Journal of Applied Analysis and Computation Website:http://jaac-online.com/
Volume 5, Number 4, November 2015, 589–599 doi:10.11948/2015046

ANALYTICAL SOLUTION OF TIME-FRACTIONAL TWO-COMPONENT EVOLUTIONARY SYSTEM OF ORDER 2 BY RESIDUAL POWER SERIES METHOD

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Abstract In this paper, we introduce and formulate a novel study of obtaining the approximate solutions to the generalized time-fractional two-component evolutionary system of order 2 subject to given constraints conditions based on the generalized Taylor series formula. Here, a very recent technique based on the so called residual power series method is extended to handle such system. The solution methodology is based on generating the multiple fractional power series expansion solutions in the form of a rabidly convergent series with minimum size of calculations. A detailed description of the method is given and the obtained results reveal that the technique is a new significant method for exploring linear and nonlinear fractional models.

Keywords Generalized Taylor series, residual power series, time-fractional two-component evolutionary system of order 2.

MSC(2010) 26A33, 35F25, 35C10.

1. Introduction

Fractional differentiation is natural generalization of notions of integer-order differentiation and includes the n-th derivative as a particular case. There is still a lack of geometric and physical interpretation of fractional-order operators compared with the simple interpretations of their integer-order counterpart. Moreover, the definitions of the fractional-order derivative are not unique and there exist several definitions, including: Grunwald-Letnikov, Riemann-Liouville, Weyl, Riesz and Caputo sense. Therefore, authors have put their efforts to construct and develop analytical and numerical methods to study the solutions of fractional differential equations trying to provide a link of their findings to known classical integer-order case [20]. A few methods were extensively used in the literature based on its well-imposed simplicity and efficiency to handle some fractional models appears in the applied sciences. Such methods are: Variational iteration method and multivariate Pade approximations [21], Iterative Laplace transform method [13], Adomian decomposition method [12, 17–19, 22], Homotopy analysis method [10, 11], Laplace-Homotopy perturbation method [14], Decomposition method [15] and Sumudu transform method [16].

In this context we offering the residual power series (RPS) method as an alternative technique to obtain analytic solutions of different types of fractional linear

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and nonlinear partial differential equations applied in mathematics, physics and engineering. The RPS method was developed as an efficient method for determining values of coefficients of the power series solution for the first-order and the second-order fuzzy deferential equations [1]. It has been successfully applied to handle the numerical solution of the generalized Lane-Emden equation -which is-a highly nonlinear singular differential equation [2], the solution of composite and non-composite fractional deferential equations [3], predicting and representing the multiplicity of solutions to boundary value problems of fractional order [4], constructing and predicting the solitary pattern solutions for nonlinear time-fractional dispersive partial deferential equations [5], the approximate solution of the nonlinear fractional KdV-Burgers equation [6], the approximate solutions of fractional population diffusion model [7], and the numerical solutions of linear non-homogeneous partial differential equations of fractional order [8]. The RPS method is effective and easy to construct power series solution for strongly linear and nonlinear equations without linearization, perturbation, or discretization. Different from the classical power series method, the RPS method does not need to compare the coefficients of the corresponding terms and a recursion relation is not required. This method computes the coefficients of the power series by a chain of algebraic equations of one or more variables. In fact, the RPS method is an alternative procedure for obtaining analytic solutions for partial differential equations of fractional order. By using residual error concept, we get a series solutions; in practice truncated series solutions. Moreover, the obtained solutions and all their time-space-fractional derivatives are applicable for each arbitrary point and each multi-dimensional variable in the given domain. On the other aspect as well, the RPS method does not require any converting while switching from the low-order to the higher-order; as a result the method can be applied directly to the given system by choosing an appropriate initial guesses approximations.

The main objective of this paper is to present a new generalization of a two-component evolutionary homogeneous system of order 2 by replacing the first order time derivative by a fractional derivative of order α, $0 \leq \alpha \leq 1$, and takes the form

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = -3v_{xx}(x,t), \quad \frac{\partial^{\alpha} v(x,t)}{\partial t^{\alpha}} = u_{xx}(x,t) + 4u^2(x,t), \tag{1.1}$$

where x and t are the space and time variables. Theoretically, α can be any positive number. Note that for $\alpha = 1$, Equation (1.1) represents the standard two-component evolutionary homogeneous system of order 2. To the best of our knowledge this system is new and to be explored in this study by means of residual power series method. The paper has been organized as follows. In Section 2, the fractional power series is described. In Section 3, we derive a residual power series solution to the time-fractional two-component evolutionary homogeneous system of order 2. Graphical results regards the proposed system is presented in Section 4.

