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GLOBAL ATTRACTABILITY AND PERMANENCE FOR A NEW STAGE-STRUCTURED DELAY IMPULSIVE ECOSYSTEM*

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Abstract In this paper, a new stage-structured delay ecosystem with impulsive effect is formulated and some dynamical properties of this system is investigated.By using comparison theorem and the stroboscopic technique, we prove the existence of the predator-extinction periodic solution of this system and obtain some sufficient conditions to guarantee the global attractivity of the prey-extinction periodic solution. In the final, we also obtain the permanence of this system. It should be pointed out that the new mathematical method used in this paper can also be applied to investigate such other ecosystems corresponding to both impulsive and delay differential equations.

Keywords Predator-prey system, impulsive effect, delay of digestion, global attractivity, permanence.

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1. Introduction

In the real world, the growth period of many species usually goes through two different life stages, namely: immature and mature. The immature needs τ units of time to become mature, which has now been considered as not to predate prey in the predator-prey system. Since time delays have important biological meanings in predator-prey models, many researchers, such as in paper [1,4–6,16,17,19,20,23,26,33] and their references, have studied the predator-prey models with time delays.

What need points out is, in the growth process of biological species, the releasing natural enemies and spraying pesticides always happen in a short time or instantaneously, which has now been showed as impulsive perturbations. Impulsive differential equations have been extensively used as models in biology and other sciences, with particular emphasis on population dynamics [3,12–14,21,25,27–29].

In this paper, time delay and impulse are both introduced into predator-prey models, which can enrich the biological background, but the system becomes non-autonomous, which will increase the difficulty of our research work. Research on impulsive differential equations with delay, mainly focuses on the theoretical analysis

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[11, 22, 24], however the results are relatively few. Recently , the predator-prey model with time delay and impulsive perturbations on predators have been studied [10,15,30,31], and the influence of stage structure and impulses is also demonstrated in detail.

To make the ecosystem more practical, a functional response was always introduced into predator-prey system to express the change in the density of prey attached per unit time per predator as the prey density changes. In 1965, Holling [7] proposed a functional response for vertebrate,which is called Holling III functional response. In this paper, we will generalize the Holling III functional response to be $\varphi(x) = \frac{x^{2k}}{1+mx^{2k}}$, here k is an arbitrary positive integer, m is a positive constant. In order to study the effect of spraying pesticides and releasing natural enemies at different moments, we consider in this paper the generalized impulsive stage-structured differential system:

$$
\begin{cases}
\left.\begin{aligned}
\dot{x}(t) &= rx(t)\ln\frac{K}{x(t)} - \frac{x^{2k}(t)y_2(t)}{1+mx^{2k}(t)},\\
\dot{y_1}(t) &= \frac{\lambda\beta x^{2k}(t)y_2(t)}{1+mx^{2k}(t)}\\
&\quad -e^{-d_1\tau}\frac{\lambda\beta x^{2k}(t-\tau)y_2(t-\tau)}{1+mx^{2k}(t-\tau)} - d_1y_1(t),\\
\dot{y_2}(t) &= e^{-d_1\tau}\frac{\lambda\beta x^{2k}(t-\tau)y_2(t-\tau)}{1+mx^{2k}(t-\tau)} - d_2y_2(t),
\end{aligned}\right\} & t \neq nT, (n+l-1)T,\\
\left.\begin{aligned}
x(t^+) &= (1-p)x(t),\\
y_1(t^+) &= y_1(t),\\
y_2(t^+) &= y_2(t),
\end{aligned}\right\} & t = (n+l-1)T,\\
\left.\begin{aligned}
x(t^+) &= x(t),\\
y_1(t^+) &= y_1(t)+\mu,\\
y_2(t^+) &= y_2(t),
\end{aligned}\right\} & t = nT,
\end{cases}
\tag{1.1}
$$

where $x(t)$ expresses the density of prey, $y_1(t), y_2(t)$ denote the densities of immature and mature predator, respectively. k is any a positive integer. constant r denotes the Gompertz intrinsic growth rate of the prey in the absence of the predator, K represents the environment carrying capacity of saturation level, λ is the rate of conversing prey into predator, β is the maximum numbers of the prey eaten by a predator per unit of time. τ represents a constant time for immature to become mature. d_1, d_2 denote the mortality rates of the immature predator and mature predator. $\mu \geq 0$ represents the releasing amount of the immature predator at $t = nT$, $p(0 \leq p < 1)$represents the fraction of prey that die due to pesticide at $t = (n+l-1)T, l$ is a constant $(0 < l < 1)$, and $n \in Z_+, Z_+ = \{1, 2, \ldots\}$. T denotes the period of the impulsive effect. For the econological significance of these constants $r, K, \lambda, \beta, \tau, d_1, d_2, \mu, p$, one can refer to [29].

