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# DESIGN OF $N$-DIMENSIONAL MULTI-SCROLL JERK CHAOTIC SYSTEM AND ITS PERFORMANCES*

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**Abstract** Based on three-order Jerk and high-order Jerk chaotic systems, a general approach is proposed to generate $n$-dimensional multi-scroll Jerk chaotic attractors via nonlinear control. Dynamics of the $n$-dimensional multi-scroll Jerk chaotic systems are analyzed by means of the largest Lyapunov exponent and multi-scale permutation entropy complexity. As an experimental verification, four-dimensional Jerk chaotic attractors are implemented by analog circuits. Results of the numerical simulation are consistent with that of the hardware experiments. It shows that the method of obtaining complex Jerk chaotic attractors is effective.

**Keywords** Chaos, Jerk chaotic system, multi-scroll chaotic attractor, nonlinear control.

**MSC(2010)** 93c15, 94c30.

## 1. Introduction

Over the past several decades, chaos has been intensively studied in the fields of physics, mathematics, chemistry, information technology, engineering and so on. Many chaotic systems have been proposed, like the Lorenz system [14], Chua system [6, 20], three-order Jerk systems [18, 29] and higher-order Jerk systems [5]. As multi-scroll attractors have higher complexity and better adjustability, their generation and applications have been studied with increasingly interest, which becomes a hot topic at present. There are a large number of publications devoted in this field to generating multi-scroll chaotic attractors from various chaotic systems [1, 12, 13, 15, 24, 27, 28, 30, 31]. Tang etc [24] presented a sine-function approach for generating $n$-scroll chaotic attractors from the Chua system. Yu etc [31] obtained grid multi-scroll chaotic attractors under the Chua-circuit framework. Yalcin etc [28] introduced an approach to generate multi-scroll chaotic attractors based on Jerk system using Josephson junctions. Multi-scroll chaotic attractors were obtained via symbolic functions from Jerk system in [13]. Liu etc [12] generated multi-scroll chaotic attractors from the Chen system. In addition, some new chaotic systems with multi-scrolls were introduced in recent years [3, 17, 19, 32], and several new control approaches were applied to generate multi-scroll chaotic at-

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tractors [2, 9, 16, 23]. There is an interesting problem whether there is a general approach to generating multi-dimensional multi-scroll chaotic attractors based on one-dimensional multi-scroll chaotic system.

Meanwhile, it is necessary to analyze performances of multi-scroll chaotic systems. The largest Lyapunov exponent [26] provides the predictability of chaos in a dynamical system. A system with a positive Lyapunov exponent is usually chaotic, and holding a larger value means the system is more complex. On the other hand, complexity-measure algorithms can also be employed to analyze chaotic dynamics. There are several algorithms for estimating the complexity of chaotic systems, such as the permutation entropy (PE) [25]] fuzzy entropy (FuzzyEn) [4], statistical complexity measure (SCM) [22] and spectral entropy (SE) [21], $C_0$ algorithm [10]. Recently, He etc [11] analyzed the complexity of multi-wing chaotic systems by means of SE and SCM. Here, the PE algorithm is chosen to estimate the complexity of the designed multi-dimensional multi-scroll chaotic systems for their fast estimation speed and high accuracy.

Motivated by the above discussions, we focus on the design of an $n$-dimensional multi-scroll chaotic system and its performances analysis. The rest of the paper is organized as follows. Multi-dimensional multi-scroll Jerk attractors are generated based on a one-dimensional multi-scroll Jerk system in Section 2. In Section 3, the performances of the multi-scroll chaotic attractors are analyzed. In Section 4, a hardware circuit for a four-dimensional Jerk chaotic system is designed. Finally, conclusions are summarized.

## 2. Design of multi-dimensional multi-scroll chaotic attractors

### 2.1. One-dimensional multi-scroll attractor model

To obtain multi-scroll attractor, the Jerk chaotic system [13, 18, 28] is modified to

$$\begin{cases} \dot{x} = y, \\ \dot{y} = z, \\ \dot{z} = 0.6(-x - y - z + f(x)), \end{cases} \tag{2.1}$$

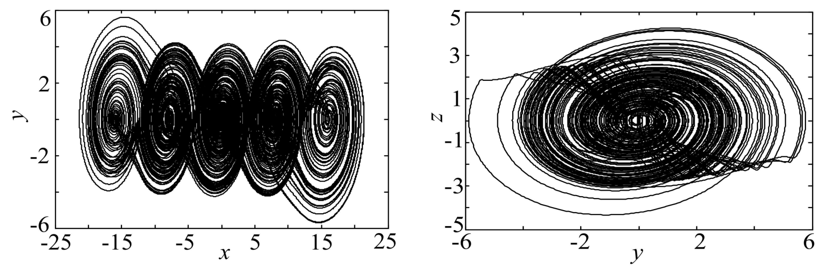
in which $f(x)$ is the nonlinear function, and it is defined by

