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INTERACTION SOLUTIONS AND ABUNDANT EXACT SOLUTIONS FOR THE NEW (3+1)-DIMENSIONAL GENERALIZED KADOMTSEV-PETVIASHVILI EQUATION IN FLUID MECHANICS

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Abstract In this work, we present the interaction solutions and abundant exact solutions for the new (3+1)-dimensional generalized Kadomtsev-Petviashvili equation based on the Hirota's bilinear form and a direct function. The obtained interaction solutions contain the interaction between the rational function and the tanh function and the interaction between the rational function and the cos function. The dynamical properties of these resulting solutions are analyzed and shown in three-dimensional plots, corresponding contour graphs and plane figures.

Keywords Interaction solutions, bilinear form, exact solutions, dynamical properties.

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1. Introduction

Profound changes happening in modern natural science, nonlinear science through the mathematical science, life science, space science and earth science, becomes an important frontier field for contemporary scientific research. Solitary wave and soliton are one of the important concepts to promote the development of nonlinear science [1–5]. Soliton originated in solitary wave, it has been applied in a series of high tech fields such as nonlinear optical, magnetic flux sub-device, biology, plasma and optical fiber isolation, and many of these applications can be represented by nonlinear partial differential equation (NPDE) [6–14]. So it is necessary to study the solitary wave solution or the soliton solution for the NPDE. Various method are proposed by the researchers [15–25].

Recently, the rational function solution called lump solution as a kind of soliton solution has attracted the attention of many scholars, especially interaction solution between the rational function and other functions, such as trigonometric functions, exponential functions, hyperbolic functions, and so on [26, 27, 30–32]. In

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this paper, we aim to research the new $(3+1)$-dimensional generalized Kadomtsev-Petviashvili(ngKP) equation [28].

$$u_{ty}+u_{tx}+u_{tz}-u_{zz}+3(u_x\,u_y)_x+u_{xxxy}=0. \tag{1.1}$$

The multiple soliton solutions were investigated by Wazwaz in Ref. [28]. New exact periodic solitary-wave solutions were obtained in Ref. [29]. As far as we know, interaction solutions among the rational function, the tanh function and the cos function have not been found in other literatures.

The organization of this paper is as follows: Section 2 lists the Hirota's bilinear form and obtains the interaction solutions between the rational function and the tanh function. Section 3 presents the interaction solutions between the rational function and the cos function. Section 4 obtains the abundant exact solutions. The dynamical properties of these obtained solutions are analyzed and shown in some corresponding figures. Finally, the conclusions are presented.

2. Hirota's bilinear form and interaction solutions between the rational function and the tanh function

Substituting $u=2[ln\zeta]_x$ into Eq. (1.1) and simplifying, we have the following Hirota's bilinear form [29]

$$(D_tD_x+D_tD_y+D_tD_z+D_x^3D_y-D_z^2)\zeta\cdot\zeta=0. \tag{2.1}$$

This is equivalent to:

$$\begin{aligned}&(\zeta_{xxxy}+\zeta_{tx}+\zeta_{ty}+\zeta_{tz}-\zeta_{zz})\,\zeta-3\zeta_{xxy}\zeta_x+3\zeta_{xy}\,\zeta_{xx}\\&-\zeta_y\,\zeta_{xxx}-\zeta_t\zeta_x-\zeta_t\zeta_y-\zeta_t\zeta_z+\zeta_z^2=0.\end{aligned} \tag{2.2}$$

Considering Eq. (2.2) has the following interaction solutions between the rational function and the tanh function

$$\begin{aligned}&\varrho=\iota_1x+\iota_2y+\iota_3z+\iota_4t+\iota_5,\\&\varsigma=\iota_6x+\iota_7y+\iota_8z+\iota_9t+\iota_{10},\\&\zeta_1=\varrho^2+\varsigma^2+k\tanh(j_1x+j_2y+j_3z+j_4t)+\iota_{11},\end{aligned} \tag{2.3}$$

where $\iota_i(1\le i\le 11)$ and $j_i(1\le i\le 4)$ are undetermined constants. Substituting Eq. (2.3) into Eq. (2.2), we have the following relational expression

$$\begin{aligned}&\iota_2=\iota_7=j_1=0,\iota_8=\frac{\iota_3\iota_6}{\iota_1},\iota_4=\frac{\iota_3^2}{\iota_1+\iota_3},j_2=\frac{\iota_1j_3}{\iota_3},\\&j_4=\frac{\iota_3j_3}{\iota_1+\iota_3},\iota_9=\frac{\iota_3^2\iota_6}{\iota_1^2+\iota_3\iota_1},\end{aligned} \tag{2.4}$$

where $\iota_1\ne0,\iota_3\ne0,\iota_1+\iota_3\ne0$. Therefore, we have

$$\begin{aligned}\zeta_1=&\,k\tanh(\frac{\iota_3j_3t}{\iota_1+\iota_3}+\frac{\iota_1j_3y}{\iota_3}+j_3z)+(\iota_5+\frac{\iota_3^2t}{\iota_1+\iota_3}+\iota_1x+\iota_3z)^2\\&+(\iota_{10}+\frac{\iota_6\iota_3^2t}{\iota_1^2+\iota_3\iota_1}+\iota_6x+\frac{\iota_6\iota_3z}{\iota_1})^2+\iota_{11}.\end{aligned} \tag{2.5}$$