The initial conditions of system (1.1) are

$$
(\phi_1, \phi_2, \phi_3) \in C([-\tau, 0], R^3_+),\ \phi_i(0) > 0, i = 1, 2, 3,\ R^3_+ = \{x \in R^3 : x \geq 0\}. \tag{1.2}
$$

In the present paper, we assume that the prey is pest, and the predator is the beneficial enemy to inhibit the pest. To avoid pesticide from harming beneficial enemy, one should choose an appropriate spraying pesticide periods the beneficial enemies are not exposed. This implies that spraying pesticides and releasing enemies should be at different fixed time. In this paper, we will use comparison theorem and the stroboscopic map to analyze the global attractivity and the permanence of system (1.1).

From the biological point of view, we only consider system (1.1) in the biological meaning region: $\Omega = \{(x, y_1, y_2) \mid x \geq 0, y_1 \geq 0, y_2 \geq 0\}$.

2. Preliminaries

For the convenience, let us first cite several known results, the proof in detail, please refer to the corresponding literature.

Lemma 2.1 (see [2, 18]). *Consider the impulsive differential inequations:*

$$\begin{cases} w'(t) \leq (\geq) p(t)w(t) + q(t), & t \neq t_k, \\ w(t_k^+) \leq (\geq) d_k w(t_k) + b_k, & t = t_k, k \in Z_+, \end{cases}$$

here $p(t), q(t) \in C(R_+, R), d_k \geq 0$ and b_k are constants.

Assume that:

(1) *The sequence $\{t_k\}$ satisfies $0 \leq t_0 < t_1 < t_2 < \cdots$, and $\lim_{k\to\infty} t_k = \infty$;*

(2) *Let $PC'(R_+, R)$ be the set of functions $\omega : R_+ \to R$, if $\omega \in PC'(R_+, R)$ and ω is continuously differential in $(t_{k-1}, t_k]$ and left-continuous at $t_k, k \in Z_+$, here $R_+ = [0, \infty)$, then*

$$\omega(t) \leq (\geq) w(t_0) \prod_{t_0<t_k<t} d_k \exp\left(\int_{t_0}^{t} p(s)ds\right) + \sum_{t_0<t_k<t} \left(\prod_{t_k<t_j<t} d_j \exp\left(\int_{t_k}^{t} p(s)ds\right)\right) b_k$$
$$+ \int_{t_0}^{t} \prod_{s<t_k<t} d_k \exp\left(\int_{s}^{t} p(\theta)d\theta\right) q(s)ds, \quad t \geq t_0.$$

Lemma 2.2 (see [32]). *Consider the delay equation*

$$x'(t) = ax(t-\tau) - bx(t) - cx^2(t),$$

where a, b, c and τ are positive constants, $x(t) > 0$ for $t \in [-\tau, 0]$. Then

(1) *If $a > b$, then $\lim_{t\to+\infty} x(t) = \frac{a-b}{c}$;*

(2) *If $a < b$, then $\lim_{t\to+\infty} x(t) = 0$.*

Lemma 2.3 (see [9]). *The impulsive equation*

$$\begin{cases} x'(t) = rx(t) \ln \dfrac{K}{x(t)}, & t \neq nT, \\ x(t^+) = (1-p)x(t), & t = nT \end{cases} \tag{2.1}$$

has a globally asymptotically stable positive periodic solution

$$x^*(t) = \exp\{\ln K + \frac{\ln(1-p)e^{-r(t-(n-1)T)}}{1 - e^{-rT}}\}, \quad t \in ((n-1)T, nT].$$

Where both r and K are positive constants, and $0 < p < 1$.

Lemma 2.4 (see [8]). *The impulsive system*

$$\begin{cases} x'(t) = c - wx(t), & t \neq nT, \\ x(t^+) = x(t) + \mu, & t = nT. \end{cases} \tag{2.2}$$

has a globally asymptotically stable positive periodic solution

$$x^*(t) = \frac{c}{w} + \frac{\mu e^{-w(t-nT)}}{1-e^{-wT}}, \quad nT < t \le (n+1)T.$$

Let $(x(t), y_1(t), y_2(t))$ be any a solution of system (1.1), we further have:

Lemma 2.5. *For any solution $(x(t), y_1(t), y_2(t))$ of system (1.1), there exists a positive constant M such that $x(t) \le K, y_1(t) \le M$ and $y_2(t) \le M$ for t large enough.*

Proof. Let $(x(t), y_1(t), y_2(t))$ be any a solution of system (1.1) with the initial conditions (1.2).

Firstly, from the first equation of system (1.1), we have that $\dot{x}(t) \le rx(t) \ln \frac{K}{x(t)}$, and then $\dot{x}|_{x=K} \le 0$. Note that $0 < p < 1$, which implies that $x(nT^+) < x(nT)$, and hence $x(t) \le K$ for t large enough.