2. Fractional power series

In this section we re-introduce some needed parts from [3, 6, 7] regards fractional power series. First, we should pointed here that the fractional derivative considered in this study is of Caputo type. In Caputo case, the derivative of a constant is zero and one can define, properly, the initial conditions for the fractional differential equations which can be handled by using an analogy with the classical integer case [9].

Definition 2.1. For m to be the smallest integer that exceeds α, the Caputo fractional derivatives of order $\alpha > 0$ is defined as

$$D^{\alpha}u(x,t) = \frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}} = \begin{cases} \frac{1}{\Gamma(m-\alpha)}\int_0^t (t-\tau)^{m-\alpha-1}\frac{\partial^m u(x,\tau)}{\partial \tau^m}d\tau, & m-1 < \alpha < m, \\ \frac{\partial^m u(x,t)}{\partial t^m}, & \alpha = m \in N. \end{cases}$$

Definition 2.2. A power series expansion of the form

$$\sum_{m=0}^{\infty} c_m(t-t_0)^{m\alpha} = c_0 + c_1(t-t_0)^{\alpha} + c_2(t-t_0)^{2\alpha} + ... \quad 0 \le n-1 < \alpha \le n,\ t \le t_0$$

is called fractional power series PS about $t = t_0$.

Theorem 2.1. *Suppose that f has a fractional PS representation at $t = t_0$ of the form*

$$f(t) = \sum_{m=0}^{\infty} c_m(t-t_0)^{m\alpha}, \quad t_0 \le t < t_0 + R.$$

If $D^{m\alpha}f(t),\quad m = 0,1,2,..$ are continuous on (t_0, t_0+R), then $c_m = \frac{D^{m\alpha}f(t_0)}{\Gamma(1+m\alpha)}$.

Definition 2.3. A power series expansion of the form

$$\sum_{m=0}^{\infty} f_m(x)(t-t_0)^{m\alpha}$$

is called multiple fractional power series PS about $t = t_0$.

Theorem 2.2. *Suppose that $u(x,t)$ has a multiple fractional PS representation at $t = t_0$ of the form*

$$u(x,t) = \sum_{m=0}^{\infty} f_m(x)(t-t_0)^{m\alpha}, \quad x \in I, \quad t_0 \le t < t_0 + R.$$

If $D_t^{m\alpha}u(x,t),\quad m = 0,1,2,..$ are continuous on $I \times (t_0, t_0+R)$, then

$$f_m(x) = \frac{D_t^{m\alpha}u(x,t_0)}{\Gamma(1+m\alpha)}.$$

From the last theorem, it is clear that if $n+1$-dimensional function has a multiple fractional PS representation at $t = t_0$, then it can be derived in the same manner. i.e.

Corollary 2.1. *Suppose that $u(x,y,t)$ has a multiple fractional PS representation at $t = t_0$ of the form*

$$u(x,y,t) = \sum_{m=0}^{\infty} g_m(x,y)(t-t_0)^{m\alpha}, \quad (x,y) \in I_1 \times I_2, \quad t_0 \le t < t_0 + R.$$

If $D_t^{m\alpha}u(x,y,t),\quad m = 0,1,2,..$ are continuous on $I_1 \times I_2 \times (t_0, t_0+R)$, then

$$g_m(x,y) = \frac{D_t^{m\alpha}u(x,y,t_0)}{\Gamma(1+m\alpha)}.$$

3. Residual power series (RPS) for time-fractional evolutionary homogeneous system of order 2

Consider the time-fractional two-component evolutionary system

$$\begin{aligned} D_t^{\alpha} u(x,t) &= -3v_{xx}(x,t), \\ D_t^{\alpha} v(x,t) &= u_{xx}(x,t) + 4u^2(x,t), \end{aligned} \tag{3.1}$$

subject to the initial conditions:

$$\begin{aligned} u(x,0) &= f(x), \\ v(x,0) &= g(x). \end{aligned} \tag{3.2}$$

We aim to derive the solution of the above system by substituting its power series (PS) expansion among its truncated residual function [1–3]. From the resulting equation a recursion formula for the computation of the coefficients is derived, while the coefficients in the fractional PS expansion can be computed recursively by recurrent fractional differentiation of the truncated residual function.