Substituting Eq. (2.5) into the transformation $u = 2[ln\xi]_x$, we derive the interaction solutions of the ngKP equation

$$
\begin{aligned}
u = {} & [2[2\iota_1(\iota_5 + \frac{\iota_3^2 t}{\iota_1+\iota_3} + \iota_1 x + \iota_3 z) + 2\iota_6(\iota_{10} + \frac{\iota_6\iota_3^2 t}{\iota_1^2+\iota_3\iota_1} + \iota_6 x \\
& + \frac{\iota_6\iota_3 z}{\iota_1})]]/[\iota_{11} + k\tanh(\frac{\iota_3 j_3 t}{\iota_1+\iota_3} + \frac{\iota_1 j_3 y}{\iota_3} + j_3 z) + (\iota_5 + \frac{\iota_3^2 t}{\iota_1+\iota_3} \\
& + \iota_1 x + \iota_3 z)^2 + (\iota_{10} + \frac{\iota_6\iota_3^2 t}{\iota_1^2+\iota_3\iota_1} + \iota_6 x + \frac{\iota_6\iota_3 z}{\iota_1})^2]. \qquad (2.6)
\end{aligned}
$$

The dynamical properties for interaction solution between rational function and tanh function are displayed in Fig. 1. Figures 1(a), 1(b) and 1(c) list the 3d graphs in the (y, z)-plane when $x = -5, 0, 5$ respectively, Figures 1(d), 1(e) and 1(f) present the corresponding contour diagrams of Figures 1(a), 1(b) and 1(c), and Figures 1(g), 1(h) and 1(i) show the corresponding plane figures of Figures 1(a), 1(b) and 1(c) with $y = -8$.

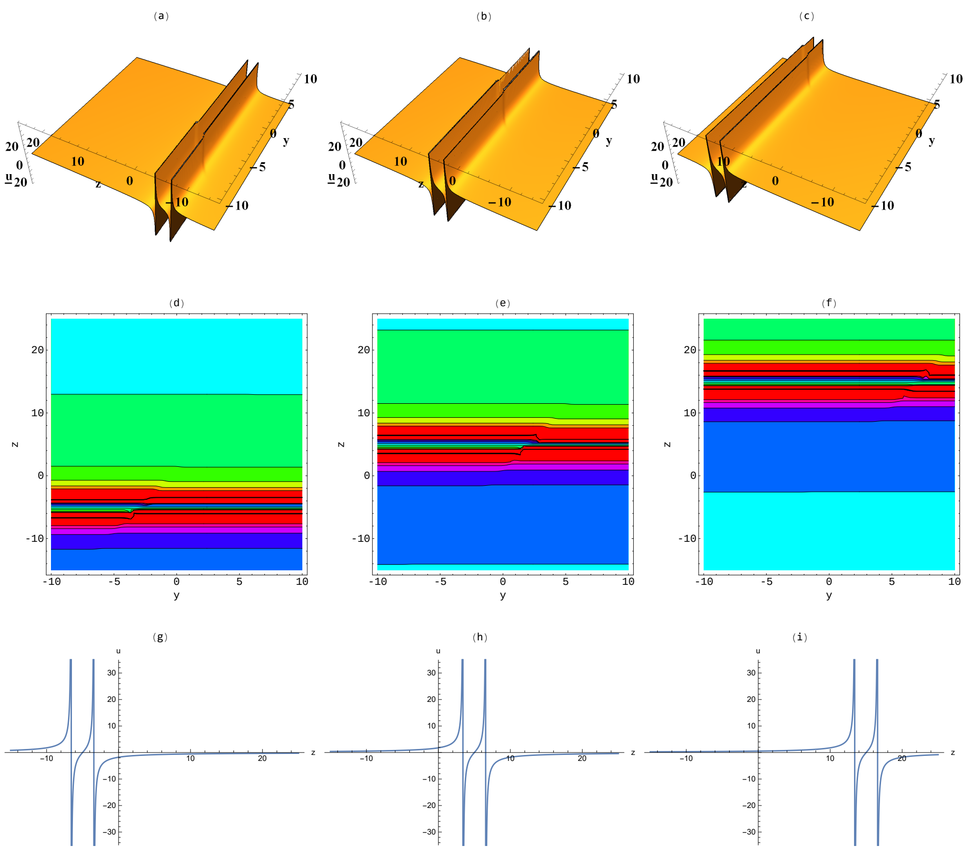

Figure 1. Plots of the interaction solutions (2.6) for $\iota_1 = \iota_6 = 2$, $k = t = 1$, $\iota_3 = -1$, $\iota_5 = 5$, $\iota_{11} = -6$, $\iota_{10} = 3$, $j_3 = -2$, when $x = -5$ in (a) (d) (g), $x = 0$ in (b) (e) (h) and $x = 5$ in (c) (f) (i). $y = -8$ in (g) (h) and (i).