Next, we will prove that $y_1(t) \le M, y_2(t) \le M$. Let

$$V(t) = \lambda\beta x(t) + y_1(t) + y_2(t).$$

It is clear that $V \in V_0$ (detailed proof can be found in [29]). Note that the function $x \ln \frac{Ke^{\frac{d}{r}}}{x}$ has a maximum value at point $Ke^{\frac{d}{r}-1}$. Then, by direct calculation, for $t \neq (n+l-1)T, nT$, we have that the upper right derivative of $V(t)$ along a solution of the impulsive differential system (1.1) is

$$\begin{aligned} V'(t) &= \lambda\beta r x(t) \ln \frac{K}{x(t)} - d_1 y_1(t) - d_2 y_2(t) \\ &\le \lambda\beta r x(t) \ln \frac{Ke^{\frac{d}{r}}}{x(t)} - dV(t) \\ &\le \lambda\beta r K e^{\frac{d}{r}-1} - dV(t) \\ &= M_0 - dV(t), \end{aligned}$$

where $d = \min(d_1, d_2)$, $M_0 = \lambda\beta r K e^{\frac{d}{r}-1}$.

When $t = nT$, we have that

$$V(nT^+) = \lambda\beta x(t) + y_1(t) + y_2(t) + \mu = V(t) + \mu.$$

When $t = (n+l-1)T$, we have that

$$V((n+l-1)T^+) = \lambda\beta(1-p)x(t) + y_1(t) + y_2(t) \le \lambda\beta x(t) + y_1(t) + y_2(t) = V(t).$$

Therefore, we only need consider the following auxiliary system

$$\begin{cases} U'(t) = -dU(t) + M_0, & t \neq nT, \\ U(t^+) = U(t) + \mu, & t = nT, \end{cases}$$

where $U(0^+) = V(0^+)$. It is easy to know that $V(t) \le U(t)$ for $t \ge 0$. When $t > 0$, from Lemma 2.1 we have for $t \to \infty$ that

$$\begin{aligned} V(t) &\le V(0^+)e^{-dt} + \frac{M_0}{d}(1-e^{-dt}) + \frac{\mu(e^{-d(t-T)} - e^{dT})}{1-e^{dT}} \\ &\longrightarrow \frac{M_0}{d} + \frac{\mu e^{dT}}{e^{dT}-1}. \end{aligned}$$

This implies that $V(t)$ is uniformly ultimately bounded. Let

$$M = \frac{M_0}{d} + \frac{\mu e^{dT}}{e^{dT} - 1} > 0,$$

then from the definition of $V(t)$, we can claim that $y_1(t) \le M, y_2(t) \le M$ for t large enough. $\square$

3. Global Attractivity and Permanence

Firstly, it is easy to know that $y_2^* = 0$ is a trivial solution for the variable $y_2(t)$, as it leaves $y_2'(t) = 0$. This motivates us to demonstrate the following result on the existence and global attractivity of the predator-extinction periodic solution $(x^*(t), y_1^*(t), 0)$ to system (1.1), where

$$x^*(t) = \exp\{\ \ln K + \frac{\ln(1-p)}{1 - e^{-rT}}\}, \qquad y_1^*(t) = \frac{ue^{-d_1(t-nT)}}{1 - e^{-d_1 T}}.$$

Theorem 3.1. *The predator-extinction periodic solution* $(x^*(t), y_1^*(t), 0)$ *of system (1.1) is globally attractive, provided that* $\frac{\lambda\beta K^{2k} e^{d_1\tau} e^{\frac{2kln(1-p)}{e^{rT}-1}}}{1+mK^{2k} e^{\frac{2kln(1-p)}{e^{rT}-1}}} < d_2$ *holds.*

Proof. Assume that $(x(t), y_1(t), y_2(t))$ is any a solution of (1.1). From the first and forth equations of (1.1), we can obtain that

$$\begin{cases} x'(t) \le rx(t)\ln\dfrac{K}{x(t)}, & t \ne (n+l-1)T, \\ x(t^+) = (1-p)x(t), & t = (n+l-1)T. \end{cases}$$

Consider the following auxiliary system

$$\begin{cases} u_1'(t) = ru_1(t)\ln\dfrac{K}{u_1(t)}, & t \ne (n+l-1)T, \\ u_1(t^+) = (1-p)u_1(t), & t = (n+l-1)T. \end{cases}$$

By lemma 2.3, we can obtain the positive periodic solution of system (2.1)

$$u_1^*(t) = \exp\{\ln K + \frac{\ln(1-p)e^{-r[t-(n+l)T]}}{1 - e^{-rT}}\} = x^*(t), \qquad t \in ((n+l)T, (n+l+1)T],$$

and it is globally asymptotically stable. According to the comparison theorem of impulsive equation [27], there will exist a positive integer n_1 and a positive constant ε_1 small enough such that for $t > (n_1 + l)T$

$$x(t) < u_1^*(t) + \varepsilon_1 = x^*(t) + \varepsilon_1 = \exp\{\ln K + \tfrac{\ln(1-p)e^{-rT}}{1-e^{-rT}}\} + \varepsilon_1 \triangleq h. \tag{3.1}$$

From (2.1) and the third equation of system (1.1), we obtain that

$$y_2'(t) \le e^{-d_1\tau}\frac{\lambda\beta h^{2k} y_2(t-\tau)}{1 + mh^{2k}} - d_2 y_2, \qquad t > (n_1 + l)T + \tau.$$