$$f(x) = kp[-\operatorname{sgn}(x) + \sum_{i=0}^{N} \operatorname{sgn}(x + 2ip) \ + \sum_{i=0}^{N} \operatorname{sgn}(x - 2ip)], \tag{2.2}$$

for $2N + 2$-scroll chaotic attractors, and

$$f(x) = kp[\sum_{i=0}^{M} \operatorname{sgn}(x + (2i+1)p) \ + \sum_{i=0}^{M} \operatorname{sgn}(x - (2i+1)p)], \tag{2.3}$$

for $(2N + 3)$-scroll chaotic attractors. When $k = 1$ and $p = 4$, 5-scroll chaotic attractors are shown in Fig.1, which is obtained by system (2.1) and Eq.(2.3) for the case $N = 1$.



(a) $x - y$ plane (b) $y - z$ plane

**Figure 1.** One-dimensional 5 scrolls chaotic attractors.

## 2.2. Two-dimensional multi-scroll attractor model

By introducing a nonlinear control function $g_1(y)$ in the $y$ dimension, two-dimensional multi-scroll chaotic attractors are generated. The role of the controller is to extend the one-scroll attractor into multi-scroll attractor in $y$ dimension. Then, the Jerk chaotic system is denoted by

$$\begin{cases} \dot{x} = y - g_1(y), \\ \dot{y} = z, \\ \dot{z} = 0.6[-x - (y - g_1(y)) - z + f(x)], \end{cases} \tag{2.4}$$

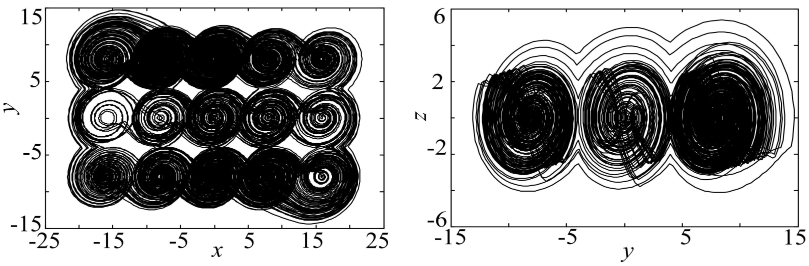
where $g_1(y)$ is defined as

$$g_1(y) = \sum_{i=1}^{M_1} B[1 + \mathrm{sgn}(y - y_i)] - \sum_{j=1}^{M_2} B[1 - \mathrm{sgn}(y - y_j)], \tag{2.5}$$

in which $f(x)$ is the same as above. $M_1$ and $M_2$ are non-negative integers. System (2.4) can generate $(2N+2) \times (M_1 + M_2 + 1)$ scrolls or $(2N+3) \times (M_1 + M_2 + 1)$ scrolls chaotic attractors. The parameters $B$, $y_i$ and $y_j$ are obtained by trial-and-error method. The value of parameter $B$ is related to the size of the attractor, while the values of parameters $y_i$ and $y_j$ are linked to the position of the attractor. The numerical simulation of two-dimensional multi-scroll chaotic attractors is displayed in Fig.2. $g_1(y)$ is defined as $g_1(y) = (1 + \mathrm{sgn}(y-4)) - 4(1 - \mathrm{sgn}(y+4))$ for $5 \times 3$ scrolls chaotic attractors. It shows that the nonlinear controller extends the $y$ dimension into multi-scroll attractor from one-scroll attractor through the comparison of Fig.1 and Fig.2, and it illustrates that the designed approach for generating two-dimensional multi-scroll attractor is correct.

## 2.3. Three-dimensional multi-scroll attractor model

Similarly, a nonlinear control function $g_2(z)$ is introduced into the $z$ dimension to generate three-dimensional multi-scroll chaotic attractors. The role of the controller is to extend the one-scroll attractor into multi-scroll attractor in $z$ dimension. Then,




(a) $x - y$ plane (b) $y - z$ plane

**Figure 2.** Two-dimensional $5 \times 3$ scrolls chaotic attractors.

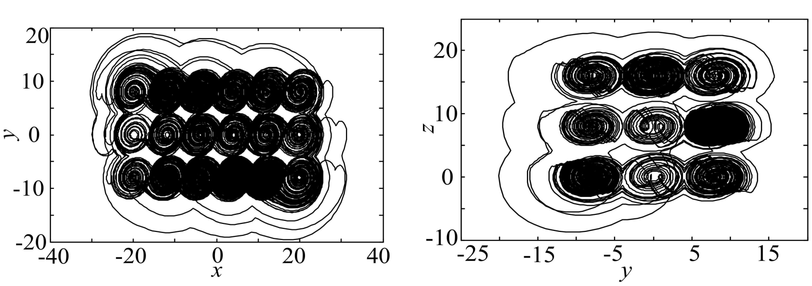
the system is described by

$$\begin{cases} \dot{x} = y - g_1(y), \\ \dot{y} = z - g_2(z), \\ \dot{z} = 0.6[-x - (y - g_1(y)) - (z - g_2(z)) + f(x)], \end{cases} \tag{2.6}$$

