $\sup_{t\in[0,b]} E \| x(t) \|_H^p < \infty$.

We introduce the space $\mathcal{PC}(H)$ formed by all $\mathcal{F}_t$-adapted measurable, H-valued stochastic processes $\{x(t) : t \in [0,b]\}$ such that x is continuous at $t \neq t_i$, $x(t_i) =$

$x(t_i^-)$ and $x(t_i^+)$ exists for all $i = 1, ..., N$. In this paper, we always assume that $\mathcal{PC}(H)$ is endowed with the norm $\| x \|_{\mathcal{PC}} = (\sup_{0\le t\le b} E \| x(t) \|_H^p)^{\frac{1}{p}}$. Then $(\mathcal{PC}(H), \| \cdot \|_{\mathcal{PC}})$ is a Banach space. The notation $B_r(x, H)$ stands for the closed ball with center at x and radius $r > 0$ in H.

In this paper, we assume that the phase space $(\mathcal{B}, \| \cdot \|_{\mathcal{B}})$ is a seminormed linear space of $\mathcal{F}_0$-measurable functions mapping $(-\infty, 0]$ into H, and satisfying the following fundamental axioms due to Hale and Kato (see e.g., in [17]).

(A) If $x : (-\infty, \sigma + b] \to H$, $b > 0$, is such that $x|_{[\sigma,\sigma+b]} \in C([\sigma, \sigma + b], H)$ and $x_\sigma \in \mathcal{B}$, then for every $t \in [\sigma, \sigma + b]$ the following conditions hold:

 (i) x_t is in $\mathcal{B}$;

 (ii) $\| x(t) \|_H \le \tilde{H} \| x_t \|_{\mathcal{B}}$;

 (iii) $\| x_t \|_{\mathcal{B}} \le K(t - \sigma) \sup\{\| x(s) \|_H : \sigma \le s \le t\} + M(t - \sigma) \| x_\sigma \|_{\mathcal{B}}$, where $\tilde{H} \ge 0$ is a constant; $K, M : [0, \infty) \to [1, \infty)$, K is continuous and M is locally bounded, and $\tilde{H}, K, M$ are independent of $x(\cdot)$.

(B) For the function $x(\cdot)$ in (A), the function $t \to x_t$ is continuous from $[\sigma, \sigma + b]$ into $\mathcal{B}$.

(C) The space $\mathcal{B}$ is complete.

A closed and linear operator A is said to be sectorial of type ω if there exist $0 < \theta < \pi/2, M > 0$ and $\omega \in \mathbb{R}$ such that its resolvent exists outside the sector $\omega + S_\theta := \{\omega + \lambda : \lambda \in \mathbb{C}, |\arg(-\lambda)| < \theta\}$ and $\| (\lambda - A)^{-1} \|_H \le \frac{M}{|\lambda-\omega|}, \lambda \notin \omega + S_\theta$. Sectorial operator are well studied in the literature. For a recent reference including several examples and properties we refer the reader to Haase [16]. In order to give an operator theoretical approach we recall the following definition (cf. [6, 7]).

Definition 2.1. Let A be a closed and linear operator with domain $D(A)$ defined on a Hilbert space H. We call A the generator of a solution operator if there exist $\omega \in \mathbb{R}$ and a strongly continuous function $S_\alpha : \mathbb{R}^+ \to L(H)$ such that $\{\lambda^\alpha : \operatorname{Re}(\lambda) > \omega\} \subset \rho(A)$ and $\lambda^{\alpha-1}(\lambda^\alpha - A)^{-1}x = \int_0^\infty e^{-\lambda t} S_\alpha(t)dt, \operatorname{Re}(\lambda) > \omega, x \in H$. In this case, $S_\alpha(\cdot)$ is called the solution operator generated by A.

We note that, if A is sectorial of type ω with $0 < \theta < \pi(1 - \frac{\alpha}{2})$ then A is the generator of a solution operator given by

$$S_\alpha(t) = \frac{1}{2\pi i} \int_\Sigma e^{-\lambda t} \lambda^{\alpha-1} (\lambda^\alpha - A)^{-1} d\lambda, \tag{2.1}$$

where Σ is a suitable path lying outside the sector $\omega + S_\alpha$.

Cuesta [5] has proved that, if A is a sectorial operator of type $\omega < 0$, for some $M > 0$ and $0 < \theta < \pi(1 - \frac{\alpha}{2})$, there is $C > 0$ such that

$$\| S_\alpha(t) \|_{L(H)} \le \frac{CM}{1 + |\omega| t^\alpha}, \ t \ge 0. \tag{2.2}$$

Remark 2.1. ([23]) We note that solution operators, as well as resolvent families, are a particular case of (a, k)-regularized families introduced in [23]. According to [23] a solution operator $S_\alpha(t)$ corresponds to a $(1, \frac{t^{\alpha-1}}{\Gamma(\alpha)})$-regularized family. As in the situation of C_0-semigroups we have diverse relations between a solution operator and its generator. Moreover, the following result is a direct consequence of [23] for Proposition 3.1 and Lemma 2.2.

Lemma 2.1. *Let $S_\alpha(t)$ be a solution operator on H with generator A. Then, we have*

(a) $S_\alpha(t)D(A) \subset D(A)$ *and* $AS_\alpha(t)x = S_\alpha(t)Ax$ *for all* $x \in D(A), t \geq 0$.

(b) Let $x \in D(A)$ *and* $t \geq 0$. *Then* $S_\alpha(t)x = x + \int_0^t \frac{(t-s)^\alpha}{\Gamma(\alpha)} AS_\alpha(s)xds$.

(c) Let $x \in H$ *and* $t \geq 0$. *Then* $\int_0^t \frac{(t-s)^\alpha}{\Gamma(\alpha)} S_\alpha(s)xds \in D(A)$ *and*

$$S_\alpha(t)x = x + A\int_0^t \frac{(t-s)^\alpha}{\Gamma(\alpha)} S_\alpha(s)xds.$$

Remark 2.2. ([6]) A characterization of generators of solution operators, analogous to the Hille-Yosida Theorem for C_0-semigroups, can be directly deduced from Theorem 3.4 in [23]. Results on perturbation, approximation, representation as well as ergodic type theorems can be also deduced from the more general context of (a, k) regularized resolvents (see [24, 31]).

Note that the Laplace transform of abstract functions $\sigma \in L^p(\mathbb{R}^+, H), f \in L^p(\mathbb{R}^+, L(K, H))$ are defined by

$$\hat{\sigma}(\rho) = \int_0^\infty e^{-\rho t}\sigma(t)dt,$$
$$\hat{f}(\rho) = \int_0^\infty e^{-\rho t} f(t)dw(t).$$

We consider the following problem

$$dx(t) = \int_0^t \frac{(t-s)^{\alpha-2}}{\Gamma(\alpha-1)} Ax(s)dsdt + \sigma(t)dt + f(t)dw(t), \quad t > 0, 1 < \alpha < 2, \tag{2.3}$$
$$x_0 = \varphi \in H. \tag{2.4}$$

Formally applying the Laplace transform in (2.3)-(2.4), we obtain

$$\lambda\hat{x}(\rho) - \varphi = \lambda^{1-\alpha}A\hat{x}(\rho) + \hat{\sigma}(\lambda)d\lambda + \hat{f}(\lambda)dw(\lambda),$$

which establishes the following result

$$\lambda\hat{x}(\rho) = \lambda^{\alpha-1}R(\lambda^\alpha, A)\varphi + \lambda^{\alpha-1}R(\lambda^\alpha, A)\hat{\sigma}(\lambda)d\lambda + \lambda^{\alpha-1}R(\lambda^\alpha, A)\hat{f}(\lambda)dw(\lambda).$$

This means that

$$x(t) = S_\alpha(t)\varphi + \int_0^t S_\alpha(t-s)\sigma(s)ds + \int_0^t S_\alpha(t-s)f(s)dw(s).$$

Motivated by the above discuss, we give the following definition.