Then we consider the following auxiliary system

$$u_2'(t) = e^{-d_1\tau}\frac{\lambda\beta h^{2k}u_2(t-\tau)}{1+mh^{2k}} - d_2u_2.$$

Note that $\frac{x^{2k}}{1+mx^{2k}}$ is monotonically increasing with respect to x, and

$$\frac{e^{d_1\tau}\lambda\beta K^{2k}e^{\frac{2k\ln(1-p)}{e^{rT}-1}}}{1+mK^{2k}e^{\frac{2k\ln(1-p)}{e^{rT}-1}}} < d_2,$$

we can choose $\varepsilon_1 > 0$ small enough such that

$$\frac{e^{d_1\tau}\lambda\beta(Ke^{\frac{\ln(1-p)}{e^{rT}-1}}+\varepsilon_1)^{2k}}{1+m(Ke^{\frac{\ln(1-p)}{e^{rT}-1}}+\varepsilon_1)^{2k}} < d_2,$$

this means that $e^{d_1\tau}\frac{\lambda\beta\eta^{2k}}{1+m\eta^{2k}} < d_2$. From Lemma 2.2, we have that $\lim_{t\to\infty} u_2(t) = 0$. Note that $y_2(t) = u_2(t) = \varphi_3(t) > 0$, for all $t \in [-\tau, 0]$, by the comparison theorem of impulsive differential equation, we have that

$$\lim_{t\to\infty} y_2(t) \le \lim_{t\to\infty} u_2(t) = 0.$$

Without loss of generality, we choose $\varepsilon > 0$ small enough, such that

$$0 < y_2(t) < \varepsilon. \tag{3.2}$$

By the first equation of (1.1) and Lemma 2.5, we get

$$x'(t) \ge rx(t)\ln\frac{Ke^{-\frac{K^{2k-1}\varepsilon}{r}}}{x(t)}.$$

By the same way as above, we further consider the following auxiliary system

$$\begin{cases} u_3'(t) = ru_3(t)\ln\dfrac{Ke^{-\frac{-K^{2k-1}\varepsilon}{r}}}{u_3(t)}, & t \ne (n+l-1)T, \\ u_3(t^+) = (1-p)u_3(t), & t = (n+l-1)T. \end{cases} \tag{3.3}$$

From Lemma 2.3, we obtain the following unique positive periodic solution of (3.3)

$$z_3^*(t) = \exp\{\ln Ke^{-\frac{K^{2k-1}\varepsilon}{r}} + \frac{\ln(1-p)e^{-r[t-(n+l)T]}}{1-e^{-rT}}\}, \quad t \in ((n+l)T, (n+l+1)T].$$

Then from comparison theorem again, we have that, for any an arbitrarily small positive constant $\varepsilon_2 > 0$, there exists a positive integer n_2 such that $x(t) > u_3^*(t) - \varepsilon_2$ for $t > (n_2+l)T$. Now, let $\varepsilon \to 0$, then $u_3^*(t) \to x^*(t)$. Therefore, from (3.1), we can obtain that $x(t) < u_3^*(t) + \varepsilon_2$ for t sufficiently large , this means that $x(t) \to x^*(t)$ when $t \to \infty$.

By (3.1), (3.2) and the second and eighth equations of system (1.1), we get

$$\begin{cases} y_1'(t) \le \lambda\beta h^{2k}\varepsilon - d_1y_1(t), & t \ne nT, \\ y_1(t^+) = y_1(t) + \mu, & t = nT. \end{cases} \tag{3.4}$$

Consider the auxiliary system of (3.4)

$$\begin{cases} u_4'(t) = \lambda\beta h^{2k}\varepsilon - d_1 u_4(t), & t \neq nT, \\ u_4(t^+) = u_4(t) + \mu, & t = nT. \end{cases} \tag{3.5}$$

From Lemma 2.4, we have

$$u_4^*(t) = \frac{\lambda\beta h^{2k}\varepsilon}{d_1} + \frac{\mu e^{-d_1(t-nT)}}{1-e^{-d_1 T}}, \qquad t \in (nT, (n+1)T],$$

which is a unique positive periodic solution of system (3.5). By the comparison theorem, we have that, for any an arbitrarily small positive constant ε_3, there exists a positive integer n_3 such that $y_1(t) \leq u_4^*(t) + \varepsilon_3$ for $t > n_3 T$. Let $\varepsilon \to 0$, then $u_4^*(t) \to y_1^*(t)$ and

$$y_1(t) \leq y_1^*(t) + \varepsilon_3. \tag{3.6}$$

Hence, it follows from system (1.1) and (3.1), (3.2), that

$$\begin{cases} y_1'(t) \geq -e^{-d_1\tau}\lambda\beta h^{2k}\varepsilon - d_1 y_1(t), & t \neq nT, \\ y_1(t^+) = y_1(t) + \mu, & t = nT. \end{cases} \tag{3.7}$$