where $f(x)$ and $g_1(y)$ are the same as above. $g_2(z)$ is given by

$$g_2(z) = \sum_{i=1}^{M_3} B[1 + \operatorname{sgn}(z - z_i)] - \sum_{j=1}^{M_4} B[1 - \operatorname{sgn}(z - z_j)], \tag{2.7}$$

where $M_3$ and $M_4$ are non-negative integers. Similarly, the value of parameter $B$ is related to the size of the attractor, and the values of parameter $z_i$ and $z_j$ are linked to the position of the attractor. System (2.6) can generate $(2N+2)\times(M_1+M_2+1)\times(M_3+M_4+1)$ scrolls chaotic attractors or $(2N+3)\times(M_1+M_2+1)\times(M_3+M_4+1)$ scrolls chaotic attractors. When $g_1(y) = 4(1 + \operatorname{sgn}(y - 4)) - 4(1 - \operatorname{sgn}(y + 4))$ and $g_2(z) = 4(1 + \operatorname{sgn}(z - 4)) + 4(1 + \operatorname{sgn}(z - 11.9))$, the numerical simulation results are displayed in Fig.3(a) and (b). The distinct and good symmetry attractors show that three-dimensional multi-scroll chaotic attractors are generated based on system (2.6). The spans of $x$, $y$ and $z$ dimensions are consistent.




(a) $x - y$ plane (b) $y - z$ plane

**Figure 3.** Three-dimensional $6 \times 3 \times 3$ scrolls chaotic attractors.

### 2.4. Four-dimensional multi-scroll attractor model

The hyperjerk systems were reported in Ref. [5], and one of the high order Jerk systems is described as

$$a_n \frac{d^n x}{d\tau^n} + a_{n-1} \frac{d^{n-1} x}{d\tau^{n-1}} + \cdots + a_1 \frac{dx}{d\tau} + a_0 x = H(x), \tag{2.8}$$

where $n$ is the order of the system, and $H(x)$ is a nonlinear function. To generate $(2N+2)$-scroll chaotic attractors, $H(x)$ is modified as

$$H(x) = kp[-\operatorname{sgn}(x) + \sum_{i=0}^{N} \operatorname{sgn}(x + 2ip) + \sum_{i=0}^{N} \operatorname{sgn}(x - 2ip)]. \tag{2.9}$$

To obtain $(2N+3)$-scroll chaotic attractors, $H(x)$ is defined by

$$H(x) = kp[\sum_{i=0}^{N} \operatorname{sgn}(x + (2i+1)p) + \sum_{i=0}^{N} \operatorname{sgn}(x - (2i+1)p)]. \tag{2.10}$$

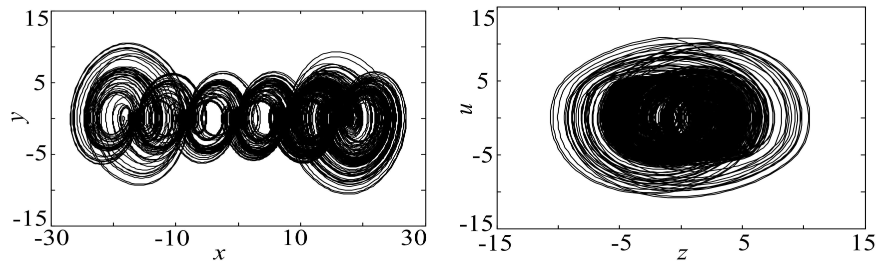
When $k = 8$, $p = 4$, the 6-scroll chaotic attractors from 4-order Jerk system and 5-order Jerk system are plotted in Fig.4 and Fig.5, respectively. Fig.4 is obtained by system (2.8) and Eq. (2.9) for the case $n = 4$ and $N = 2$. Fig.5(a) and Fig.5(b) are obtained by system (2.8) and Eq.(2.9) for the case $n = 5$ and $N = 2$. It shows that both 4-order Jerk system and 5-order Jerk system generate multi-scroll in $x$ dimension, while the other dimensions present only one-scroll. To generate four-dimensional multi-scroll attractor, three nonlinear control functions $g_1(y)$, $g_2(z)$ and $g_3(u)$ are introduced into system (2.8). When $n = 4$, the system becomes to

$$\begin{cases} \dot{x} = y - g_1(y), \\ \dot{y} = z - g_2(z), \\ \dot{z} = u - g_3(u), \\ \dot{u} = F_1(x, y, z, u), \end{cases} \tag{2.11}$$

where $F_1(x, y, z, u)$ is described as

$$F_1(x, y, z, u) = -[7.278(u - g_3(u)) + 4(z - g_2(z)) \tag{2.12}$$
$$+ 9.19(y - g_1(y) + 7.9x)] + H(x), \tag{2.13}$$