Definition 2.2. An $\mathcal{F}_t$-adapted stochastic process $x : (-\infty, b] \to H$ is called a mild solution of the system (1.1)-(1.3) if $x_0 = \varphi \in \mathcal{B}$ on $(-\infty, 0]$, $x|_{[0,b]} \in \mathcal{PC}([0, b], H)$; and

(i) $x(t)$ is measurable and adapted to $\mathcal{F}_t, t \geq 0$.

(ii) $x(t) \in H$ has càdlàg paths on $t \in [0,b]$ a.s and for each $t \in [0,b]$, $x(t)$ satisfies $x(t) = g_j(t, x(t))$ for all $t \in (t_j, s_j], j = 1, ..., N$, and

$$\begin{aligned} x(t) &= S_\alpha(t)[\varphi(0) - q(0,\varphi)] + q(t, x_t) \\ &\quad + \int_0^t S_\alpha(t-s)\sigma(s, x_s)ds + \int_0^t S_\alpha(t-s)f(s,x_s)dw(s), \end{aligned}$$

for all $t \in [0, t_1]$ and

$$\begin{aligned} x(t) &= S_\alpha(t-s_i)[g_i(s_i, x(s_i)) - q(s_i, x_{s_i})] + q(t, x_t) \\ &\quad + \int_{s_i}^t S_\alpha(t-s)\sigma(s, x_s)ds + \int_{s_i}^t S_\alpha(t-s)f(s,x_s)dw(s), \end{aligned}$$

for all $t \in (s_i, t_{i+1}]$, $i = 1, ..., N$.

The next result is a consequence of the phase space axioms.

Lemma 2.2. *Let $x : (-\infty, b] \to H$ be an $\mathcal{F}_t$-adapted measurable process such that the $\mathcal{F}_0$-adapted process $x_0 = \varphi(t) \in L_2^0(\Omega, \mathcal{B})$ and $x|_{[0,b]} \in \mathcal{PC}([0,b], H)$, then*

$$\| x_s \|_{\mathcal{B}} \leq M_b E \| \varphi \|_{\mathcal{B}} + K_b \sup_{0 \leq s \leq b} E \| x(s) \|_H,$$

where $K_b = \sup\{K(t) : 0 \leq t \leq b\}, M_b = \sup\{M(t) : 0 \leq t \leq b\}$.

Lemma 2.3. ([10]) *For any $p \geq 1$ and for arbitrary $L_2^0(K, H)$-valued predictable process $\phi(\cdot)$ such that*

$$\sup_{s \in [0,t]} E \left\| \int_0^s \phi(v)dw(v) \right\|_H^{2p} \leq (p(2p-1))^p \left(\int_0^t (E \| \phi(s) \|_{L_2^0}^{2p})^{1/p} ds \right)^p, \ t \in [0, \infty).$$

In the rest of this paper, we denote by $C_p = (p(p-1)/2)^{p/2}$. Further, we introduce the following assumptions to establish our results:

(H1) The function $q : [0,b] \times \mathcal{B} \to H$ is continuous and there exists a constant $L_q > 0$ such that

$$E \| q(t, \psi_1) - q(t, \psi_2) \|_H^p \leq L_q \| \psi_1 - \psi_2 \|_{\mathcal{B}}^p, \quad \psi_1, \psi_2 \in \mathcal{B}.$$

(H2) The function $\sigma : [0,b] \times \mathcal{B} \to H$ is continuous and there exists a constant $L_\sigma > 0$ such that

$$E \| \sigma(t, \psi_1) - \sigma(t, \psi_2) \|_H^p \leq L_\sigma \| \psi_1 - \psi_2 \|_{\mathcal{B}}^p, \quad \psi_1, \psi_2 \in \mathcal{B}.$$

(H3) The function $f : [0,b] \times \mathcal{B} \to L(K, H)$ is continuous and there exists a constant $L_f > 0$ such that

$$E \| f(t, \psi_1) - f(t, \psi_2) \|_H^p \leq L_f \| \psi_1 - \psi_2 \|_{\mathcal{B}}^p, \quad \psi_1, \psi_2 \in \mathcal{B}.$$

(H4) The functions $g_i : (t_i, s_i] \times \mathcal{B} \to H, i = 1, ..., N$, are continuous and there exist constants $\gamma_i > 0, i = 1, ..., N$, such that

$$E \| g_i(t, \psi_1) - g_i(t, \psi_2) \|_H^p \leq \gamma_i \| \psi_1 - \psi_2 \|_{\mathcal{B}}^p, \quad \psi_1, \psi_2 \in \mathcal{B}.$$

3. Existence and uniqueness of mild solution

Theorem 3.1. *Assume that A is sectorial of type $\omega < 0$. If the assumptions* (H1)-(H4) *are satisfied and*

$$\max_{1\le i\le N}\{2^{p-1}K_b^p[2^{p-1}(8^{p-1}(CM)^p+1)\gamma_i + 8^{p-1}(2^{p-1}(CM)^p+1)L_q + 3^{p-1}(CM)^p(b^pL_\sigma + C_p b^{p/2}L_f)]\} < 1. \tag{3.1}$$

Then the problem (1.1)-(1.3) *has a unique mild solution on* $[0,b]$, *and there exists a constant* $\widetilde{K} > 0$ *such that*

$$E \parallel x(t) \parallel_H^p \le \widetilde{K} \quad \text{for all } t\in[0,b].$$

Proof. Consider the space $\mathcal{Y} = \{x \in \mathcal{PC}(H) : x(0) = \varphi(0)\}$ endowed with the uniform convergence topology. We define the operator $\Psi : \mathcal{Y} \to \mathcal{Y}$ by

$$(\Psi x)(t) = \begin{cases} S_\alpha(t)[\varphi(0) - q(0,\varphi)] + q(t,\bar{x}_t) & \\ \quad + \int_0^t S_\alpha(t-s)\sigma(s,\bar{x}_s)ds & \\ \quad + \int_0^t S_\alpha(t-s)f(s,\bar{x}_s)dw(s), & t\in[0,t_1], i=0,\\ g_i(t,\bar{x}_t), & t\in(t_i,s_i], i\ge 1,\\ S_\alpha(t-s_i)[g_i(s_i,\bar{x}_{s_i}) - q(s_i,\bar{x}_{s_i})] + q(t,\bar{x}_t) & \\ \quad + \int_{s_i}^t S_\alpha(t-s)\sigma(s,\bar{x}_s)ds & \\ \quad + \int_{s_i}^t S_\alpha(t-s)f(s,\bar{x}_s)dw(s), & t\in(s_i,t_{i+1}], i\ge 1, \end{cases}$$

and $\bar{x} : (-\infty, 0] \to H$ is such that $\bar{x}_0 = \varphi$ and $\bar{x} = x$ on $[0,b]$. It is clear that Ψ is a well-defined operator from $\mathcal{Y}$ into $\mathcal{Y}$. We show that Ψ has a fixed point, which in turn is a mild solution of the problem (1.1)-(1.3).