Consider the auxiliary system of (3.7)

$$\begin{cases} u_5'(t) = -e^{-d_1\tau}\lambda\beta h^{2k}\varepsilon - d_1 u_5(t), & t \neq nT, \\ u_5(t^+) = u_5(t) + \mu, & t = nT. \end{cases} \tag{3.8}$$

Again from Lemma 2.4, we have that (3.8) has a unique positive periodic solution

$$u_5^*(t) = \frac{-e^{-d_1\tau}\lambda\beta h^{2k}\varepsilon}{d_1} + \frac{\mu e^{-d_1(t-nT)}}{1-e^{-d_1 T}}, \qquad t \in (nT, (n+1)T].$$

Then, we have that, for $\varepsilon_4 > 0$ small enough, there exists $n_4 > 0$ such that $y_1(t) \geq u_5^*(t) - \varepsilon_4$ for $t > n_4 T$. Let $\varepsilon \to 0$, then $u_5^*(t) \to y_1^*(t)$ and

$$y_1(t) \geq y_1^*(t) - \varepsilon_4. \tag{3.9}$$

Finally, from (3.6) and (3.9), we can obtain that $y_1(t) \to y_1^*(t)$ as $t \to \infty$. □

Next, to facilitate the following study, we first give the following definition of permanence.

Definition 3.1. Assume that, for each positive solution $(x(t), y_1(t), y_2(t))$ of system (1.1) with the initial values $x(0^+) > 0,\ y_1(0^+) > 0, y_2(0^+) > 0$, there exist $m_1 > 0$ and $m_2 > 0$, and a finite time T_0 such that the following inequalities:

$$m_1 \leq x(t), y_1(t), y_2(t) \leq m_2$$

hold for all $t \geq T_0$, then we can say that system (1.1) is uniformly permanent.

Now we give the study of permanence of system (1.1).

Theorem 3.2. *Assume that the condition $e^{-d_1\tau}\frac{\lambda\beta\delta^{2k}}{1+m\delta^{2k}} \geq d_2$ holds, then system (1.1) is uniformly permanent, where δ is given in (3.14).*

Proof. According to Definition 3.1, We only need to find two positive constants m_1 and M such that $m_1 \leq x(t) \leq M, m_1 \leq y_1(t) \leq M, m_1 \leq y_2(t) \leq M$.

From Lemma 2.5, we can obtain $x(t) \leq K, y_1(t) \leq M, y_2(t) \leq M$. Therefore, in the following, we only need to find m_1, satisying $m_1 < M$ such that $x(t) \geq m_1, y_1(t) \geq m_1, y_2(t) \geq m_1$.

By the first equation of system (1.1) we can obtain

$$x'(t) \geq rx(t) \ln \frac{Ke^{-\frac{K^{2k-1}M}{r}}}{x(t)}.$$

Consider the auxiliary system

$$\begin{cases} u_6'(t) = ru_6(t) \ln \dfrac{Ke^{-\frac{K^{2k-1}M}{r}}}{u_6(t)}, \quad t \neq (n+l-1)T, \\ u_6(t^+) = (1-p)u_6(t), \quad t = (n+l-1)T. \end{cases} \tag{3.10}$$

From Lemma 2.3, we have

$$u_6^*(t) = \exp\{\ln Ke^{-\frac{K^{2k-1}M}{r}} + \frac{\ln(1-p)e^{-r[t-(n+l)T]}}{1-e^{-rT}}\}, \quad t \in ((n+l)T, (n+l+1)T],$$

which is the unique positive periodic solution of system (3.10).

Again from the comparison theorem, we have that, for $\varepsilon_5 > 0$ small enough, there exists a $n_5 > 0$ such that for $t > (n_5 + l)T$, we have

$$x(t) \geq u_6^*(t) - \varepsilon_5 \geq \exp\{\ln Ke^{-\frac{K^{2k-1}M}{r}} + \frac{\ln(1-p)e^{-r[t-(n+l)T]}}{1-e^{-rT}}\} - \varepsilon_5 \triangleq U. \tag{3.11}$$

On the other hand, let

$$V(t) = y_2(t) + e^{-d_1\tau} \int_{t-\tau}^{t} \frac{\lambda\beta x^{2k}(\theta)y_2(\theta)}{1+mx^{2k}(\theta)} d\theta.$$

Then from system (1.1), we have that

$$V'(t)|_{(1.1)} = [e^{-d_1\tau} \frac{\lambda\beta x^{2k}(t)}{1+mx^{2k}(t)} - d_2]y_2(t). \tag{3.12}$$

Note that $e^{-d_1\tau}\frac{\lambda\beta\delta^{2k}}{1+m\delta^{2k}} \geq d_2$, there exists $\varepsilon_6 > 0$ small enough such that

$$e^{-d_1\tau} \frac{\lambda\beta(\delta-\varepsilon_6)^{2k}}{1+m(\delta-\varepsilon_6)^{2k}} \geq d_2. \tag{3.13}$$

Now, let us choose a positive constant y_2^*, such that

$$y_2^* \leq \frac{r}{2K}[\frac{2\ln(1-p)}{1-e^{-rT}} + \ln(K^2(\frac{\lambda\beta}{d} - m))],$$

then we can claim that for any $t_0 > 0$, the inequality $y_2(t) < y_2^*$ is not always holds for all $t > t_0$.