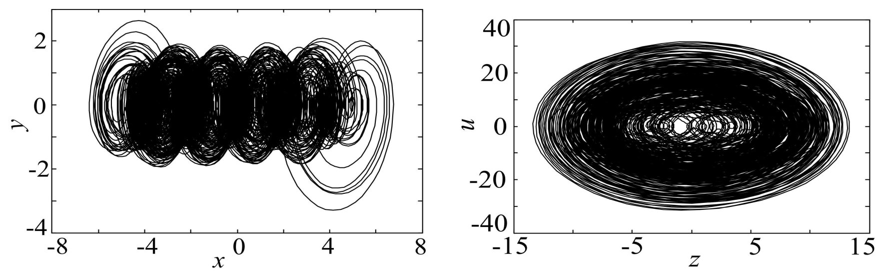
$g_1(y)$, $g_2(z)$ and $g_3(u)$ have the same form with the Eq.(2.5). The parameters of the controllers are related to the size and position of the attractors. When $g_1(y) = 4(1 + \operatorname{sgn}(y - 4))$, $g_2(z) = 4(1 + \operatorname{sgn}(z - 4))$, $g_3(u) = 4(1 + \operatorname{sgn}(u - 4))$ and $H(x) = 32(\operatorname{sgn}(x) + \operatorname{sgn}(x + 8) + \operatorname{sgn}(x - 8))$, the numerical simulations of four-dimensional $4 \times 2 \times 2 \times 2$-scroll chaotic attractors are shown in Fig.6. Compared with Fig.4, it illustrates that not only $x$ dimension generates multi-scroll attractor, but also the other dimensions do show multi-scroll attractor in the system (2.11). It means that one-scroll attractor is extended into multi-scroll attractor in $y$, $z$ and $u$ dimensions by employing the nonlinear controllers $g_1(y)$, $g_2(z)$ and $g_3(u)$.




(a) $x-y$ plane

(b) $z-u$ plane

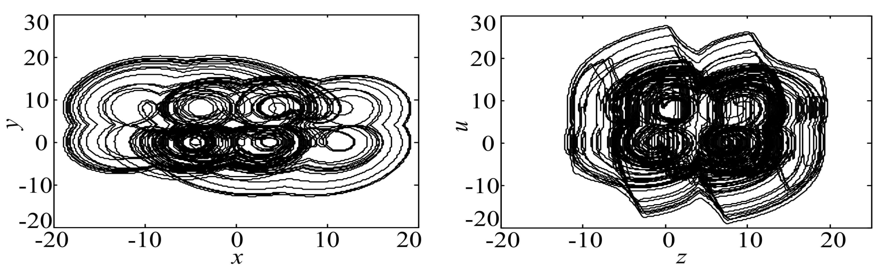
**Figure 4.** Chaotic attractors of 4-order Jerk system.




(a) $x-y$ plane

(b) $z-u$ plane

**Figure 5.** Chaotic attractors of 5-order Jerk system.




(a) $x-y$ plane

(b) $z-u$ plane

**Figure 6.** Four-dimensional $4 \times 2 \times 2 \times 2$ scrolls chaotic attractors.

## 2.5. Five-dimensional multi-scroll attractor model

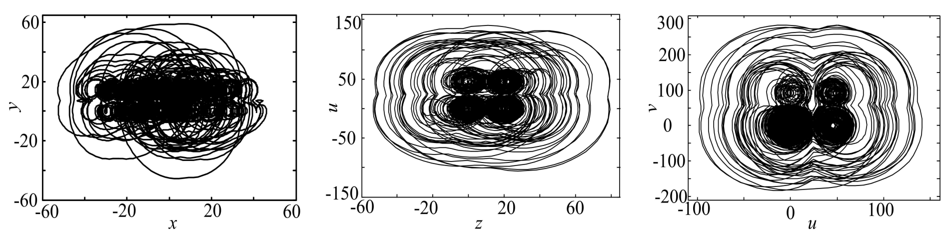
Similarly, by introducing four nonlinear control functions $g_1(y)$, $g_2(z)$, $g_3(u)$ and $g_4(v)$, 5-dimensional multi-scroll chaotic system is written as

$$\begin{cases} \dot{x} = y - g_1(y), \\ \dot{y} = z - g_2(z), \\ \dot{z} = u - g_3(u), \\ \dot{u} = v - g_4(v), \\ \dot{v} = F_1(x, y, z, u, v), \end{cases} \tag{2.14}$$

in which $F_1(x,y,z,u,v)$ is described as

$$F_1(x,y,z,u) = -[v - g_4(v) + 7.278(u - g_3(u)) + 4(z - g_2(z)) + 9.19(y - g_1(y) + 7.9x)] + H(x), \tag{2.15}$$