For any $t\in[0,t_1]$, and $x^*, x^{**}\in\mathcal{Y}$. From (H1)-(H4) and Lemmas 2.2, 2.3, we have

$$\begin{aligned}
&E \parallel (\Psi x^*)(t) - (\Psi x^{**})(t) \parallel_H^p\\
\le& 3^{p-1}E \parallel q(t,\overline{x^*}_t) - q(t,\overline{x^{**}}_t) \parallel_H^p\\
&+ 3^{p-1}E\left\| \int_0^t S_\alpha(t-s)[\sigma(s,\overline{x^*}_s) - \sigma(s,\overline{x^{**}}_s)]ds \right\|_H^p\\
&+ 3^{p-1}E\left\| \int_0^t S_\alpha(t-s)[f(s,\overline{x^*}_s) - f(s,\overline{x^{**}}_s)]dw(s) \right\|_H^p\\
\le& 3^{p-1}L_q \parallel \overline{x^*}_t - \overline{x^{**}}_t \parallel_{\mathcal{B}}^p + 3^{p-1}(CM)^p t_1^{p-1}\int_0^t \left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p\\
&\times E \parallel \sigma(s,\overline{x^*}_s) - \sigma(s,\overline{x^{**}}_s) \parallel_H^p ds + 3^{p-1}C_p(CM)^p\left[\int_0^t \left[\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p\right.\right.\\
&\left.\left.\times E \parallel f(s,\overline{x^*}_s) - f(s,\overline{x^{**}}_s) \parallel_H^p\right]^{2/p} ds\right]^{p/2}\\
\le& 3^{p-1}L_q \parallel \overline{x^*}_t - \overline{x^{**}}_t \parallel_{\mathcal{B}}^p\\
&+ 3^{p-1}(CM)^p t_1^{p-1}\int_0^t \left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p L_\sigma \parallel \overline{x^*}_s - \overline{x^{**}}_s \parallel_{\mathcal{B}}^p ds\\
&+ 3^{p-1}C_p(CM)^p t_1^{p/2-1}\int_0^t \left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p L_f \parallel \overline{x^*}_s - \overline{x^{**}}_s \parallel_{\mathcal{B}}^p ds
\end{aligned}$$

$$
\begin{aligned}
&\le 6^{p-1}K_b^p[L_q+(CM)^p(t_1^pL_\sigma+C_pt_1^{p/2}L_f)]\sup_{s\in[0,b]}E\parallel \overline{x^*}(s)-\overline{x^{**}}(s)\parallel_H^p\\
&= 6^{p-1}K_b^p[L_q+(CM)^p(t_1^pL_\sigma+C_pt_1^{p/2}L_f)]\sup_{s\in[0,b]}E\parallel x^*(s)-x^{**}(s)\parallel_H^p\\
&\quad \text{(since } \bar{x}=x \text{ on } [0,b])\\
&\le 6^{p-1}K_b^p[L_q+(CM)^p(t_1^pL_\sigma+C_pt_1^{p/2}L_f)]\parallel x^*-x^{**}\parallel_{\mathcal{PC}}^p .
\end{aligned}
$$

For any $t\in(t_i,s_i], i=1,\dots,N$, we have

$$
\begin{aligned}
&E\parallel(\Psi x^*)(t)-(\Psi x^{**})(t)\parallel_H^p\\
=&E\parallel q(t_i,\overline{x^*}_{t_i})-q(t_i,\overline{x^{**}}_{t_i})\parallel_H^p\\
\le&\gamma_i\parallel \overline{x^*}_{t_i}-\overline{x^{**}}_{t_i}\parallel_{\mathcal{B}}^p\\
\le&2^{p-1}\gamma_iK_b^p\sup_{s\in[0,b]}E\parallel \overline{x^*}(s)-\overline{x^{**}}(s)\parallel_H^p\\
=&2^{p-1}\gamma_iK_b^p\sup_{s\in[0,b]}E\parallel x^*(s)-x^{**}(s)\parallel_H^p \quad \text{(since } \bar{x}=x \text{ on } [0,b])\\
\le&2^{p-1}\gamma_iK_b^p\parallel x^*-x^{**}\parallel_{\mathcal{PC}}^p .
\end{aligned}
$$

Similarly, for any $t\in(s_i,t_{i+1}], i=1,\dots,N$, we have

$$
\begin{aligned}
&E\parallel(\Psi x^*)(t)-(\Psi x^{**})(t)\parallel_H^p\\
\le&3^{p-1}E\parallel S_\alpha(t-t_i)[g_i(s_i,\overline{x^*}_{s_i})-g_i(s_i,\overline{x^{**}}_{s_i})+q(s_i,\overline{x^*}_{s_i})-q(s_i,\overline{x^{**}}_{s_i})]\parallel_H^p\\
&+3^{p-1}E\parallel q(t,\overline{x^*}_t)-q(t,\overline{x^{**}}_t)\parallel_H^p\\
&+3^{p-1}E\left\|\int_{s_i}^t S_\alpha(t-s)[\sigma(s,\overline{x^*}_s)-\sigma(s,\overline{x^{**}}_s)]ds\right\|_H^p\\
&+3^{p-1}E\left\|\int_{s_i}^t S_\alpha(t-s)[f(s,\overline{x^*}_s)-f(s,\overline{x^{**}}_s)]dw(s)\right\|_H^p\\
\le&6^{p-1}(CM)^p[\gamma_i\parallel\overline{x^*}_{s_i}-\overline{x^{**}}_{s_i}\parallel_{\mathcal{B}}^p+L_q\parallel\overline{x^*}_{s_i}-\overline{x^{**}}_{s_i}\parallel_{\mathcal{B}}^p]\\
&+3^{p-1}L_q\parallel\overline{x^*}_t-\overline{x^{**}}_t\parallel_{\mathcal{B}}^p+3^{p-1}(CM)^p(t_{i+1}-s_i)^{p-1}\int_{s_i}^t\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p\\
&\times E\parallel\sigma(s,\overline{x^*}_s)-\sigma(s,\overline{x^{**}}_s)\parallel_H^p ds+3^{p-1}C_p(CM)^p\left[\int_{s_i}^t\left[\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p\right.\right.\\
&\left.\left.\times E\parallel f(s,\overline{x^*}_s)-f(s,\overline{x^{**}}_s)\parallel_H^p\right]^{2/p}ds\right]^{p/2}\\
\le&6^{p-1}(CM)^p[\gamma_i\parallel\overline{x^*}_{s_i}-\overline{x^{**}}_{s_i}\parallel_{\mathcal{B}}^p+L_q\parallel\overline{x^*}_{s_i}-\overline{x^{**}}_{s_i}\parallel_{\mathcal{B}}^p]\\
&+3^{p-1}L_q\parallel\overline{x^*}_t-\overline{x^{**}}_t\parallel_{\mathcal{B}}^p+3^{p-1}(CM)^p(t_{i+1}-s_i)^{p-1}\int_{s_i}^t\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p\\
&\times L_\sigma\parallel\overline{x^*}_s-\overline{x^{**}}_s\parallel_{\mathcal{B}}^p ds\\
&+3^{p-1}C_p(CM)^p(t_{i+1}-s_i)^{p/2-1}\int_{s_i}^t\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p\\
&\times L_f\parallel\overline{x^*}_s-\overline{x^{**}}_s\parallel_{\mathcal{B}}^p ds\\
\le&6^{p-1}K_b^p[L_q+2^{p-1}(CM)^p(\gamma_i+L_q)+(CM)^p((t_{i+1}-s_i)^pL_\sigma\\
&+C_p(t_{i+1}-s_i)^{p/2}L_f)]\sup_{s\in[0,b]}E\parallel\overline{x^*}(s)-\overline{x^{**}}(s)\parallel_H^p
\end{aligned}
$$

$$
\begin{aligned}
&=6^{p-1}K_b^p[L_q+2^{p-1}(CM)^p(\gamma_i+L_q)+(CM)^p((t_{i+1}-s_i)^pL_\sigma \\
&\quad +C_p(t_{i+1}-s_i)^{p/2}L_f)]\sup_{s\in[0,b]}E\parallel x^*(s)-x^{**}(s)\parallel_H^p \ \text{ (since } \bar{x}=x \text{ on } [0,b])\\
&\leq 6^{p-1}K_b^p[L_q+2^{p-1}(CM)^p(\gamma_i+L_q)+(CM)^p((t_{i+1}-s_i)^pL_\sigma \\
&\quad +C_p(t_{i+1}-s_i)^{p/2}L_f)]\parallel x^*-x^{**}\parallel_{\mathcal{PC}}^p .
\end{aligned}
$$