If not, there will exist a $t_0 > 0$ such that $y_2(t) < y_2^*$ for all $t > t_0$. Then from the first equation of system (1.1), we can obtain that

$$\begin{cases} x'(t) \geq rx(t) \ln \dfrac{Ke^{-\frac{K^{2k-1}y_2^*}{r}}}{x(t)}, & t \neq (n+l-1)T, \\ x(t^+) = (1-p)x(t), & t = (n+l-1)T. \end{cases}$$

In view of Lemma 2.3, we get that

$$x(t) \geq u_7^*(t) - \varepsilon_6 \geq \exp\{\ln Ke^{-\frac{K^{2k-1}y_2^*}{r}} + \frac{\ln(1-p)}{1-e^{-rT}}\} - \varepsilon_6 \triangleq \delta - \varepsilon_6, \tag{3.14}$$

where $u_7^*(t) = \exp\{\ln Ke^{-\frac{K^{2k-1}y_2^*}{r}} + \frac{\ln(1-p)e^{-r[t-(n+l)T]}}{1-e^{-rT}}\}$ is the unique positive periodic solution of the following system:

$$\begin{cases} u_7'(t) = ru_7(t) \ln \dfrac{Ke^{-\frac{K^{2k-1}y_2^*}{r}}}{u_7(t)}, & t \neq (n+l-1)T, \\ u_7(t^+) = (1-p)u_7(t), & t = (n+l-1)T. \end{cases}$$

and it is globally asymptotically stable.

From (3.12) and the inequality (3.14), we can have for all $t \geq t_1$ that

$$V'(t) \geq [e^{-d_1\tau} \frac{\lambda\beta(\delta-\varepsilon_6)^{2k}}{1+m(\delta-\varepsilon_6)^{2k}} - d_2]y_2(t).$$

Let

$$y_2^m = \min_{t \in [t_1, t_1+\tau]} y_2(t).$$

Next we can prove, for all $t \geq t_1$, that $y_2(t) \geq y_2^m$. If not, there will exist a $T_0 > 0$ such that $y_2(t) \geq y_2^m$ for $t_1 \leq t \leq t_1 + \tau - T_0$, while $y_2(t_1 + \tau - T_0) = y_2^m$ and $y_2'(t_1 + \tau - T_0) < 0$. Then, from (3.13), (3.14) and the first equation of system (1.1), we have that

$$y_2'(t_1 + \tau - T_0) \geq [e^{-d_1\tau} \frac{\lambda\beta(\delta-\varepsilon_6)^{2k}}{1+m(\delta-\varepsilon_6)^{2k}} - d_2]y_2^m > 0,$$

this leds to a contradiction. Therefore, we have

$$V'(t) \geq [e^{-d_1\tau} \frac{\lambda\beta(\delta-\varepsilon_6)^{2k}}{1+m(\delta-\varepsilon_6)^{2k}} - d_2]y_2(t) \geq [e^{-d_1\tau} \frac{\lambda\beta(\delta-\varepsilon_6)^{2k}}{1+m(\delta-\varepsilon_6)^{2k}} - d_2]y_2^m > 0.$$

Which means that

$$\lim_{t \to \infty} V(t) = \infty. \tag{3.15}$$

On the other hand, from the definition of $V(t)$, we can easily get

$$V(t) < M(1 + \lambda\beta\tau K^{2k} e^{-d_1\tau}). \tag{3.16}$$

Obviously, (3.15) has a contradiction to (3.16), which ends the proof the claim.

According to the above claim, for all t large enough, we only need to consider the following two cases:

(a) $y_2(t) \geq y_2^*$;
(b) $y_2(t)$ oscillates about y_2^* .
Let

$$\triangle = \min\{\frac{y_2^*}{2}, \quad y_2^* e^{-d_2\tau}\}.$$

In the following, we will prove that $y_2(t) \geq \triangle$ for all t sufficiently large.

It is easy to see that the conclusion holds in case (a). For case (b), set $t^* > t_0$ and $\xi > 0$ such that $y_2(t^*) = y_2(t^* + \xi) = y_2^*$ and $y_2(t) < y_2^*$ for all $t^* < t < t^* + \xi$, where t^* is large enough, such that

$$x(t) > \delta \quad \text{for} \quad t^* < t < t^* + \xi.$$

Note that $y_2(t)$ is not affected by impulses and it is also continuous and bounded, this implies that $y_2(t)$ is uniformly continuous. Therefore, we can obtain that there exists a constant T_1, such that $y_2(t) > \frac{y_2^*}{2}$ for $t^* < t < t^* + T_1$, where $0 < T_1 < \tau$ and T_1 is dependent of the choice of t^*. If $\xi < T_1$, the conclusion holds, and nothing is needed to be proved. If $T_1 < \xi < \tau$, note that