3. Interaction solutions between the rational function and the cos function

Based on the collision of rational function and trigonometric function, we set

$$\begin{aligned}
\varrho &= \iota_1 x + \iota_2 y + \iota_3 z + \iota_4 t + \iota_5, \\
\varsigma &= \iota_6 x + \iota_7 y + \iota_8 z + \iota_9 t + \iota_{10}, \\
\zeta_2 &= \varrho^2 + \varsigma^2 + k\cos(j_1 x + j_2 y + j_3 z + j_4 t) + \iota_{11}.
\end{aligned} \tag{3.1}$$

Substituting Eq. (3.1) into Eq. (2.2), we have the following comparison expression

$$\begin{aligned}
&\iota_7 = \frac{\iota_2\iota_6}{\iota_1}, \iota_8 = \frac{\iota_3\iota_6}{\iota_1}, \iota_4 = \frac{\iota_3^2}{\iota_1+\iota_2+\iota_3}, \iota_9 = \frac{\iota_3^2\iota_6}{\iota_1\left(\iota_1+\iota_2+\iota_3\right)}, \\
&j_2 = -\frac{\iota_2 j_1}{\iota_1}, j_4 = \frac{\iota_1 j_3^2 - \iota_2 j_1^4}{\iota_1\left(j_1+j_3\right) - \iota_2 j_1}, k = \frac{2\left(\iota_1^2+\iota_6^2\right)}{j_1^2}, \\
&\iota_3 = -[\iota_1\left(\iota_1^2+\iota_6^2\right)\left(-j_1\right)[2\iota_2^2 j_1^3 + j_3\left(\iota_1^2-\iota_2^2-\iota_1\iota_2 j_1^2\right)] - \\
&\quad \sqrt{\iota_1\iota_2\left(\iota_1^2+\iota_6^2\right)^2 j_1^4[\iota_1\left(j_1+j_3\right)-\iota_2 j_1]^2[-\iota_1^2+3\iota_2^2+\iota_2\iota_1\left(j_1^2+2\right)]} \\
&\quad /[\left(\iota_1^2+\iota_6^2\right) j_1^2[\iota_1^2+\iota_2^2-\iota_2\iota_1\left(j_1^2+2\right)]],
\end{aligned} \tag{3.2}$$

where $\iota_1 \neq 0, j_1+j_3 \neq 0, \iota_1+\iota_2+\iota_3 \neq 0, j_1 \neq 0, \iota_1^2+\iota_6^2 \neq 0, \iota_1\left(j_1+j_3\right)-\iota_2 j_1 \neq 0, \iota_1^2+\iota_2^2-\iota_2\iota_1\left(j_1^2+2\right) \neq 0$. Substituting Eq. (3.1) and Eq. (3.2) into the transformation $u = 2[ln\xi]_x$, we get another interaction solutions of the ngKP equation

$$\begin{aligned}
u = {} & 4\iota_1(\iota_5 + \frac{\iota_3^2 t}{\iota_1+\iota_2+\iota_3} + \iota_1 x + \iota_2 y + \iota_3 z)/\zeta_2 - 4(\iota_1^2+\iota_6^2) \\
& \times \sin(\frac{t\left(\iota_1 j_3^2 - \iota_2 j_1^4\right)}{\iota_1\left(j_1+j_3\right)-\iota_2 j_1} + j_1 x - \frac{\iota_2 j_1 y}{\iota_1} + j_3 z)/(j_1\zeta_2) \\
& +4\iota_6[\iota_{10} + \frac{\iota_6\iota_3^2 t}{\iota_1(\iota_1+\iota_2+\iota_3)} + \iota_6 x + \frac{\iota_2\iota_6 y}{\iota_1} + \frac{\iota_6\iota_3 z}{\iota_1}]/\zeta_2.
\end{aligned} \tag{3.3}$$

The dynamical properties for interaction solution between rational function and cos function are demonstrated in Fig. 2 and Fig. 3. Figures 2(a), 2(b) and 2(c) show the 3d graphs in the (y,t)-plane when $x = -3, 0, 3$ respectively, Figures 2(d), 2(e) and 2(f) list the corresponding contour diagrams of Figures 2(a), 2(b) and 2(c). Fig. 3 display the 3d graphs and corresponding contour diagrams of interaction solution (3.3) in the (x,y)-plane when $z = -35, 0, 35$ respectively.