The RPS method propose the solution for Eqs. (3.1-3.2) as a fractional PS about the initial point $t = 0$

$$\begin{aligned} u(x,t) &= \sum_{n=0}^{\infty} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \\ v(x,t) &= \sum_{n=0}^{\infty} g_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \quad 0 < \alpha \le 1,\ x \in I,\ 0 \le t < R. \end{aligned} \tag{3.3}$$

Next, we let $u_k(x,t),\ v_k(x,t)$ to denote the k-th truncated series of $u(x,t),\ v(x,t)$, respectively, i.e.

$$\begin{aligned} u_k(x,t) &= \sum_{n=0}^{k} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \\ v_k(x,t) &= \sum_{n=0}^{k} g_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \quad 0 < \alpha \le 1,\ x \in I,\ 0 \le t < R. \end{aligned} \tag{3.4}$$

It is clear that by condition (3.2) the 0-th RPS approximate solutions of $u(x,t)$ and $v(x,t)$ are

$$\begin{aligned} u_0(x,t) &= f_0(x) = u(x,0) = f(x), \\ v_0(x,t) &= g_0(x) = v(x,0) = g(x). \end{aligned} \tag{3.5}$$

Also, Eqs. (3.4) can be written as

$$\begin{aligned} & u_k(x,t) = f(x) + \sum_{n=1}^{k} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \\ & v_k(x,t) = g(x) + \sum_{n=1}^{k} g_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \\ & 0 < \alpha \le 1,\ x \in I,\ 0 \le t < R,\ k = 1,2,3,\cdots . \end{aligned} \tag{3.6}$$

Now, we define the residual functions, $Res_u,\ Res_v$, for Eqs. (3.1)

$$\begin{aligned}Res_u(x,t) &= D_t^\alpha u(x,t) + 3v_{xx}(x,t),\\ Res_v(x,t) &= D_t^\alpha v(x,t) - u_{xx}(x,t) - 4u^2(x,t),\end{aligned} \tag{3.7}$$

and therefore, the k-th residual functions, $Res_{u,k},\ Res_{v,k}$, are

$$\begin{aligned}Res_{u,k}(x,t) &= D_t^\alpha u_k(x,t) + 3\frac{\partial^2}{\partial x^2}v_k(x,t),\\ Res_{v,k}(x,t) &= D_t^\alpha v_k(x,t) - \frac{\partial^2}{\partial x^2}u_k(x,t) - 4u_k^2(x,t).\end{aligned} \tag{3.8}$$

As in [1–3], $Res(x,t) = 0$ and $\lim_{k\to\infty} Res_k(x,t) = Res(x,t)$ for all $x \in I$ and $t \geq 0$. Therefore, $D_t^{r\alpha} Res(x,t) = 0$ -fractional derivative of a constant in the Caputo's sense is 0- and the fractional derivative $D_t^{r\alpha}$ of $Res(x,t)$ and $Res_k(x,t)$ are matching at $t = 0$ for each $r = 0, 1, 2, \cdots, k$. To clarify the RPS technique: First, we substitute the k-th truncated series of $u(x,t),\ v(x,t)$ into Eqs. (3.8). Second, we find the fractional derivative formula $D_t^{(k-1)\alpha}$ of both $Res_{u,k}(x,t),\ Res_{v,k},\ \ k = 1, 2, 3, \cdots$, and finally, we solve the obtained algebraic system

$$\begin{aligned}&D_t^{(k-1)\alpha} Res_{u,k}(x,0) = 0,\\ &D_t^{(k-1)\alpha} Res_{v,k}(x,0) = 0,\ 0 < \alpha \leq 1,\ x \in I,\ k = 1, 2, 3, \cdots\end{aligned} \tag{3.9}$$

to get the required coefficients $f_n(x),\ g_n(x),\ \ n = 1, 2, 3, \cdots, k$ in Eqs. (3.6). Now, we construct the following steps.