Thus, for all $t\in[0,b]$, we have

$$E\parallel(\Psi x^*)(t)-(\Psi x^{**})(t)\parallel_H^p\leq L_0\parallel x^*-x^{**}\parallel_{\mathcal{PC}}^p .$$

Taking supremum over t,

$$\parallel \Psi x^*-\Psi x^{**}\parallel_{\mathcal{PC}}^p\leq L_0\parallel x^*-x^{**}\parallel_{\mathcal{PC}}^p,$$

where

$$
\begin{aligned}
L_0=\max_{1\leq i\leq N}\{&2^{p-1}K_b^p[2^{p-1}(8^{p-1}(CM)^p+1)\gamma_i\\
&+8^{p-1}(2^{p-1}(CM)^p+1)L_q+3^{p-1}(CM)^p(b^pL_\sigma+C_pb^{p/2}L_f)]\}<1.
\end{aligned}
$$

Hence, Ψ is a contraction on $\mathcal{Y}$ and has a unique fixed point $x\in\mathcal{Y}$, which is obviously a unique mild solution of the system (1.1)-(1.3) on $[0,b]$. Then, we have

$$
x(t)=\begin{cases}
S_\alpha(t)[\varphi(0)-q(0,\varphi)]+q(t,\bar{x}_t) & \\
\quad +\int_0^t S_\alpha(t-s)\sigma(s,\bar{x}_s)ds & \\
\quad +\int_0^t S_\alpha(t-s)f(s,\bar{x}_s)dw(s), & t\in[0,t_1], i=0,\\
g_i(t,\bar{x}_t), & t\in(t_i,s_i], i\geq 1,\\
S_\alpha(t-s_i)[g_i(s_i,\bar{x}_{s_i})-q(s_i,\bar{x}_{s_i})]+q(t,\bar{x}_t) & \\
\quad +\int_{s_i}^t S_\alpha(t-s)\sigma(s,\bar{x}_s)ds & \\
\quad +\int_{s_i}^t S_\alpha(t-s)f(s,\bar{x}_s)dw(s), & t\in(s_i,t_{i+1}], i\geq 1.
\end{cases}
$$

By (H1)-(H4), we have for any $t\in[0,t_1]$,

$$
\begin{aligned}
&E\parallel x(t)\parallel_H^p\\
&\leq 4^{p-1}E\parallel S_\alpha(t)[\varphi(0)-q(0,\varphi)]\parallel_H^p+4^{p-1}E\parallel q(t,\bar{x}_t)\parallel_H^p\\
&\quad +4^{p-1}E\left\|\int_0^t S_\alpha(t-s)\sigma(s,\bar{x}_s)ds\right\|_H^p+4^{p-1}E\left\|\int_0^t S_\alpha(t-s)f(s,\bar{x}_s)dw(s)\right\|_H^p\\
&\leq 8^{p-1}(CM)^p[(\tilde{H}\parallel\varphi\parallel_{\mathcal{B}})^p+2^{p-1}(L_q\parallel\varphi\parallel_{\mathcal{B}}^p+a_q)]+8^{p-1}(L_q\parallel\bar{x}_t\parallel_{\mathcal{B}}^p+a_q)\\
&\quad +4^{p-1}(CM)^pt_1^{p-1}\int_0^t\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^pE\parallel\sigma(s,\bar{x}_s)\parallel_H^p ds\\
&\quad +4^{p-1}C_p(CM)^p\left[\int_0^t\left[\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^pE\parallel f(s,\bar{x}_s)\parallel_H^p\right]^{2/p}ds\right]^{p/2}\\
&\leq 8^{p-1}(CM)^p[(\tilde{H}\parallel\varphi\parallel_{\mathcal{B}})^p+2^{p-1}(L_q\parallel\varphi\parallel_{\mathcal{B}}^p+a_q)]\\
&\quad +8^{p-1}(L_q\parallel\bar{x}_t\parallel_{\mathcal{B}}^p+a_q)\\
&\quad +8^{p-1}(CM)^pt_1^{p-1}\int_0^t\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p(L_\sigma\parallel\bar{x}_s\parallel_{\mathcal{B}}^p+a_\sigma)ds\\
&\quad +8^{p-1}C_p(CM)^pt_1^{p/2-1}\int_0^t\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p(L_f\parallel\bar{x}_t\parallel_{\mathcal{B}}^p+a_f)ds,
\end{aligned}
$$

where $a_\sigma = \max_{t\in[0,b]} E \parallel \sigma(t,0) \parallel_H^p$, $a_f = \max_{t\in[0,b]} E \parallel f(t,0) \parallel_H^p$.

For any $t \in (t_i, s_i], i = 1, \ldots, N$, we have

$$E \parallel x(t) \parallel_H^p \leq 2^{p-1}(\gamma_i \parallel \bar{x}_t \parallel_{\mathcal{B}}^p + \nu_i),$$

where $\nu_i = \max_{t\in[0,b]} E \parallel g_i(t,0) \parallel_H^p, i = 1, ..., N$.

Similarly, for any $t \in (s_i, t_{i+1}], i = 1, \ldots, N$, we have

$$\begin{aligned}
&E \parallel x(t) \parallel_H^p \\
\leq & 4^{p-1} E \parallel S_\alpha(t-s_i)[g_i(s_i,\bar{x}_{s_i}) - q(s_i,\bar{x}_{s_i})] \parallel_H^p \\
&+ 4^{p-1} E \parallel q(t,\bar{x}_t) \parallel_H^p + 4^{p-1} E \left\| \int_{s_i}^t S_\alpha(t-s)\sigma(s,\bar{x}_s)ds \right\|_H^p \\
&+ 4^{p-1} E \left\| \int_{s_i}^t S_\alpha(t-s) f(s,\bar{x}_s)dw(s) \right\|_H^p \\
\leq & 16^{p-1}(CM)^p[\gamma_i \parallel \bar{x}_{s_i} \parallel_{\mathcal{B}}^p + \nu_i + L_q \parallel \bar{x}_{s_i} \parallel_{\mathcal{B}}^p + a_q] + 8^{p-1}(L_q \parallel \bar{x}_t \parallel_{\mathcal{B}}^p + a_q) \\
&+ 4^{p-1}(CM)^p(t_{i+1}-s_i)^{p-1} \int_{s_i}^t \left(\frac{1}{1+|\omega|(t-s)^\alpha} \right)^p E \parallel \sigma(s,\bar{x}_s) \parallel_H^p ds \\
&+ 4^{p-1} C_p (CM)^p \left[\int_{s_i}^t \left[\left(\frac{1}{1+|\omega|(t-s)^\alpha} \right)^p E \parallel f(s,\bar{x}_s) \parallel_H^p \right]^{2/p} ds \right]^{p/2} \\
\leq & 16^{p-1}(CM)^p[\gamma_i \parallel \bar{x}_{s_i} \parallel_{\mathcal{B}}^p + \nu_i + L_q \parallel \bar{x}_{s_i} \parallel_{\mathcal{B}}^p + a_q] + 8^{p-1}(L_q \parallel \bar{x}_t \parallel_{\mathcal{B}}^p + a_q) \\
&+ 8^{p-1}(CM)^p(t_{i+1}-s_i)^{p-1} \int_{s_i}^t \left(\frac{1}{1+|\omega|(t-s)^\alpha} \right)^p (L_\sigma \parallel \bar{x}_s \parallel_{\mathcal{B}}^p + a_\sigma) ds \\
&+ 8^{p-1} C_p (CM)^p (t_{i+1}-s_i)^{p/2-1} \int_{s_i}^t \left(\frac{1}{1+|\omega|(t-s)^\alpha} \right)^p \\
&\times (L_f \parallel \bar{x}_s \parallel_{\mathcal{B}}^p + a_f) ds.
\end{aligned}$$