4. Abundant exact solutions

To study the exact solutions for Eq. (1.1), a direct test function is selected as follows in Eq. (2.2)

$$\zeta = \Lambda_1\, e^{\Psi_1} + e^{-\Psi_1} + \Lambda_2\, \tan\left(\Psi_2\right) + \Lambda_3\, \tanh\left(\Psi_3\right), \tag{4.1}$$

where $\Psi_i = \kappa_i\, x + \lambda_i\, y + \mu_i\, z + \nu_i\, t, i = 1,2,3,4$ and κ_i, λ_i, μ_i, ν_i are undetermined constants. Substituting Eq. (4.1) into Eq. (2.2) and equating all the coefficients

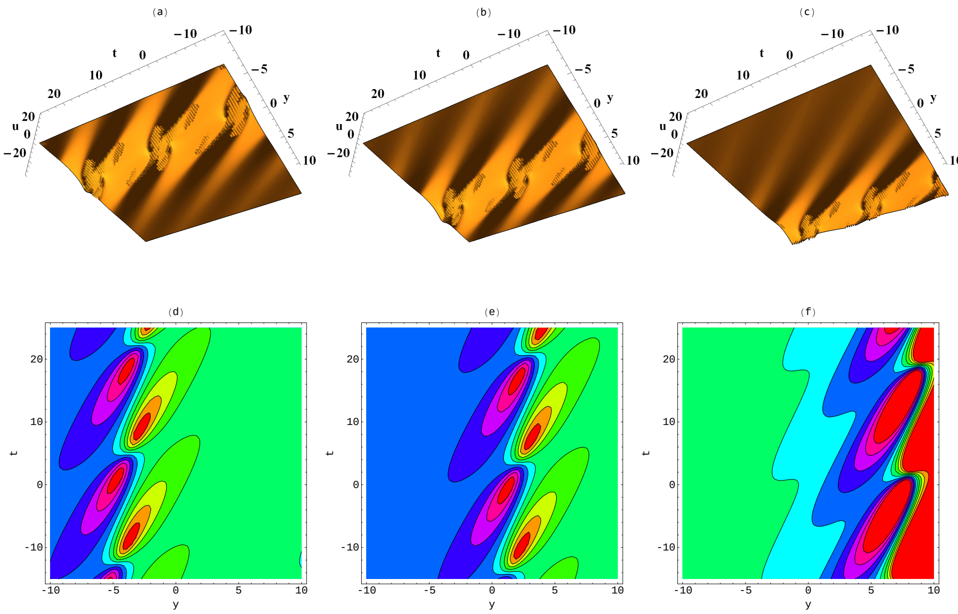

Figure 2. Plots of the interaction solutions (3.3) for $\iota_1 = \iota_5 = j_1 = 2$, $\iota_2 = -1$, $z = \iota_6 = 0$, $\iota_{11} = -6$, $\iota_{10} = 3$, $j_3 = -2$, when $x = -3$ in (a) (d), $x = 0$ in (b) (e) and $x = 3$ in (c) (f).

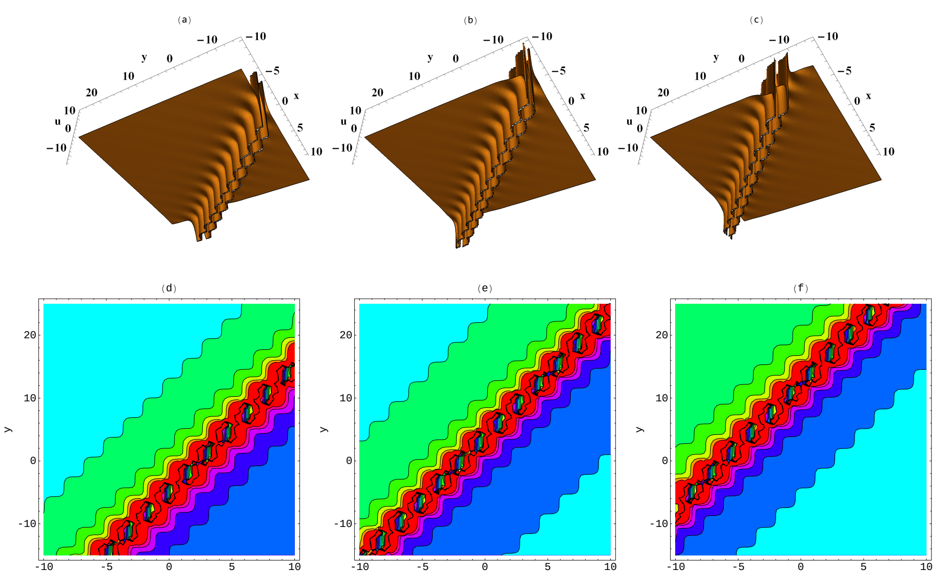

Figure 3. Plots of the interaction solutions (3.3) for $\iota_1 = \iota_5 = j_1 = 2$, $\iota_2 = -1$, $\iota_6 = 1$, $t = 20$ $\iota_{11} = -6$, $\iota_{10} = 3$, $j_3 = -2$, when $z = -35$ in (a) (d), $z = 0$ in (b) (e) and $z = 35$ in (c) (f).

of different powers of e^{Ψ_1}, $e^{-\Psi_1}$, $\tan(\Psi_2)$, $\tanh(\Psi_3)$ and constant term to zero via symbolic computation [33-39], Eq. (1.1) has the following exact solutions