Step 1. To determine $f_1(x),\ g_1(x)$, we consider $(k = 1)$ in (3.8)

$$\begin{aligned}Res_{u,1}(x,t) &= D_t^\alpha u_1(x,t) + 3\frac{\partial^2}{\partial x^2}v_1(x,t),\\ Res_{v,1}(x,t) &= D_t^\alpha v_1(x,t) - \frac{\partial^2}{\partial x^2}u_1(x,t) - 4u_1^2(x,t).\end{aligned} \tag{3.10}$$

But, $u_1(x,t) = f(x) + f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}$ and $v_1(x,t) = g(x) + g_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}$. Therefore,

$$\begin{aligned}Res_{u,1}(x,t) &= f_1(x) + 3g''(x) + 3g_1''(x)\frac{t^\alpha}{\Gamma(1+\alpha)},\\ Res_{v,1}(x,t) &= g_1(x) - f''(x) - f_1''(x)\frac{t^\alpha}{\Gamma(1+\alpha)} - 4\left(f(x) + f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)^2.\end{aligned} \tag{3.11}$$

From Eqs. (3.9) we deduce that $Res_{u,1}(x,0) = 0,\ Res_{v,1}(x,0) = 0$ and thus,

$$\begin{aligned}f_1(x) &= -3g''(x),\\ g_1(x) &= f''(x) + 4f^2(x).\end{aligned} \tag{3.12}$$

Therefore, the 1-st RPS approximate solutions are

$$\begin{aligned}u_1(x,t) &= f(x) - 3g''(x)\frac{t^\alpha}{\Gamma(1+\alpha)},\\ v_1(x,t) &= g(x) + \left(f''(x) + 4f^2(x)\right)\frac{t^\alpha}{\Gamma(1+\alpha)}.\end{aligned} \tag{3.13}$$

Step 2. To obtain $f_2(x)$, $g_2(x)$, we substitute the 2-nd truncated series $u_2(x,t) = f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}+f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}$ and $v_2(x,t) = g(x)+g_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}+g_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}$ into the 2-nd residual function $Res_{u,2}(x,t)$ and $Res_{v,2}(x,t)$, i.e.

$$\begin{aligned}
&Res_{u,2}(x,t) = D_t^\alpha u_2(x,t) + 3\frac{\partial^2}{\partial x^2}v_2(x,t) \\
&= f_1(x) + f_2(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + 3\left(g''(x) + g_1''(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + g_2''(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)
\end{aligned} \tag{3.14}$$

and

$$\begin{aligned}
&Res_{v,2}(x,t) = D_t^\alpha v_2(x,t) - \frac{\partial^2}{\partial x^2}u_2(x,t) - 4u_2^2(x,t) \\
&= g_1(x) + g_2(x)\frac{t^\alpha}{\Gamma(1+\alpha)} - \left(f''(x) + f_1''(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + f_2''(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right) \\
&\quad - 4\left(f(x) + f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)^2.
\end{aligned} \tag{3.15}$$

Applying D_t^α on both sides of Eqs. (3.14) and (3.15) gives

$$\begin{aligned}
D_t^\alpha Res_{u,2}(x,t) &= f_2(x) + 3\left(g_1''(x) + g_2''(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right), \\
D_t^\alpha Res_{v,2}(x,t) &= g_2(x) - \left(f_1''(x) + f_2''(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right) \\
&\quad - 8\left(f(x) + f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right) \\
&\quad \cdot \left(f_1(x) + f_2(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right).
\end{aligned} \tag{3.16}$$

By the fact that $D_t^\alpha Res_{u,2}(x,0) = 0 = D_t^\alpha Res_{v,2}(x,0)$ and solving the resulting system in (3.16) for the unknown coefficient functions $f_2(x), g_2(x)$, we get

$$\begin{aligned}
f_2(x) &= -3g_1''(x), \\
g_2(x) &= f_1''(x) + 8f(x)f_1(x).
\end{aligned} \tag{3.17}$$

Therefore, the 2-nd RPS approximate solutions of system (3.1-3.2) has the form

$$\begin{aligned}
u_2(x,t) &= f(x) - 3g''(x)\frac{t^\alpha}{\Gamma(1+\alpha)} - 3g_1''(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}, \\
v_2(x,t) &= g(x) + (f''(x) + 4f^2(x))\frac{t^\alpha}{\Gamma(1+\alpha)} + (f_1''(x) + 8f(x)f_1(x))\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}.
\end{aligned} \tag{3.18}$$

Step 3. In this step we first derive $f_3(x)$. Substitute the 3-rd truncated series $u_3(x,t) = f(x) + f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}$ and $v_3(x,t) = g(x) + g_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + g_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + g_3(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}$ into the 3-rd residual function $Res_{u,3}(x,t)$, i.e.

$$
\begin{aligned}
&Res_{u,3}(x,t)\\
&= D_t^{\alpha}u_3(x,t) + 3\frac{\partial^2}{\partial x^2}v_3(x,t)\\
&= f_1(x) + f_2(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_3(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\\
&+ 3\left(g''(x) + g_1''(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + g_2''(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + g_3''(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right). \qquad (3.19)
\end{aligned}
$$