$H(x)$ is the same as above. The role of the controllers is to extend the one-scroll attractor into multi-scroll attractor. $g_1(y)$, $g_2(z)$, $g_3(u)$ and $g_4(v)$ have the same form with the Eq.(2.5). The parameters of the controllers are related to the size and position of the attractors. When $g_1(y) = 7.2(1 + \mathrm{sgn}(y-7))$, $g_2(z) = 10(1 + \mathrm{sgn}(z-10))$, $g_3(u) = 23(1 + \mathrm{sgn}(u-24))$ and $g_4(v) = 46(1 + \mathrm{sgn}(v-58))$, the numerical simulations of five-dimensional chaotic attractors are shown in Fig.7. Thus, the five-dimensional multi-scroll attractor is generated, and it means that the controller works well.




(a) $x-y$ plane (b) $z-u$ plane (c) $u-v$ plane

**Figure 7.** Five-dimensional $10 \times 2 \times 2 \times 2 \times 2$ scrolls chaotic attractors.

## 2.6. $n$-dimensional multi-scroll attractor model

To generate $n$-dimensional multi-scroll attractors, $n-1$ nonlinear control functions $g_1(y)$, $g_2(z)$, ..., $g_{n-1}(\mu)$ are introduced into system (2.8). The $n$-dimensional multi-scroll chaotic system is denoted as

$$\begin{cases} \dot{x} = y - g_1(y), \\ \dot{y} = z - g_2(z), \\ \qquad \vdots \\ \dot{w} = \mu - g_{n-1}(\mu), \\ \dot{\mu} = F_1(x,y,\cdots,w,\mu), \end{cases} \tag{2.16}$$

where $g_1(y)$, $g_2(z)$, ..., $g_{n-1}(\mu)$ have the same form with Eq.(2.5) and

$$F_1(x,y,\cdots,w,\mu) = -[a_1(v - g_{n-1}(\mu)) + a_2(w - g_{n-2}(w)) + \cdots + a_{n-1}(y - g_1(y) + a_n x)] + H(x). \tag{2.17}$$

According to above model, parameters of the $n$-dimensional multi-scroll system can be obtained by applying trial-and-error method. Thus we find a general approach to generate multi-dimensional multi-scroll chaotic attractor based on one-dimensional $n$-order hyperjerk system.

# 3. Performance analysis

## 3.1. The largest Lyapunov exponent

The largest Lyapunov exponents (LE) values of the systems with different scrolls are calculated based on the algorithm proposed in [26], and the results are listed in Tabs.1, 2 and 3.

**Table 1.** The largest LE of three-dimensional Jerk system.

| $(M,N)$ | $6\times M\times N$ | $8\times M\times N$ | $10\times M\times N$ |
|---|---|---|---|
| (1,1) | 0.0988 | 0.0988 | 0.0988 |
| (3,1) | 0.0988 | 0.0988 | 0.0988 |
| (4,1) | 0.0988 | 0.0988 | 0.0988 |
| (3,3) | 0.0988 | 0.0991 | 0.0988 |
| (4,4) | 0.0988 | 0.0987 | 0.0988 |

**Table 2.** The largest LE of four-dimensional Jerk system.

| $(M,N,P)$ | $6\times M\times N\times P$ | $8\times M\times N\times P$ | $10\times M\times N\times P$ |
|---|---|---|---|
| (1,1,1) | 0.1883 | 0.1883 | 0.1883 |
| (3,1,1) | 0.1867 | 0.1876 | 0.1876 |
| (3,3,1) | 0.1842 | 0.1842 | 0.1842 |
| (3,3,3) | 0.1856 | 0.1850 | 0.1868 |

**Table 3.** The largest LE of five-dimensional Jerk system.

| $(M,N,P,Q)$ | $6\times M\times N\times P\times Q$ | $8\times M\times N\times P\times Q$ | $10\times M\times N\times P\times Q$ |
|---|---|---|---|
| (1,1,1,1) | 0.2283 | 0.2283 | 0.2283 |
| (2,1,1,1) | 0.2303 | 0.2303 | 0.2303 |
| (2,3,1,1) | 0.2501 | 0.2486 | 0.2500 |
| (2,2,1,1) | 0.2486 | 0.2486 | 0.2499 |
| (2,2,2,2) | 0.2356 | 0.2356 | 0.2492 |

According to Tabs.1, 2 and 3, all the values are greater than zero, and it indicates that those systems are chaotic. In addition, the results also show that the largest Lyapunov exponents are located on the same level of magnitude for different scrolls for the same order system, and it means the largest Lyapunov exponent does not change with scrolls for the same dimensional Jerk system. The largest exponent values of the five-order Jerk system are the largest, while those of the three-dimensional Jerk system are the smallest. It shows that higher dimensional system is more complicated.