Case(1)

$$\begin{aligned}
&\kappa_2=\kappa_3=0,\lambda_2=\frac{\mu_2^2-\mu_2\nu_2}{\nu_2},\lambda_3=\frac{\mu_3^2-\mu_3\nu_3}{\nu_3},\mu_3=\frac{\mu_2\nu_3}{\nu_2},\\
&\lambda_1=-\frac{\mu_3^2\left(\kappa_1^3+\nu_1\right)}{\nu_3^2}+\frac{\mu_3\left(\kappa_1^3+2\mu_1\right)}{\nu_3}-\kappa_1-\mu_1,\nu_1=\frac{\mu_1^2}{\kappa_1+\mu_1},\\
&\nu_2=\frac{\mu_2\left(-\sqrt{\kappa_1^2-4}\kappa_1^2\epsilon_1+\kappa_1^3+2\mu_1\right)}{2\left(\kappa_1+\mu_1\right)},
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
u_1=&[2\left(\kappa_1\Lambda_1e^{\frac{\mu_1^2t}{\kappa_1+\mu_1}+\kappa_1x+\lambda_1y+\mu_1z}-\kappa_1e^{-\frac{\mu_1^2t}{\kappa_1+\mu_1}-\kappa_1x-\lambda_1y-\mu_1z}\right)]\\
&/[\Lambda_1e^{\frac{\mu_1^2t}{\kappa_1+\mu_1}+\kappa_1x+\lambda_1y+\mu_1z}+e^{-\frac{\mu_1^2t}{\kappa_1+\mu_1}-\kappa_1x-\lambda_1y-\mu_1z}+\Lambda_2\tan[\nu_2t+\mu_2z\\
&+\mu_2y\left(\frac{\mu_2}{\nu_2}-1\right)]+\Lambda_3\tanh[\nu_3t+\frac{\mu_2\nu_3y\left(\frac{\mu_2}{\nu_2}-1\right)}{\nu_2}+\frac{\mu_2\nu_3z}{\nu_2}]],
\end{aligned} \tag{4.3}$$

where $\epsilon_1=\pm1$.

Case(2)

$$\begin{aligned}
&\kappa_2=\kappa_3=\lambda_1=0,\lambda_2=\frac{\mu_2^2-\mu_2\nu_2}{\nu_2},\lambda_3=\frac{\mu_3^2-\mu_3\nu_3}{\nu_3},\kappa_1=2\epsilon_2,\\
&\mu_3=\frac{\nu_3\left(\mu_1+2\epsilon_2\right)}{\mu_1+4\epsilon_2},\nu_1=\frac{\mu_1^2}{\kappa_1+\mu_1},\nu_2=\frac{\mu_2\left(\mu_1+4\epsilon_2\right)}{\mu_1+2\epsilon_2},
\end{aligned} \tag{4.4}$$

$$\begin{aligned}
u_2=&[2\left(2\epsilon_2\Lambda_1e^{\frac{\mu_1^2t}{\mu_1+2\epsilon_2}+2x\epsilon_2+\mu_1z}-2\epsilon_2e^{-\frac{\mu_1^2t}{\mu_1+2\epsilon_2}-2x\epsilon_2-\mu_1z}\right)]/[\Lambda_1e^{\frac{\mu_1^2t}{\mu_1+2\epsilon_2}+2x\epsilon_2+\mu_1z}\\
&+e^{-\frac{\mu_1^2t}{\mu_1+2\epsilon_2}-2x\epsilon_2-\mu_1z}+\Lambda_3\tanh[\nu_3t-\frac{2\nu_3y\epsilon_2\left(\mu_1+2\epsilon_2\right)}{\left(\mu_1+4\epsilon_2\right)^2}+\frac{\nu_3z\left(\mu_1+2\epsilon_2\right)}{\mu_1+4\epsilon_2}]\\
&+\Lambda_2\tan\left(\frac{\mu_2t\left(\mu_1+4\epsilon_2\right)}{\mu_1+2\epsilon_2}-\frac{2\mu_2y\epsilon_2}{\mu_1+4\epsilon_2}+\mu_2z\right)],
\end{aligned} \tag{4.5}$$

where $\epsilon_2=\pm1$.

Case(3)

$$\begin{aligned}
&\kappa_3=\Lambda_2=\lambda_1=0,\lambda_3=\frac{\mu_3^2-\mu_3\nu_3}{\nu_3},\kappa_1=\frac{\mu_1\left(\mu_1-\nu_1\right)}{\nu_1},\\
&\nu_3=-[\mu_3[\mu_1\epsilon_3\left(\mu_1-\nu_1\right)^2\sqrt{-2\mu_1^3\nu_1+\left(\mu_1^2-4\right)\nu_1^2+\mu_1^4}+3\mu_1^4\nu_1\\
&\quad-3\mu_1^3\nu_1^2+\mu_1^2\nu_1^3-\mu_1^5-2\nu_1^3]]/(2\mu_1\nu_1^2),
\end{aligned} \tag{4.6}$$

$$u_3=\frac{2\mu_1\left(\mu_1-\nu_1\right)}{\nu_1[\frac{\Lambda_3e^{\nu_1t+\frac{\mu_1^2x}{\nu_1}+\mu_1(z-x)}\tanh(\nu_3t+\lambda_3y+\mu_3z)+2}{\Lambda_1e^{2\left(\nu_1t+\frac{\mu_1^2x}{\nu_1}+\mu_1(z-x)\right)}-1}+1]}, \tag{4.7}$$

where $\epsilon_3=\pm1$.