Applying $D_t^{2\alpha}$ on both sides of Eq. (3.19) yields

$$
D_t^{2\alpha}Res_{u,3}(x,t) = f_3(x) + 3\left(g_2''(x) + g_3''(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)}\right). \qquad (3.20)
$$

By the fact that $D_t^{2\alpha}Res_{u,3}(x,0) = 0$ and solving the resulting equation in (3.20) for the unknown coefficient function $f_3(x)$, we get

$$
f_3(x) = -3g_2''(x). \qquad (3.21)
$$

Now, to compute $g_3(x)$, we consider

$$
\begin{aligned}
&Res_{v,3}(x,t) = D_t^{\alpha}v_3(x,t) - 3\frac{\partial^2}{\partial x^2}u_3(x,t) - 4u_4^2(x,t)\\
&= g_1(x) + g_2(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + g_3(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\\
&- 3\left(f''(x) + f_1''(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2''(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + f_3''(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right)\\
&- 4\left(f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right)^2. \qquad (3.22)
\end{aligned}
$$

Applying $D_t^{2\alpha}$ on both sides of Eq. (3.22) yields

$$
\begin{aligned}
&D_t^{2\alpha}Res_{v,3}(x,t) = D_t^{\alpha}\left(D_t^{\alpha}\left(Res_{v,3}(x,t)\right)\right)\\
&= g_3(x) - \left(f_2''(x) + f_3''(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)}\right) - 8\left(f_1(x) + ... + f_3(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)^2\\
&- 8\left(f(x) + ... + f_3(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right)\left(f_2(x) + f_3(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)}\right). \qquad (3.23)
\end{aligned}
$$

By solving $D_t^{2\alpha}Res_{v,3}(x,0) = 0$ we get

$$
g_3(x) = f_2''(x) + 8(f(x)f_2(x) + f_1^2(x)). \qquad (3.24)
$$

Finally, we solve the system $D_t^{3\alpha}Res_{u,4}(x,0) = 0, \quad D_t^{3\alpha}Res_{v,4}(x,0) = 0$ to obtain the following results

$$
\begin{aligned}
f_4(x) &= -3g_3''(x),\\
g_4(x) &= f_3''(x) + 8(f(x)f_3(x) + 3f_1(x)f_2(x)). \qquad (3.25)
\end{aligned}
$$

By the above recurrence relations, we are ready to present some graphical results regards the time-fractional two-component evolutionary system.

4. Validity of the RPS

The purpose of this portion is to test the derivation of the residual power series solutions of the system:

$$D_t^{\alpha} u(x,t) = -3v_{xx}(x,t), \quad D_t^{\alpha} v(x,t) = u_{xx}(x,t) + 4u^2(x,t), \tag{4.1}$$

subject to the initial conditions:

$$u(x,0) = f(x) = -\frac{3}{4(1+\cos(x))}, \quad v(x,0) = g(x) = \frac{\sqrt{3}}{4}\tan(\frac{x}{2}). \tag{4.2}$$

Figure 1, represents the 4-th RPS approximate solution of the function $u(x,t)$ for