### 3.2. Complexity analysis

According to the definition of probability distribution $d$ associated with the time series $\{x(i) : i = 1, 2, ..., N\}$, permutation entropy (PE) algorithm [21] is briefly presented as follows.

Step 1: Defining probability distribution. For different dimension $d$, the probability distribution $p(\pi)$ is defined by

$$p(\pi) = \frac{\#\{i|i \leqslant T - d + 1, \text{has type } \pi\}}{T - d + 1}, \tag{3.1}$$

where the symbol # stands for number and $\pi$ is the possible order pattern.

Step 2: Calculating PE. According to the definition of Shannon entropy, PE is denoted as

$$\mathrm{PE}(x, d) = -\sum_{j=1}^{d!} p_j(\pi) \ln(p_j(\pi))/\ln(d!). \tag{3.2}$$

The normalized entropy is evaluated for this permutation probability distribution. The range of $d$ is $\{2, 4, ..., 7\}$ [11]. In this paper, we choose $d = 5$ to calculate PE. Obviously, larger PE value means the time series is more complex.

Multiscale permutation entropy (MPE) was developed to analyze complexity of time series for a range of scales [7, 8]. The MPE algorithm is briefly described as follows.

Step 1: Coarse-grained procedure. Given a one-dimensional discrete time series $\{x(i) : i = 1, 2, ..., N\}$, the consecutive coarse-grained time series are constructed by

$$y^\tau(j) = \frac{1}{\tau} \sum_{i=(j-1)\tau+1}^{j\tau} x(i), \tag{3.3}$$

where $1 \le j \le \lfloor N/\tau \rfloor$, and $\lfloor\cdot\rfloor$ denotes the floor function.

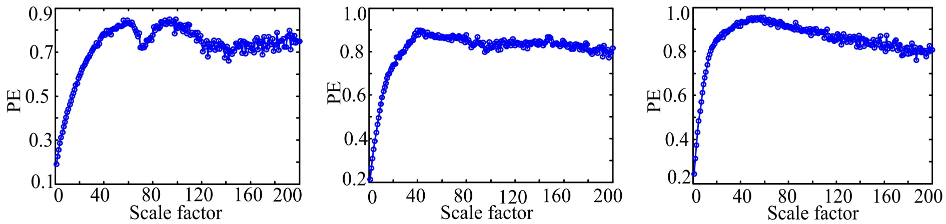
Step 2: Calculating MPE. It is defined as

$$\mathrm{MPE}(x, \tau, d) = \mathrm{PE}(y^\tau, d). \tag{3.4}$$

Set the max-scale $\tau$=200, the complexity of three-order, four-order and five-order Jerk chaotic sequences are calculated by multi-scale PE algorithm, and the results are shown in Fig.8. It shows that complexity value of the chaotic sequence increases rapidly, and then begins to flatten out with scale factor. The average of the PE values with different scales represents the average complexity. The larger of the average complexity value of the chaotic sequence is, the more complex of the chaotic attractor is. The average of the complexity of three-dimensional, four-dimensional and five-dimensional Jerk system with different scrolls is shown in Tabs.4, 5 and 6 respectively. It shows that the values of five-dimensional multi-scroll system are the largest, while the values of three-dimensional multi-scroll system are the smallest. It means that higher dimensional multi-scroll Jerk chaotic system is more complicated, and the result is similar with that of the largest Lyapunov exponent.

## 4. Circuit implementation

In this section, the circuit of four-dimensional multi-scroll chaotic system is designed. As the range of state variables beyond the scope of the amplifiers, all the




(a) three-order Jerk system (b) four-order Jerk system (c) five-order Jerk system

**Figure 8.** Multi-scale PE complexity of different order Jerk system.

**Table 4.** Complexity of three-dimensional multi-scroll system.

| $(M, N)$ | $6 \times M \times N$ | $8 \times M \times N$ | $10 \times M \times N$ |
|---|---|---|---|
| (1,1) | 0.6983 | 0.7127 | 0.7190 |
| (3,1) | 0.6977 | 0.7122 | 0.7143 |
| (3,3) | 0.6995 | 0.7232 | 0.7250 |

**Table 5.** Complexity of four-dimensional multi-scroll system

| $(M, N, P)$ | $6 \times M \times N \times P$ | $8 \times M \times N \times P$ | $10 \times M \times N \times P$ |
|---|---|---|---|
| (1,1,1) | 0.7930 | 0.8407 | 0.8309 |
| (3,1,1) | 0.7896 | 0.8358 | 0.8334 |
| (3,3,1) | 0.7842 | 0.8642 | 0.7950 |
| (3,3,3) | 0.7856 | 0.8077 | 0.8069 |