Thus, for all $t \in [0,b]$, we have

$$\begin{aligned}
E \parallel x(t) \parallel_H^p \leq & \widetilde{M} + 16^{p-1}(CM)^p[\gamma_i \parallel \bar{x}_{s_i} \parallel_{\mathcal{B}}^p + L_q \parallel \bar{x}_{s_i} \parallel_{\mathcal{B}}^p] + 2^{p-1}\gamma_i \parallel \bar{x}_t \parallel_{\mathcal{B}}^p \\
&+ 8^{p-1} L_q \parallel \bar{x}_t \parallel_{\mathcal{B}}^p + 8^{p-1}(CM)^p b^{p-1} \int_0^t L_\sigma \parallel \bar{x}_s \parallel_{\mathcal{B}}^p ds \\
&+ 8^{p-1} C_p (CM)^p b^{p/2-1} \int_0^t L_f \parallel \bar{x}_s \parallel_{\mathcal{B}}^p ds,
\end{aligned}$$

where

$$\begin{aligned}
\widetilde{M} = & 8^{p-1}(CM)^p[(\tilde{H} \parallel \varphi \parallel_{\mathcal{B}})^p + 2^{p-1}(L_q \parallel \varphi \parallel_{\mathcal{B}}^p + a_q)] \\
&+ \max_{1\leq i\leq N} 2^{p-1}[8^{p-1}(CM)^p + 1]\nu_i + 8^{p-1}[2^{p-1}(CM)^p + 1]a_q \\
&+ 8^{p-1}(CM)^p b^p a_\sigma + 8^{p-1} C_p (CM)^p b^{p/2} a_f.
\end{aligned}$$

By Lemma 2.2, it follows that

$$\begin{aligned}
\sup\{\parallel \bar{x}_s \parallel_{\mathcal{B}}^p : 0 \leq s \leq t\} \leq & 2^{p-1}(M_b E \parallel \varphi \parallel_{\mathcal{B}})^p \\
&+ 2^{p-1} K_b^p \sup\{E \parallel x(s) \parallel_H^p : 0 \leq s \leq t\}.
\end{aligned}$$

Consider the function defined by

$$\zeta(t) = 2^{p-1}(M_b E \parallel \varphi \parallel_{\mathcal{B}})^p + 2^{p-1}K_b^p \sup\{E \parallel x(s) \parallel_H^p : 0 \le s \le t\},\ 0 \le t \le b.$$

For all $t \in [0, b]$, we have

$$\begin{aligned}\zeta(t) \le\ & 2^{p-1}(M_b E \parallel \varphi \parallel_{\mathcal{B}})^p + 2^{p-1}K_b^p \widetilde{M} + 32^{p-1}K_b^p(CM)^p[\gamma_i\zeta(t) + L_q\zeta(t)] \\ & + 4^{p-1}K_b^p\gamma_i\zeta(t) + 16^{p-1}K_b^p L_q\zeta(t) + 16^{p-1}K_b^p(CM)^p b^{p-1}L_\sigma \int_0^t \zeta(s)ds \\ & + 16^{p-1}K_b^p C_p(CM)^p b^{p/2-1}L_f \int_0^t \zeta(s)ds.\end{aligned}$$

Since $L_* = \max_{1\le i\le N}\{4^{p-1}K_b^p[(8^{p-1}(CM)^p+1)\gamma_i + 4^{p-1}(2^{p-1}(CM)^p+1)L_q]\} < 1$, we have

$$\zeta(t) \le \frac{1}{1-L_*}[2^{p-1}(M_b E \parallel \varphi \parallel_{\mathcal{B}})^p + 2^{p-1}K_b^p\widetilde{M}] + (\tilde{K}_1 + \tilde{K}_2)\int_0^t \zeta(s)ds,$$

where

$$\begin{aligned}\tilde{K}_1 &= 16^{p-1}\frac{1}{1-L_*}K_b^p(CM)^p b^{p-1}L_\sigma, \\ \tilde{K}_2 &= 16^{p-1}\frac{1}{1-L_*}K_b^p C_p(CM)^p b^{p/2-1}L_f.\end{aligned}$$

Applying Gronwall's inequality in the above expression, we obtain

$$\zeta(t) \le \frac{1}{1-L_*}[2^{p-1}(M_b E \parallel \varphi \parallel_{\mathcal{B}})^p + 2^{p-1}K_b^p\widetilde{M}]\exp\{(\tilde{K}_1 + \tilde{K}_2)b\} := \widetilde{K}.$$

Then for all $t \in [0, b]$, we get that $E \parallel x(t) \parallel_H^p \le \widetilde{K}$. This completes the proof. □

4. Continuous dependence of mild solutions

Theorem 4.1. *Assume that A is sectorial of type $\omega < 0$. If the assumptions* (H1)-(H4) *are satisfied and*

$$\max_{1\le i\le N}\{2^{p-1}K_b^p[(8^{p-1}CM)^p + 1)\gamma_i + 8^{p-1}((CM)^p + 1)L_q]\} < 1. \tag{4.1}$$

Then there exists a constant $C_* > 0$ such that for each $\varphi^1, \varphi^2 \in \mathcal{B}$ and $x^1(t), x^2(t)$ be the corresponding mild solutions of the problem (1.1)-(1.3) *with $x_0^1 = \varphi^1, x_0^2 = \varphi^1$ satisfy*

$$\parallel x^1 - x^2 \parallel_{\mathcal{PC}}^p \le C_* \parallel \varphi^1 - \varphi^2 \parallel_{\mathcal{B}}^p .$$

Proof. Let $\mathcal{Y}$ be defined as in the proof of Theorem 3.1. By Lemma 2.2, it follows that

$$\begin{aligned}&\sup\{\parallel \overline{x^1}_s - \overline{x^2}_s \parallel_{\mathcal{B}}^p : 0 \le s \le t\} \\ \le\ & 2^{p-1}M_b^p E \parallel \varphi^1 - \varphi^2 \parallel_{\mathcal{B}}^p + 2^{p-1}K_b^p \sup\{E \parallel x^1(s) - x^2(s) \parallel_H^p : 0 \le s \le t\}, t \in [0, b].\end{aligned}$$