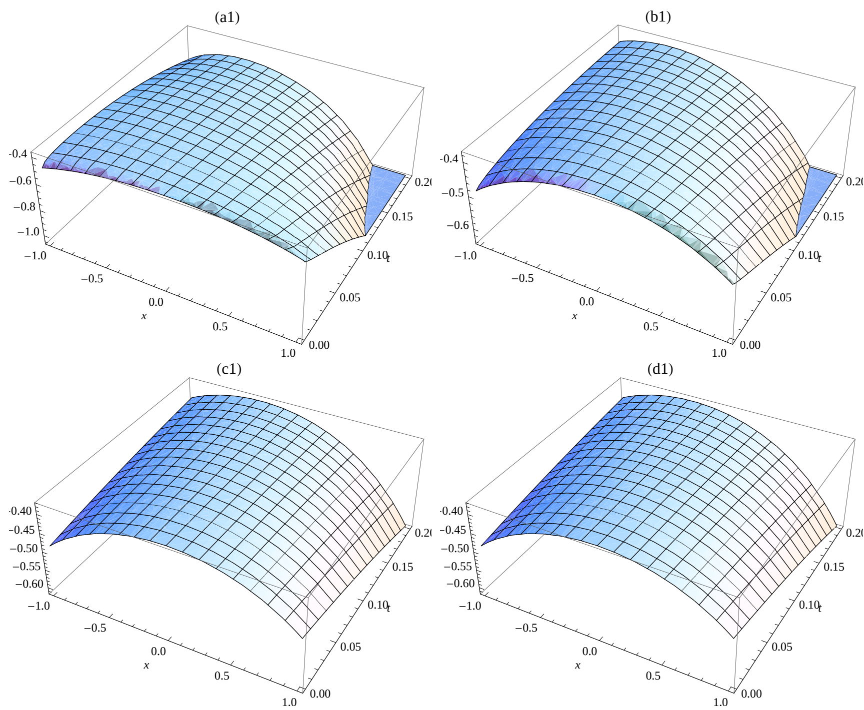

Figure 1. The 4-th RPS approximate solution of the function $u(x,t)$: (a1) $u_4(x,t,\alpha=0.5)$, (b1) $u_4(x,t,\alpha=0.75)$, (c1) $u_4(x,t,\alpha=1)$, (d1) $u(x,t)$ for $\alpha=1$, $-1<x<1$, $0<t<0.2$.

different values of the fractional order α. Figure 2, represents the corresponding 4-th RPS approximate solutions of the function $v(x,t)$.

5. Conclusions

In this paper, a relatively new analytical iterative technique based on the residual power series (RPS) is proposed to obtain an approximate solution to a nonlinear time-fractional two-component evolutionary system of order 2. This method can be used as an alternative to obtain analytic solutions of different types of fractional linear and nonlinear partial differential equations applied in mathematics, physics,

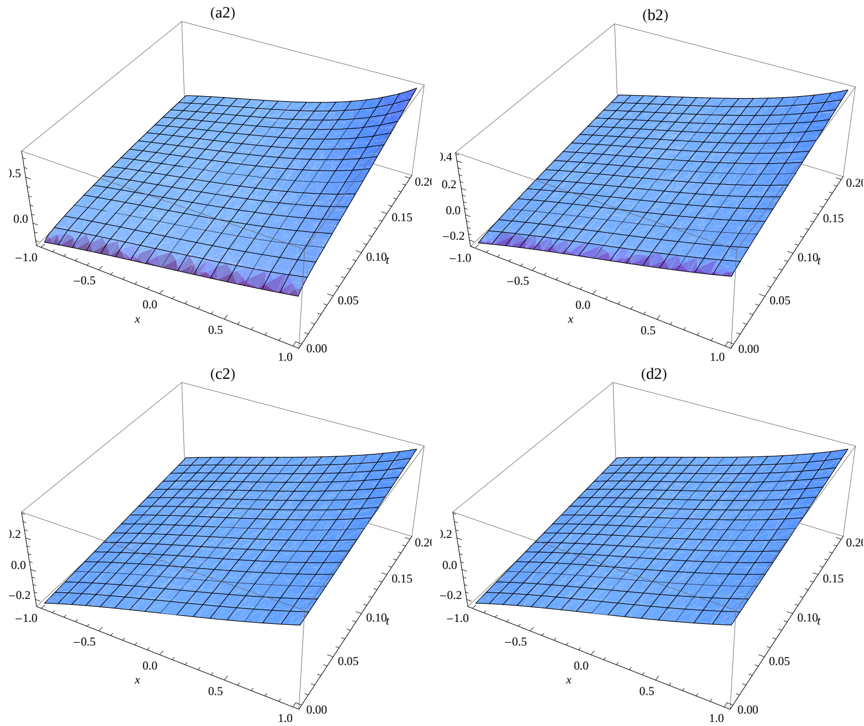

Figure 2. The 4-th RPS approximate solution of the function $v(x,t)$: (a2) $v_4(x,t,\alpha=0.5)$, (b2) $v_4(x,t,\alpha=0.75)$, (c2) $v_4(x,t,\alpha=1)$, (d2) $v(x,t)$ for $\alpha=1$, $-1<x<1$, $0<t<0.2$

and engineering. Efficacious computational algorithm is provided to guarantee the procedure and to illustrate the theoretical statements of the present method in order to show its potentiality, generality, and superiority for solving such systems. Graphical results and numerical descriptions are presented to illustrate the solutions. As future work, we will extend the RPS method to handle $(2+1)$-dimensional linear and nonlinear space- and time-fractional physical models.

Acknowledgements

The author would like to thank the reviewers for their comments that help improve the manuscript.

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