**Table 6.** Complexity of five-dimensional multi-scroll system.

| $(M, N, P, Q)$ | $6 \times M \times N \times P \times Q$ | $8 \times M \times N \times P \times Q$ | $10 \times M \times N \times P \times Q$ |
|---|---|---|---|
| (1,1,1,1) | 0.8392 | 0.8493 | 0.8477 |
| (2,1,1,1) | 0.8475 | 0.8389 | 0.8498 |
| (2,3,1,1) | 0.8531 | 0.8531 | 0.8439 |
| (2,2,1,1) | 0.8402 | 0.8398 | 0.8439 |
| (2,2,2,2) | 0.8410 | 0.8377 | 0.8665 |

variables are compressed. Let $x \to 8x$, $y \to 8y$, $z \to 8z$, $u \to 8u$, the transformed system is described as

$$\begin{cases} \dot{x} = y - g_1(y), \\ \dot{y} = z - g_2(z), \\ \dot{z} = u - g_3(u), \\ \dot{u} = f(x) - a_1 x - a_2(y - g_1(y)) - a_3(z - g_2(z)) - a_4(u - g_3(u)), \end{cases} \tag{4.1}$$

where $a_{1,2,3,4} = (7.9, 9.19, 4, 7.278)$, and the nonlinear function are $f(x) = 4[\mathrm{sgn}(x)+\mathrm{sgn}(x+1)+\mathrm{sgn}(x-1)]$, $g_1(y) = 0.5[1+\mathrm{sgn}(y-0.5)]$, $g_2(z) = 0.5[1+\mathrm{sgn}(z-0.5)]$, $g_3(u) = 0.5[1+\mathrm{sgn}(u-0.5)]$ for $4 \times 2 \times 2 \times 2$ scrolls attractor.

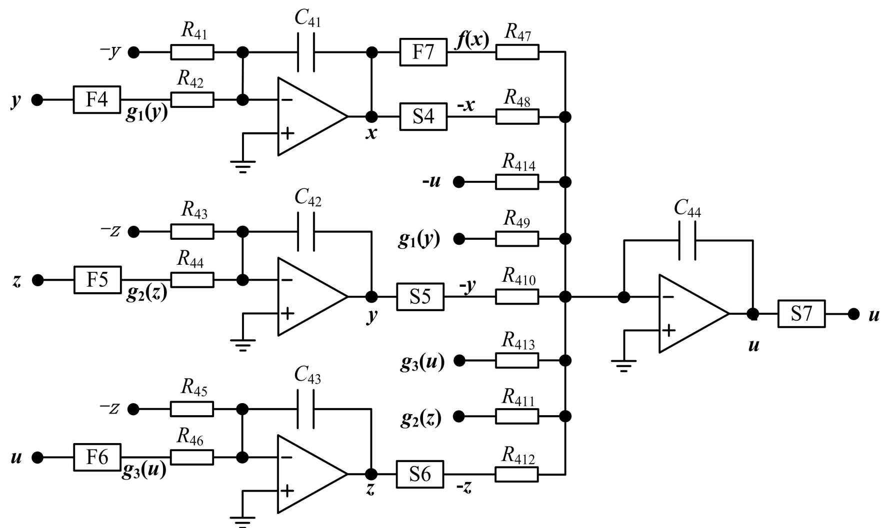
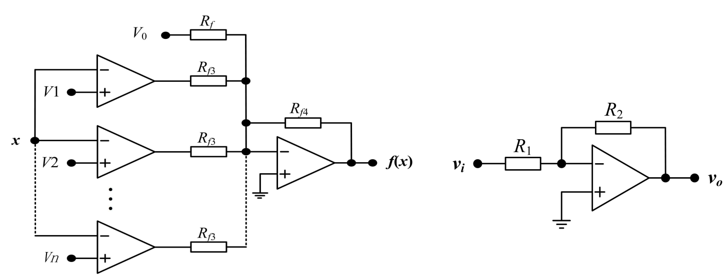



**Figure 9.** Main circuit of the 4-dimensional multi-scroll chaotic system.



(a) the step wave sequence generator (b) the reverse proportional amplifier

**Figure 10.** Circuit modules.

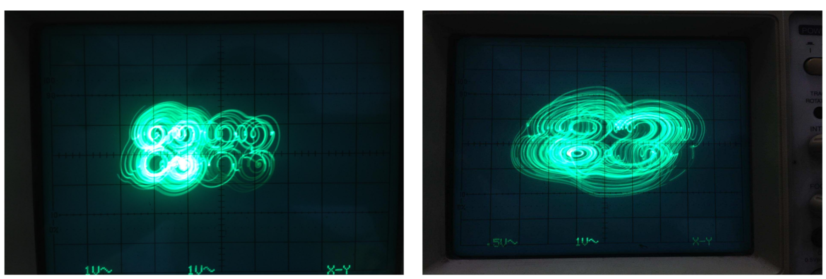
**Table 7.** Values of resistors and ceramic capacitors for $4 \times 2 \times 2 \times 2$ scrolls attractor