For any $t \in [0, t_1]$, and $x^1, x^2 \in \mathcal{Y}$. Using (H1)-(H4), we have

$$E \parallel x^1(t) - x^2(t) \parallel_H^p$$

$$
\begin{aligned}
\leq & 4^{p-1}E \parallel S_\alpha(t)[\varphi^1(0)-\varphi^2(0)-q(0,\varphi^1)+q(0,\varphi^2)] \parallel_H^p \\
& + 4^{p-1}E \parallel q(t,\overline{x^1}_t)-q(t,\overline{x^2}_t) \parallel_H^p \\
& + 4^{p-1}E\left\| \int_0^t S_\alpha(t-s)[\sigma(s,\overline{x^1}_s)-\sigma(s,\overline{x^2}_s)]ds \right\|_H^p \\
& + 4^{p-1}E\left\| \int_0^t S_\alpha(t-s)[f(s,\overline{x^1}_s)-f(s,\overline{x^2}_s)]dw(s) \right\|_H^p \\
\leq & 8^{p-1}(CM)^p[E \parallel \varphi^1(0)-\varphi^2(0) \parallel_H^p + L_q \parallel \varphi^1-\varphi^2 \parallel_{\mathcal{B}}^p] + 4^{p-1}L_q \parallel \overline{x^1}_t-\overline{x^2}_t \parallel_{\mathcal{B}}^p \\
& + 4^{p-1}(CM)^p t_1^{p-1}\int_0^t \left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p E \parallel \sigma(s,\overline{x^1}_s)-\sigma(s,\overline{x^2}_s) \parallel_H^p ds \\
& + 4^{p-1}C_p(CM)^p\left[\int_0^t \left[\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p\right.\right. \\
& \left.\left. \times E \parallel f(s,\overline{x^1}_s)-f(s,\overline{x^2}_s) \parallel_H^p\right]^{2/p} ds\right]^{p/2} \\
\leq & 8^{p-1}(CM)^p(\tilde{H}^p+L_q) \parallel \varphi^1-\varphi^2 \parallel_{\mathcal{B}}^p + 4^{p-1}L_q \parallel \overline{x^1}_t-\overline{x^2}_t \parallel_{\mathcal{B}}^p \\
& + 4^{p-1}(CM)^p t_1^{p-1}\int_0^t \left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p L_\sigma \parallel \overline{x^1}_s-\overline{x^2}_s \parallel_{\mathcal{B}}^p ds \\
& + 4^{p-1}C_p(CM)^p t_1^{p/2-1}\int_0^t \left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p L_f \parallel \overline{x^1}_s-\overline{x^2}_s \parallel_{\mathcal{B}}^p ds \\
\leq & 8^{p-1}(CM)^p(\tilde{H}^p+L_q) \parallel \varphi^1-\varphi^2 \parallel_{\mathcal{B}}^p \\
& + 8^{p-1}L_q[M_b^p E \parallel \varphi^1-\varphi^2 \parallel_{\mathcal{B}}^p + K_b^p \sup_{s\in[0,t]} E \parallel x^1(s)-x^2(s) \parallel_H^p] \\
& + 8^{p-1}(CM)^p t_1^p L_\sigma M_b^p E \parallel \varphi^1-\varphi^2 \parallel_{\mathcal{B}}^p \\
& + 8^{p-1}K_b^p(CM)^p t_1^{p-1}L_\sigma \int_0^t \sup_{s\in[0,t]} E \parallel x^1(s)-x^2(s) \parallel_H^p ds \\
& + 8^{p-1}C_p(CM)^p t_1^{p/2}L_f M_b^p E \parallel \varphi^1-\varphi^2 \parallel_{\mathcal{B}}^p \\
& + 8^{p-1}K_b^p C_p(CM)^p t_1^{p/2-1}\int_0^t \sup_{s\in[0,t]} E \parallel x^1(s)-x^2(s) \parallel_H^p ds \\
= & 8^{p-1}[(CM)^p(\tilde{H}^p+L_q)+L_qM_b^p+(CM)^p t_1^p L_\sigma M_b^p \\
& + C_p(CM)^p t_1^{p/2}L_f M_b^p] \parallel \varphi^1-\varphi^2 \parallel_{\mathcal{B}}^p + 8^{p-1}L_qK_b^p \sup_{s\in[0,t]} E \parallel x^1(s)-x^2(s) \parallel_H^p \\
& + 8^{p-1}K_b^p(CM)^p[t_1^{p-1}L_\sigma + C_p t_1^{p/2-1}L_f]\int_0^t \sup_{s\in[0,t]} E \parallel x^1(s)-x^2(s) \parallel_H^p ds.
\end{aligned}
$$

For any $t\in(t_i,s_i], i=1,\ldots,N$, we have

$$
\begin{aligned}
& E \parallel x^1(t)-x^2(t) \parallel_H^p \\
= & E \parallel q(t_i,\overline{x^1}_{t_i})-q(t_i,\overline{x^2}_{t_i})] \parallel_H^p \leq \gamma_i \parallel \overline{x^1}_{t_i}-\overline{x^2}_{t_i} \parallel_{\mathcal{B}}^p \\
\leq & 2^{p-1}\gamma_i M_b^p E \parallel \varphi^1-\varphi^2 \parallel_{\mathcal{B}}^p + 2^{p-1}\gamma_i K_b^p \sup_{s\in[t_i,t]} E \parallel \overline{x^1}(s)-\overline{x^2}(s) \parallel_H^p .
\end{aligned}
$$