Case(4)

$$\lambda_1 = \lambda_2 = \lambda_3 = 0, \mu_2 = \frac{\mu_1\nu_2}{\nu_1}, \mu_3 = \frac{\mu_2\nu_3}{\nu_2},$$
$$\kappa_1 = \frac{\mu_1(\mu_1 - \nu_1)}{\nu_1}, \kappa_2 = \frac{\mu_2(\mu_2 - \nu_2)}{\nu_2}, \kappa_3 = \frac{\mu_3(\mu_3 - \nu_3)}{\nu_3}, \tag{4.8}$$

$$u_4 = [2[\kappa_1\Lambda_1 e^{\nu_1 t+\kappa_1 x+\mu_1 z} + \kappa_2\Lambda_2 \sec^2\left(\nu_2 t + \kappa_2 x + \frac{\mu_1\nu_2 z}{\nu_1}\right)$$
$$+ \kappa_3\Lambda_3 sech^2\left(\nu_3 t + \kappa_3 x + \frac{\mu_1\nu_3 z}{\nu_1}\right) - \kappa_1 e^{-\nu_1 t-\kappa_1 x-\mu_1 z}]]/[\Lambda_1 e^{\nu_1 t+\kappa_1 x+\mu_1 z}$$
$$+ \Lambda_2 \tan\left(\nu_2 t + \kappa_2 x + \frac{\mu_1\nu_2 z}{\nu_1}\right) + \Lambda_3 \tanh\left(\nu_3 t + \kappa_3 x + \frac{\mu_1\nu_3 z}{\nu_1}\right)$$
$$+ e^{-\nu_1 t-\kappa_1 x-\mu_1 z}]. \tag{4.9}$$

Case(5)

$$\lambda_1 = \Lambda_2 = \lambda_3 = 0, \mu_3 = \frac{\mu_1\nu_3}{\nu_1},$$
$$\kappa_1 = \frac{\mu_1(\mu_1 - \nu_1)}{\nu_1}, \kappa_3 = \frac{\mu_3(\mu_3 - \nu_3)}{\nu_3}, \tag{4.10}$$
$$u_5 = [2[\kappa_1\Lambda_1 e^{\nu_1 t+\kappa_1 x+\mu_1 z} + \kappa_3\Lambda_3 \mathrm{sech}^2\left(\nu_3 t + \kappa_3 x + \frac{\mu_1\nu_3 z}{\nu_1}\right) - \kappa_1 e^{-\nu_1 t-\kappa_1 x-\mu_1 z}]]$$
$$/[\Lambda_1 e^{\nu_1 t+\kappa_1 x+\mu_1 z} + \Lambda_3 \tanh\left(\nu_3 t + \kappa_3 x + \frac{\mu_1\nu_3 z}{\nu_1}\right) + e^{-\nu_1 t-\kappa_1 x-\mu_1 z}]. \tag{4.11}$$

Case(6)

$$\lambda_1 = \Lambda_3 = \lambda_2 = 0, \kappa_1 = \frac{\mu_1(\mu_1 - \nu_1)}{\nu_1}, \kappa_2 = \frac{\mu_2(\mu_2 - \nu_2)}{\nu_2}, \nu_2 = \frac{\mu_2\nu_1}{\mu_1}, \tag{4.12}$$
$$u_6 = [2[\kappa_1\Lambda_1 e^{\nu_1 t+\kappa_1 x+\mu_1 z} + \kappa_2\Lambda_2 \sec^2(\nu_2 t + \kappa_2 x + \mu_2 z) - \kappa_1 e^{-\nu_1 t-\kappa_1 x-\mu_1 z}]]$$
$$/[\Lambda_1 e^{\nu_1 t+\kappa_1 x+\mu_1 z} + \Lambda_2 \tan(\nu_2 t + \kappa_2 x + \mu_2 z) + e^{-\nu_1 t-\kappa_1 x-\mu_1 z}]. \tag{4.13}$$