| Function | Values |
|---|---|
| $f(x)$ | $V_0$=$V_1$=0V, $V_2$=1V, $V_3$=-1V, $R_{f3}$=2\| $V_{sat1}$ \| kΩ, $R_{f4}$=8kΩ |
| $g_1(y)$ | $V_0$=-1V, $V_1$=0.5V, $R_{f3}$=2\| $V_{sat1}$ \| kΩ, $R_{f4}$=1kΩ, $R_f$=2kΩ |
| $g_2(z)$ | $V_0$=-1V, $V_1$=0.5V, $R_{f3}$=2\| $V_{sat1}$ \| kΩ, $R_{f4}$=1kΩ, $R_f$=2kΩ |
| $g_3(u)$ | $V_0$=-1V, $V_1$=0.5V, $R_{f3}$=2\| $V_{sat1}$ \| kΩ, $R_{f4}$=1kΩ, $R_f$=2kΩ |

According to Eq.(4.1), the design of the main circuit is shown in Fig.9. F4, F5, F6 and F7 are the step wave sequence generation. S3, S4, S5, S6 are the reverse proportional amplifier. The circuit for step wave sequence generation is designed in Fig.10(a). The input signals are $y$, $z$, $u$ and $x$, and the output signals are $g_1(y)$, $g_2(z)$, $g_3(u)$ and $f(x)$, respectively. The design of reverse proportional amplifier can refer to the circuit as shown in Fig.10(b). The input signals are $x$, $y$, $z$, $-u$, and the output signals are $-x$, $-y$, $-z$ and $u$, respectively. According to the circuit as shown in Fig.9, the circuit state equation is expressed by

$$\begin{cases} \dot{x} = \frac{1}{C_{41}}\left(\frac{y}{R_{41}} - \frac{g_1(y)}{R_{42}}\right), \\ \dot{y} = \frac{1}{C_{42}}\left(\frac{z}{R_{43}} - \frac{g_2(z)}{R_{44}}\right), \\ \dot{z} = \frac{1}{C_{43}}\left(\frac{u}{R_{45}} - \frac{g_3(u)}{R_{46}}\right), \\ \dot{u} = \frac{1}{C_{44}}\left(\frac{f(x)}{R_{47}} - \frac{x}{R_{48}} + \frac{g_1(y)}{R_{49}} - \frac{y}{R_{410}} + \frac{g_2(z)}{R_{411}} - \frac{z}{R_{412}} + \frac{g_3(u)}{R_{413}} - \frac{u}{R_{414}}\right), \end{cases} \tag{4.2}$$

where $R_{41} = R_{42} = R_{43} = R_{44} = R_{45} = R_{46} = R_{47} = 2k\Omega$, $R_{48} = 253\Omega$, $R_{49} = 218\Omega$, $R_{410} = 218\Omega$, $R_{411} = R_{412} = 500\Omega$, and $R_{413}$ and $R_{414}$ are adjustable resistors. $C_{41} = C_{42} = C_{43} = C_{44} = 10nF$. The values of resistors and ceramic capacitors in the nonlinear functions are illustrated in Tab.7 for $4 \times 2 \times 2 \times 2$ scrolls attractor. The operational amplifier is TL082. When $R_{413} = 270\Omega$ and $R_{414} = 274\Omega$, the $4 \times 2 \times 2 \times 2$ scrolls attractors captured by the analog oscilloscope are shown in Fig.11, and it proves the existence of the designed system and the physical realization. That shows the potential applications in the field of secure communication and information encryption.



(a) $x - y$ plane (b) $z - u$ plane

**Figure 11.** Phase portraits of the $4 \times 2 \times 2 \times 2$ scrolls attractor realized on circuits.

# 5. Conclusion

Multi-scroll chaotic attractors are generated based on Jerk system and hyperjerk system by employing nonlinear functions. By introducing different nonlinear control functions in the different dimensions, we obtained one-, two-, three-, four- five- and $n$-dimensional multi-scroll chaotic systems respectively. Performances of the $n$-dimensional multi-scroll Jerk chaotic attractors are investigated by means of phase diagrams, the largest Lyapunov exponent and PE complexity. We found that complexity does not change with scrolls for the same order Jerk system, and the higher order multi-scroll Jerk chaotic system is more complex. By employing modular design approach and operational amplifiers TL082, we designed the circuit of $4 \times 2 \times 2 \times 2$ scrolls attractors, including circuit modules of the control functions $g_1$, $g_2$, $g_3$ and $f$. Hardware implementation results match well with that of numerical simulation. Thus, we proposed a general approach for the generation of multi-dimensional multi-scroll chaotic attractors. Our next work will focus on applications of these multi-dimensional multi-scroll chaotic systems.

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