Similarly, for any $t\in(s_i,t_{i+1}], i=1,\ldots,N$, we have

$$E \parallel x^1(t)-x^2(t) \parallel_H^p$$

$$
\begin{aligned}
&\leq 4^{p-1}E \parallel S_\alpha(t-t_i)[g_i(s_i,\overline{x^1}_{s_i})-g_i(s_i,\overline{x^2}_{s_i})\\
&\quad + q(s_i,\overline{x^1}_{s_i})-q(s_i,\overline{x^2}_{s_i})] \parallel_H^p\\
&\quad + 4^{p-1}E \parallel q(t,\overline{x^1}_t)-q(t,\overline{x^2}_t) \parallel_H^p\\
&\quad + 4^{p-1}E\left\| \int_{s_i}^t S_\alpha(t-s)[\sigma(s,\overline{x^1}_s)-\sigma(s,\overline{x^2}_s)]ds\right\|_H^p\\
&\quad + 4^{p-1}E\left\| \int_{s_i}^t S_\alpha(t-s)[f(s,\overline{x^1}_s)-f(s,\overline{x^2}_s)]dw(s)\right\|_H^p\\
&\leq 8^{p-1}(CM)^p[\gamma_i \parallel \overline{x^1}_{s_i}-\overline{x^2}_{s_i} \parallel_{\mathcal{B}}^p + L_q \parallel \overline{x^1}_{s_i}-\overline{x^2}_{s_i} \parallel_{\mathcal{B}}^p]\\
&\quad + 4^{p-1}L_q \parallel \overline{x^1}_t-\overline{x^2}_t \parallel_{\mathcal{B}}^p\\
&\quad + 4^{p-1}(CM)^p(t_{i+1}-s_i)^{p-1}\int_{s_i}^t\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p\\
&\quad \times E \parallel \sigma(s,\overline{x^1}_s)-\sigma(s,\overline{x^2}_s) \parallel_H^p ds\\
&\quad + 4^{p-1}C_p(CM)^p\Bigg[\int_{s_i}^t\Bigg[\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p\\
&\quad \times E \parallel f(s,\overline{x^1}_s)-f(s,\overline{x^2}_s) \parallel_H^p\Bigg]^{2/p}ds\Bigg]^{p/2}\\
&\leq 8^{p-1}(CM)^p[\gamma_i \parallel \overline{x^1}_{s_i}-\overline{x^2}_{s_i} \parallel_{\mathcal{B}}^p + L_q \parallel \overline{x^1}_{s_i}-\overline{x^2}_{s_i} \parallel_{\mathcal{B}}^p]\\
&\quad + 4^{p-1}L_q \parallel \overline{x^1}_t-\overline{x^2}_t \parallel_{\mathcal{B}}^p\\
&\quad + 4^{p-1}(CM)^p(t_{i+1}-s_i)^{p-1}\int_{s_i}^t\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p\\
&\quad \times L_\sigma \parallel \overline{x^1}_s-\overline{x^2}_s \parallel_{\mathcal{B}}^p ds\\
&\quad + 4^{p-1}C_p(CM)^p(t_{i+1}-s_i)^{p/2-1}\int_{s_i}^t\left(\frac{1}{1+|\omega|(t-s)^\alpha}\right)^p\\
&\quad \times L_f \parallel \overline{x^1}_s-\overline{x^2}_s \parallel_{\mathcal{B}}^p ds\\
&\leq 16^{p-1}(CM)^p(\gamma_i+L_q)[M_b^pE \parallel \varphi^1-\varphi^2 \parallel_{\mathcal{B}}^p\\
&\quad + K_b^p \sup_{s\in[s_i,t]} E \parallel \overline{x^1}(s)-\overline{x^2}(s) \parallel_H^p]\\
&\quad + 8^{p-1}L_q[M_b^pE \parallel \varphi^1-\varphi^2 \parallel_{\mathcal{B}}^p + K_b^p \sup_{s\in[s_i,t]} E \parallel \overline{x^1}(s)-\overline{x^2}(s) \parallel_H^p]\\
&\quad + 8^{p-1}(CM)^p(t_{i+1}-s_i)^pL_\sigma M_b^pE \parallel \varphi^1-\varphi^2 \parallel_{\mathcal{B}}^p\\
&\quad + 8^{p-1}K_b^p(CM)^p(t_{i+1}-s_i)^{p-1}L_\sigma\int_{s_i}^t \sup_{s\in[s_i,t]} E \parallel x^1(s)-x^2(s) \parallel_H^p ds\\
&\quad + 8^{p-1}C_p(CM)^p(t_{i+1}-s_i)^{p/2}L_fM_b^pE \parallel \varphi^1-\varphi^2 \parallel_{\mathcal{B}}^p\\
&\quad + 8^{p-1}K_b^pC_p(CM)^p(t_{i+1}-s_i)^{p/2-1}L_f\int_{s_i}^t \sup_{s\in[s_i,t]} E \parallel x^1(s)-x^2(s) \parallel_H^p ds\\
&= 8^{p-1}M_b^p[2^{p-1}(CM)^p(\gamma_i+L_q)+L_q+(CM)^p(t_{i+1}-s_i)^pL_\sigma\\
&\quad + C_p(CM)^p(t_{i+1}-s_i)^{p/2}L_f] \parallel \varphi^1-\varphi^2 \parallel_{\mathcal{B}}^p\\
&\quad + 8^{p-1}K_b^p[2^{p-1}(CM)^p(\gamma_i+L_q)+L_q] \sup_{s\in[s_i,t]} E \parallel x^1(s)-x^2(s) \parallel_H^p
\end{aligned}
$$

$$
\begin{aligned}
&+8^{p-1}K_b^p(CM)^p[(t_{i+1}-s_i)^{p-1}L_\sigma + C_p(t_{i+1}-s_i)^{p/2-1}L_f] \\
&\times \int_{s_i}^t \sup_{s\in[s_i,t]} E \parallel x^1(s)-x^2(s)\parallel_H^p ds.
\end{aligned}
$$

Thus, for all $t \in [0,b]$, we have

$$
\begin{aligned}
&E \parallel x^1(t)-x^2(t)\parallel_H^p \\
\le& \max_{1\le i\le N}\{8^{p-1}(CM)^p(\tilde{H}^p+L_q)+2^{p-1}\gamma_i M_b^p(1+8^{p-1}(CM)^p) \\
&+8^{p-1}M_b^p[(2^{p-1}(CM)^p+1)L_q+(CM)^p b^p L_\sigma + C_p(CM)^p b^{p/2}L_f]\} \\
&\times \parallel \varphi^1-\varphi^2\parallel_{\mathcal{B}}^p \\
&+\max_{1\le i\le N}\{2^{p-1}K_b^p[(8^{p-1}CM)^p+1)\gamma_i + 8^{p-1}((CM)^p+1)L_q]\} \\
&\times \sup_{s\in[0,t]} E\parallel x^1(s)-x^2(s)\parallel_H^p + 8^{p-1}K_b^p(CM)^p[b^{p-1}L_\sigma + C_p b^{p/2-1}L_f] \\
&\times \int_0^t \sup_{s\in[0,t]} E\parallel x^1(s)-x^2(s)\parallel_H^p ds.
\end{aligned}
$$

Since $L_{**} = \max_{1\le i\le N}\{2^{p-1}K_b^p[(8^{p-1}CM)^p+1)\gamma_i + 8^{p-1}((CM)^p+1)L_q]\} < 1$, we have

$$
\begin{aligned}
&\sup_{s\in[0,t]} E\parallel x^1(s)-x^2(s)\parallel_H^p \\
\le& \frac{1}{1-L_{**}}\max_{1\le i\le N}\{8^{p-1}(CM)^p(\tilde{H}^p+L_q)+2^{p-1}\gamma_i M_b^p(1+8^{p-1}(CM)^p) \\
&+8^{p-1}M_b^p[(2^{p-1}(CM)^p+1)L_q+(CM)^p b^p L_\sigma + C_p(CM)^p b^{p/2}L_f]\} \\
&\times \parallel \varphi^1-\varphi^2\parallel_{\mathcal{B}}^p + \frac{1}{1-L_{**}}8^{p-1}K_b^p(CM)^p[b^{p-1}L_\sigma + C_p b^{p/2-1}L_f] \\
&\times \int_0^t \sup_{s\in[0,t]} E\parallel x^1(s)-x^2(s)\parallel_H^p ds.
\end{aligned}
$$

Applying Gronwall's inequality in the above expression again, we obtain

$$
\sup_{s\in[0,t]} E\parallel x^1(s)-x^2(s)\parallel_H^p \le C_* \parallel \varphi^1-\varphi^2\parallel_{\mathcal{B}}^p,
$$

where

$$
\begin{aligned}
C_* =& \frac{1}{1-L_{**}}\max_{1\le i\le N}\{8^{p-1}(CM)^p(\tilde{H}^p+L_q)+2^{p-1}\gamma_i M_b^p(1+4^{p-1}(CM)^p) \\
&+8^{p-1}M_b^p[(2^{p-1}(CM)^p+1)L_q+(CM)^p b^p L_\sigma + C_p(CM)^p b^{p/2}L_f]\} \\
&\times \exp\left\{\frac{1}{1-L_{**}}8^{p-1}K_b^p(CM)^p[b^p L_\sigma + C_p b^{p/2}L_f]\right\},
\end{aligned}
$$

which implies that

$$
\parallel x^1-x^2\parallel_{\mathcal{PC}}^p \le C_* \parallel \varphi^1-\varphi^2\parallel_{\mathcal{B}}^p .
$$