Case(7)

$$\Lambda_2 = \Lambda_3 = 0, \lambda_1 = \frac{\mu_1^2 - \nu_1(\kappa_1 + \mu_1)}{4\kappa_1^3 + \nu_1}, \tag{4.14}$$
$$u_7 = [2[\kappa_1\Lambda_1 \exp[\nu_1 t + \kappa_1 x + \frac{y[\mu_1^2 - \nu_1(\kappa_1 + \mu_1)]}{4\kappa_1^3 + \nu_1} + \mu_1 z] - \kappa_1 \exp[-\nu_1 t$$
$$-\kappa_1 x - \frac{y[\mu_1^2 - \nu_1(\kappa_1 + \mu_1)]}{4\kappa_1^3 + \nu_1} - \mu_1 z]]]/[\Lambda_1 \exp[\nu_1 t + \kappa_1 x + \frac{y[\mu_1^2 - \nu_1(\kappa_1 + \mu_1)]}{4\kappa_1^3 + \nu_1}$$
$$+\mu_1 z] + \exp[-\nu_1 t - \kappa_1 x - \frac{y[\mu_1^2 - \nu_1(\kappa_1 + \mu_1)]}{4\kappa_1^3 + \nu_1} - \mu_1 z]]. \tag{4.15}$$

As an example, we study the dynamical behaviors for Eq. (4.3) by choosing different values of parameters as follows

$$\kappa_1 = \nu_3 = 2, \mu_1 = -1, \mu_2 = -3. \tag{4.16}$$

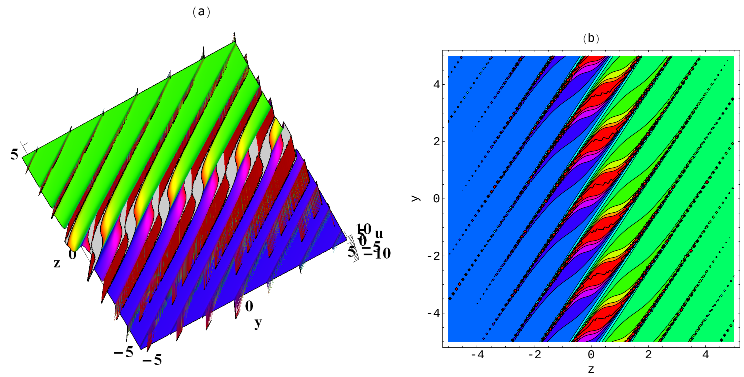

Figure 4. Solution (4.3) with $\Lambda_1 = -2$, $\Lambda_2 = 1$, $\Lambda_3 = 0$, $x = t = 0$, (a) three-dimensional graph (b) contour graph.

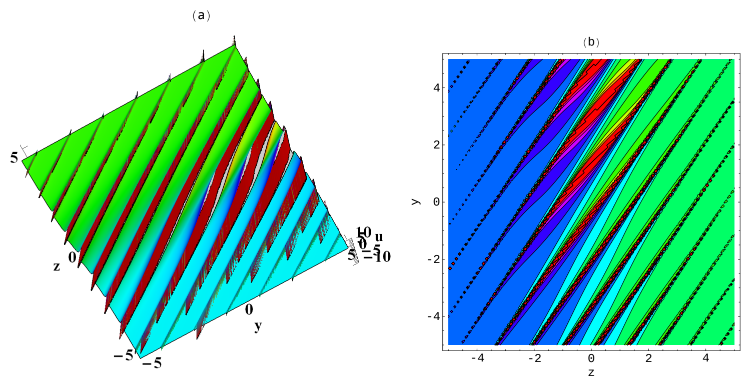

Figure 5. Solution (4.3) with $\Lambda_1 = 0$, $\Lambda_2 = 1$, $\Lambda_3 = 1$, $x = t = 0$, (a) three-dimensional graph (b) contour graph.

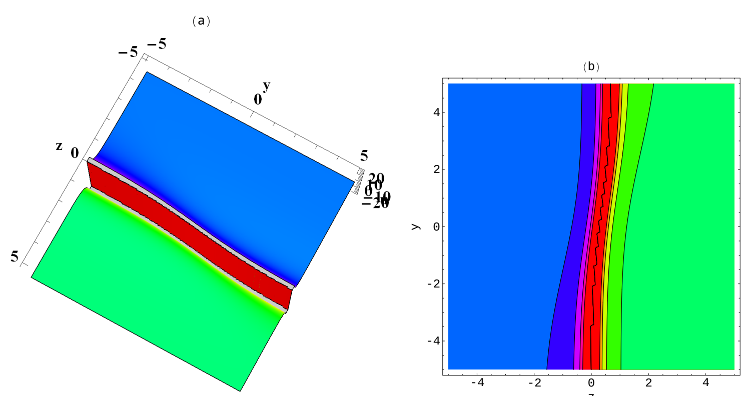

Figure 6. Solution (4.3) with $\Lambda_1 = -2$, $\Lambda_2 = 0$, $\Lambda_3 = 1$, $x = t = 0$, (a) three-dimensional graph (b) contour graph.