$$y_2'(t) > -d_2 y_2(t),$$

and $y_2(t^*) = y_2^*$, then it is easy to prove that $y_2(t) > y_2^* e^{-d_2\tau}$ for $t \in [t^*, t^* + \tau]$. By the same method used as the proof for the above claim, we can obtain that $y_2(t) > y_2^* e^{-d_2\tau}$ for $t \in [t^*, t^* + \xi]$. Note that the interval $t \in [t^*, t^* + \xi]$ is chosen arbitrarily (here, t^* is large enough),then we can obtain that for all snfficiently large t

$$y_2(t) \geq \triangle. \tag{3.17}$$

Therefore, from the second equation of system (1.1), we have that

$$y_1'(t) \geq \frac{\lambda\beta\delta^{2k}\triangle}{1 + m\delta^{2k}} - e^{-d_1\tau}\frac{\lambda\beta K^{2k} M}{1 + mK^{2k}} - d_1 y_1(t).$$

Similarly, let us consider the following auxiliary system

$$\begin{cases} u_8'(t) = \dfrac{\lambda\beta\delta^{2k}\triangle}{1 + m\delta^{2k}} - e^{-d_1\tau}\dfrac{\lambda\beta K^{2k} M}{1 + mK^{2k}} - d_1 u_8(t), & t \neq nT, \\ u_8(t^+) = u_8(t) + \mu, \quad t = nT. \end{cases} \tag{3.18}$$

Equipped with Lemma 2.4 and the comparison theorem , we have that, for the constant $\varepsilon_7 > 0$ small enough, there exists a $n_6 > 0$ such that for $t > n_6 T$

$$y_1(t) \geq u_8^*(t) - \varepsilon_7 \geq \frac{\lambda\beta\delta^{2k}\triangle}{(1+m\delta^{2k})d_1} - e^{-d_1\tau}\frac{\lambda\beta K^{2k} M}{(1+mK^{2k})d_1} + \frac{\mu e^{-d_1 T}}{1-e^{-d_1 T}} - \varepsilon_7 \triangleq R, \tag{3.19}$$

where

$$u_8^*(t) = \frac{\lambda\beta\delta^{2k}\triangle}{(1 + m\delta^{2k})d_1} - e^{-d_1\tau}\frac{\lambda\beta K^{2k} M}{(1 + mK^{2k})d_1} + \frac{\mu e^{-d_1(t-nT)}}{1 - e^{-d_1 T}}$$

is a unique positive periodic solution of system (3.18), and it is globally asymptotically stable.

Finally, let

$$m = \min\{U, \Delta, R\},$$

from the definitions of(3.11), (3.17) and (3.19), we can easily obtain that $x(t) \geq m, y_1(t) \geq m, y_2(t) \geq m$. $\square$

4. Numerical simulation and discussion

In order to verify the obtained results above, we present some numerical examples in this section, and illustrate its biological meaning.

Example 4.1. Let the initial values be $x(0) = 20, y_1(0) = 10, y_2(0) = 5$, and set $k = 2, K = 30, T = 2, l = 0.5, \mu = 0, p = 0, r = 3, m = 3, \lambda\beta = 0.01, d_1 = 0.3, d_2 = 0.2, \tau = 0.1$. It is shown that there is no impulse. The parameters cannot satisfy the condition of Theorem 3.1, and the predators decrease to zero rapidly (Fig.1).

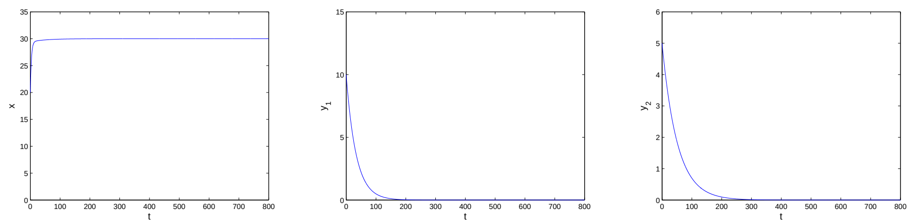

Figure 1. Time series of the prey (pest) population (x), immature predator(y_1) and mature predator(y_2) population of system (1.1) (no impulse) with $x(0) = 20, y_1(0) = 10, y_2(0) = 5$ and $k = 2, K = 30, T = 2, l = 0.5, \mu = 0, p = 0, r = 3, m = 3, \lambda\beta = 0.01, d_1 = 0.3, d_2 = 0.2, \tau = 0.1$.

Example 4.2. Let $\mu = 10, p = 0.6$ and keep the others fixed, then the conditions of Theorem 3.1 hold. From Fig.2, we can see that the mature predator-extinction periodic solution is global attractivity.