The proof is completed. $\square$

5. Application

Consider the following impulsive fractional partial neutral stochastic functional integro-differential equations of the form

$$
\begin{aligned}
dD(t,z_t)(x) =& J_t^{\alpha-1}\left(\frac{\partial^2}{\partial x^2}-\nu\right)D(t,z_t)(x)dt \\
&+\int_{-\infty}^{t}\mu_2(t,x,s-t)z(t,x)dsdt \\
&+\int_{-\infty}^{t}\mu_3(t,x,s-t)z(t,x)dsdw(t), \\
&(t,x)\in\cup_{i=1}^{N}[s_i,t_{i+1}]\times[0,\pi],
\end{aligned}
\tag{5.1}
$$

$$z(t,0)=z(t,\pi)=0, \quad t\in[0,b], \tag{5.2}$$

$$z(\tau,x)=\varphi(\tau,x), \quad x\in[0,\pi], \tag{5.3}$$

$$z(t,x)=\int_{-\infty}^{t}\eta_i(t,x,s-t)z(s,x)ds, \quad x\in[0,\pi], t\in(t_i,s_i], \tag{5.4}$$

where $0=t_0=s_0<t_1\le s_1<...<t_N\le s_N<t_{N+1}=b$ are fixed real numbers, $1<\alpha<2, \nu>0$ and φ is continuous and $w(t)$ denotes a a one-dimensional standard Wiener process in H defined on a stochastic space $(\Omega,\mathcal{F},P)$. In this system,

$$D(t,z_t)(x)=z(t,x)+\int_{-\infty}^{t}\mu_1(t,x,s-t)z(s,x)ds.$$

Let $H=L^2([0,\pi])$ with the norm $\parallel\cdot\parallel$ and define the operator $A:D(A)\subset H\to H$ by $Au=u''-\nu u$ with the domain

$$D(A):=\{u\in H: u''\in H, u(0)=u(\pi)=0\}.$$

It is well known that $\triangle u=u''$ is the infinitesimal generator of an analytic semigroup $T(t), t\ge0$ on H. Hence, A is sectorial of type $\omega=-\nu<0$.

Let $r\ge0, 1\le p<1$ and let $h:(-\infty,-r]\to\mathbb{R}$ be a nonnegative measurable function which satisfies the conditions (h-5), (h-6) in the terminology of Hino et al. [19]. Briefly, this means that h is locally integrable and there is a non-negative, locally bounded function η on $(-\infty,0]$ such that $h(\xi+\tau)\le\eta(\xi)h(\tau)$ for all $\xi\le0$ and $\theta\in(-\infty,-r)\setminus N_\xi$, where $N_\xi\subseteq(-\infty,-r)$ is a set whose Lebesgue measure zero. We denote by $\mathcal{PC}_r\times L^p(h,H)$ the set consists of all classes of functions $\varphi:(-\infty,0]\to H$ such that $\varphi_{|_{[-r,0]}}\in\mathcal{PC}([-r,0],H)$, $\varphi(\cdot)$ is Lebesgue measurable on $(-\infty,-r)$, and $h\parallel\varphi\parallel^p$ is Lebesgue integrable on $(-\infty,-r)$. The seminorm is given by

$$\parallel\varphi\parallel_{\mathcal{B}}=\sup_{-r\le\tau\le0}\parallel\varphi(\tau)\parallel+\left(\int_{-\infty}^{-r}h(\tau)\parallel\varphi\parallel^p d\tau\right)^{1/p}.$$

The space $\mathcal{B}=\mathcal{PC}_r\times L^p(h,H)$ satisfies axioms (A)-(C). Moreover, when $r=0$ and $p=2$, we can take $\tilde{H}=1$, $M(t)=\gamma(-t)^{1/2}$ and $K(t)=1+(\int_{-t}^{0}h(\tau)d\tau)^{1/2}$, for $t\ge0$ (see Theorem 1.3.8 in [19] for details).

Additionally, we will assume that

(i) The functions $\mu_1, \mu_2, \mu_3 : \mathbb{R}^3 \to \mathbb{R}$ is continuous and there exist continuous functions $\tilde{a}_j, \tilde{b}_j, \tilde{c}_j : \mathbb{R} \to \mathbb{R}, j = 1, 2$, such that

$$|\mu_1(t,x,s)| \leq \tilde{a}_1(t)\tilde{a}_2(s), \quad (t,x,s) \in \mathbb{R}^3,$$

$$|\mu_2(t,x,s)| \leq \tilde{b}_1(t)\tilde{b}_2(s), \quad (t,x,s) \in \mathbb{R}^3,$$

$$|\mu_3(t,x,s)| \leq \tilde{c}_1(t)\tilde{c}_2(s), \quad (t,x,s) \in \mathbb{R}^3$$

with $l_1 = (\int_{-\infty}^0 \frac{(\tilde{a}_2(s))^2}{h(s)} ds)^{1/2} < \infty$, $l_2 = (\int_{-\infty}^0 \frac{(\tilde{b}_2(s))^2}{h(s)} ds)^{1/2} < \infty$, $l_3 = (\int_{-\infty}^0 \frac{(\tilde{c}_2(s))^2}{h(s)} ds)^{1/2} < \infty$.

(ii) The functions $\eta_i : \mathbb{R}^3 \to \mathbb{R}, , i = 1, \dots, N$, are continuous and there exist continuous functions $\tilde{d}_i : \mathbb{R} \to \mathbb{R}$ such that

$$|\eta_i(t,x,s)| \leq \tilde{d}_i(s), \quad (t,x,s) \in \mathbb{R}^3$$

with $\tilde{L}_i = (\int_{-\infty}^0 \frac{(\tilde{d}_i(s))^2}{h(s)} ds)^{1/2} < \infty$ for every $i = 1, 2, \dots, N$.

In the sequel, $\mathcal{B}$ will be the phase space $\mathcal{PC}_0 \times L^2(h, H)$. Set $\varphi(\theta)(x) = \varphi(\theta, x) \in \mathcal{B}$, defining the maps $q, \sigma : [0, b] \times \mathcal{B} \to H$, $f : [0, b] \times \mathcal{B} \times H \to L(K, H), g_i : [0, b] \times \mathcal{B} \to H$ by

$$q(t,\varphi)(x) = \int_{-\infty}^0 \mu_1(t,x,\theta)\varphi(\theta,x)d\theta, \qquad D(t,\varphi)(x) = \varphi(0)x + q(t,\varphi)(x),$$

$$J_t^{\alpha-1}q(t) = \int_0^t \frac{(t-s)^{\alpha-2}}{\Gamma(\alpha-1)} q(s)ds, \qquad \sigma(t,\varphi)(x) = \int_{-\infty}^0 \mu_2(t,x,\theta)\varphi(\theta,x)d\theta,$$

$$f(t,\varphi)(x) = \int_{-\infty}^0 \mu_3(t,x,\theta)\varphi(\theta,x)d\theta, \quad g_i(t,\varphi)(x) = \int_{-\infty}^0 \eta_i(t,x,\theta)\varphi(\theta,x)d\theta.$$

Then the problem (5.1)-(5.4) can be written as system (1.1)-(1.3). Moreover, $q, \sigma, f, g_i (i = 1, \dots, N)$ are bounded linear operators on $\mathcal{B}$ with $E \parallel q \parallel^p \leq L_q$ and $E \parallel \sigma \parallel^p \leq L_\sigma, E \parallel f \parallel^p \leq L_f, E \parallel g_i \parallel^p \leq \gamma_i, i = 1, \dots, N$, where $L_q = [\parallel \tilde{a}_1 \parallel_\infty l_1]^p, L_\sigma = [\parallel \tilde{b}_1 \parallel_\infty l_2]^p, L_f = [\parallel \tilde{c}_1 \parallel_\infty l_3]^p, \gamma_i = [\tilde{L}_i]^p$. It is easy to see that with these choices, the assumptions (H1)-(H4) of Theorem 3.1 are satisfied. Suppose that the condition (3.1) in Section 3 holds. Hence, from Theorem 3.1, the problem (5.1)-(5.4) admits a unique mild solution on $[0, b]$. Further, if the condition (4.1) holds, then due to Theorem 4.1, we get that continuous dependence of mild solutions for the problem (5.1)-(5.4).

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