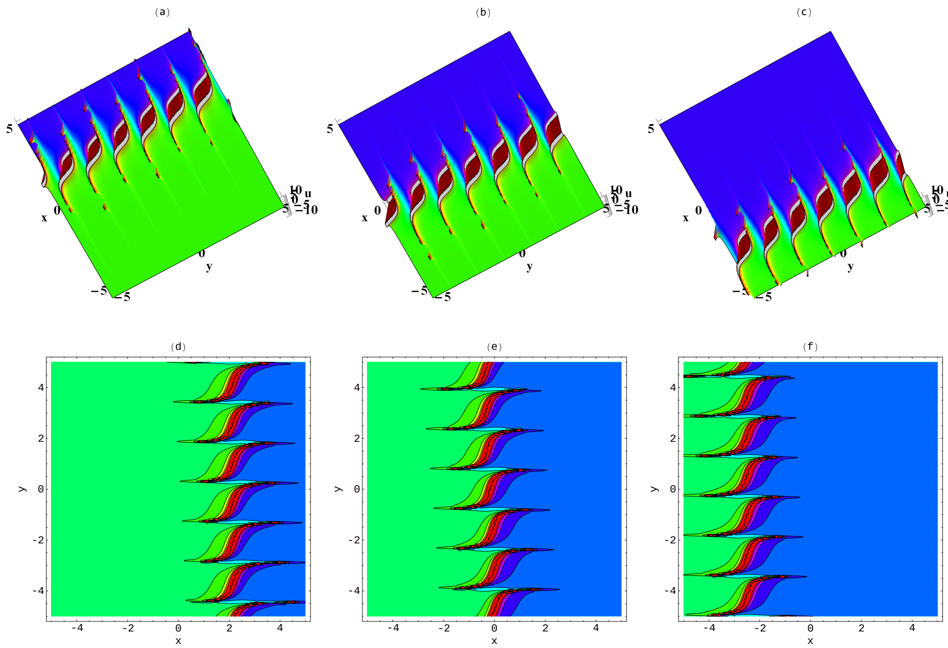

Figure 7. Solution (4.3) with $\Lambda_1 = -2$, $\Lambda_2 = 1$, $\Lambda_3 = 1$, $z = 0$, when $t = -5$ in (a) (d), $t = 0$ in (b) (e) and $t = 5$ in (c) (f).

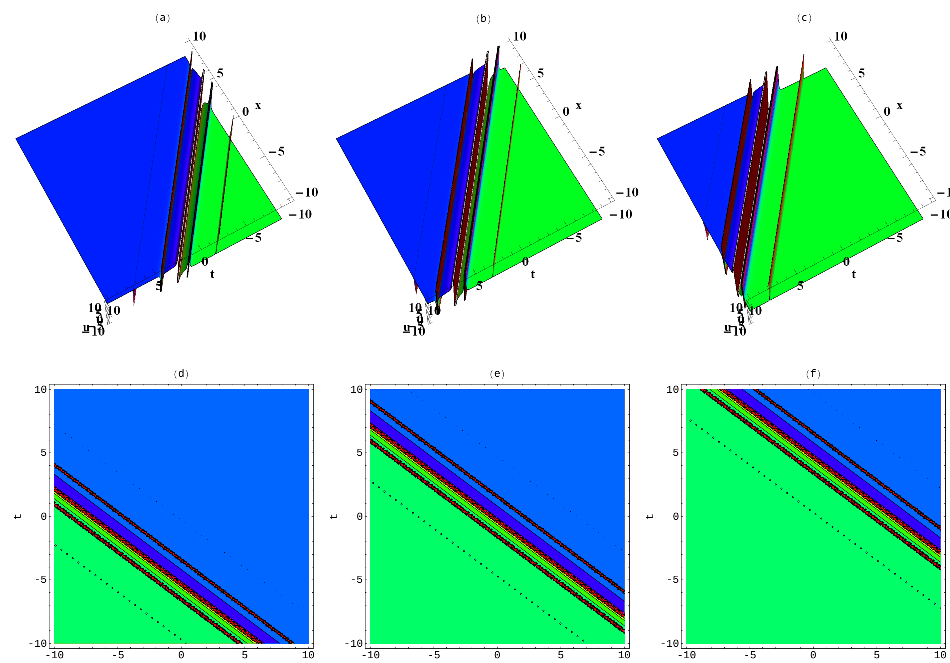

Figure 8. Solution (4.9) when $z = -10$ in (a) (d), $z = 0$ in (b) (e) and $z = 10$ in (c) (f).

Substituting Eq. (4.16) into Eq. (4.3), the dynamical behaviors for solution (4.3) are shown in Figs. 4-7.

Setting

$$\nu_1 = \nu_3 = 2, \mu_1 = -1, \nu_2 = \Lambda_2 = \Lambda_3 = 1, \Lambda_1 = -2. \tag{4.17}$$

Substituting Eq. (4.17) into Eq. (4.9), the dynamical behaviors for solution (4.9) are shown in Fig. 8.

5. Conclusion

In this paper, the interaction solutions for the ngKP equation between rational function and tanh function or cos function are presented based on the Hirota's bilinear form. Abundant exact solutions are also obtained by using a direct test function. Equations using this method need to have the Hirota's bilinear form. The dynamical properties of the obtained solutions are analyzed and shown in figures, which contain 3d plots, 2d contour plots and the plane graphs.

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