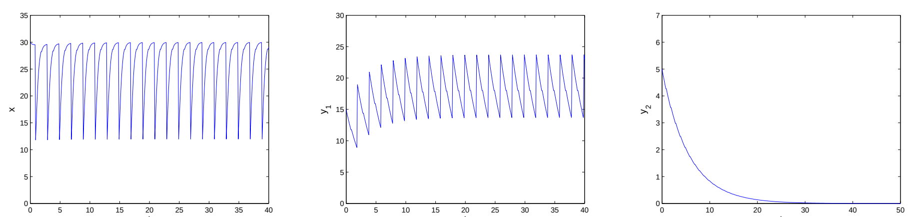

Figure 2. Time series of the prey (pest) population (x), immature predator(y_1) and mature predator(y_2) population of system (1.1) with $x(0) = 20, y_1(0) = 10, y_2(0) = 5$ and $k = 2, K = 30, T = 2, l = 0.5, \mu = 10, p = 0.6, r = 3, m = 3, \lambda\beta = 0.01, d_1 = 0.3, d_2 = 0.2, \tau = 0.1$.

Example 4.3. Let $x(0) = 30, y_1(0) = 15, y_2(0) = 5$ and $k = 2, K = 30, T = 2, l = 0.5, \lambda\beta = 0.25, \mu = 0, p = 0, r = 3, m = 0.84, d_1 = 0.3, d_2 = 0.2, \tau = 0.1$, the parameters cannot satisfy Theorem 3.2. In this case with no impulse, the predator decreases sharply(see Fig.3), this phenomenon will be harmful to the stability of ecology, which is not what we wanted.

Example 4.4. Let $\mu = 10, p = 0.6$, and keep the others fixed, the parameters will satisfy the condition of Theorem 3.2. From Fig.4, we can see that system (1.1) is permanent, which means that the prey and predator can coexist, this is what we wanted.

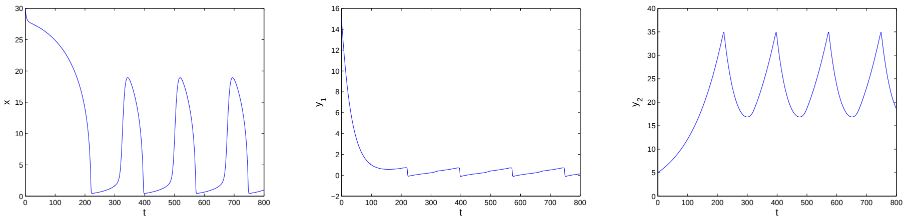

Figure 3. Time series of the prey (pest) population (x), immature predator(y_1) and mature predator(y_2) population of (1.1) (no impulse) with $x(0) = 30, y_1(0) = 15, y_2(0) = 5$ and $k = 2, K = 30, T = 2, l = 0.5, \lambda\beta = 0.25, \mu = 0, p = 0, r = 3, m = 0.84, d_1 = 0.3, d_2 = 0.2, \tau = 0.1$.

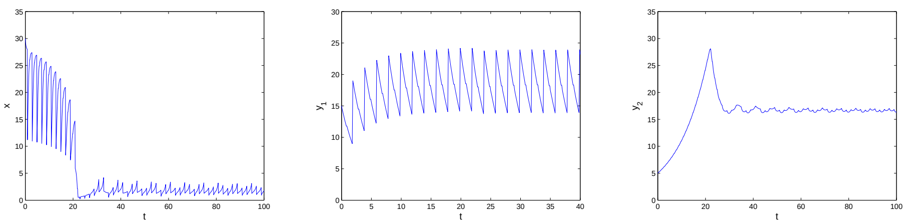

Figure 4. Time series of the prey (pest) population (x), immature predator(y_1) and mature predator(y_2) population of system (1.1) with $x(0) = 30, y_1(0) = 15, y_2(0) = 5$ and $k = 2, K = 30, T = 2, l = 0.5, \mu = 10, p = 0.6, r = 3, m = 0.84, \lambda\beta = 0.25, d_1 = 0.3, d_2 = 0.2, \tau = 0.1$.

In the present paper,we have discussed a new stage-structured delay ecosystem with impulsive effect. We have obtained not only the existence but also the global attractivity of the predator-extinction periodic solution of this system. Moreover, the permanence of this system is also obtained.

When there is no impulse, system (1.1) will be a typical prey-predator model. From Fig.1, we can see that when the density of prey (pest) exceed a certain number, the environment will be affected by it, and the number of immature predator decreases sharply in a short period of time even to zero, which is not well to the stability of ecology.

To suppress the abundance of the pest, we can choose a pulse control strategy, using a combination of biological and chemical strategies. Theorem 3.2 tell us that the impulsive period T can be determined by the effect p, which means that the densities of the pest can be controlled for suppressing the abundance of the pest, by suitable pesticide input and releasing natural enemies.

Remark 4.1. In this paper,we consider the generalized Holling III functional response $\varphi(x) = \frac{x^{2k}}{1+mx^{2k}}$. Note that the function $\varphi(x)$ is monotonous with respect to x,which was used in the proof of the Theorems. So if it cannot hold, the proof will not be completed.

The research in this paper is expected to provide valuable supplement for the studying of the ecosystems. the new mathematical method used in this paper is an efficient way to study impulsive and delay differential equations. The approach can be devoted to studying such other ecosystems corresponding to both impulsive and delay differential equations. We would like to do some valuable research about the subject.

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