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TRAVELING WAVE, LUMP WAVE, ROGUE WAVE, MULTI-KINK SOLITARY WAVE AND INTERACTION SOLUTIONS IN A (3+1)-DIMENSIONAL KADOMTSEV-PETVIASHVILI EQUATION WITH BÄCKLUND TRANSFORMATION

Shoufu Tian[1,†], Ding Guo[1], Xiubin Wang[1] and Tiantian Zhang[1,†]

Abstract We consider a (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation. By using the Hirota bilinear operators, we construct the bilinear form of the equation. Based on the resulting bilinear form, we further derive the Bäcklund transformation and the traveling wave solutions of the equation. Furthermore, lump solutions are constructed by searching the positive function from the Hirota bilinear formalism. Meanwhile, we also obtain the interaction solutions between lump solutions and the stripe solitons. We discuss the influences of each parameters on these exact solutions by using several graphics. Finally, we successfully construct its rogue wave solutions and multi-kink solitary wave solutions. It is hoped that our results can be used to enrich the dynamical behavior of the (3+1)-dimensional KP-type equations.

Keywords The (3+1)-dimensional KP equation, traveling wave solution, lump solutions, interaction solution, rogue wave solution, multi-kink solitary wave solution.

MSC(2010) 35Q51, 35Q53, 35C99.

1. Introduction

Nonlinear evolution equations (NLEEs) play a vital role in nonlinear complex physical phenomena. NLEEs have been regarded as the models to describe some nonlinear phenomena in fluid mechanics, plasma physics, optical fibers and solid state physics, etc [2,8,9,12]. With the development of science, the research of NLEEs is a importance topic [14], and their solutions play an important role in the field of nonlinear science. It is well known that the Kadomtsev-Petviashvili (KP) equation is an important model in the NLEEs, which can be used to describe water waves of long wavelength with weakly non-linear restoring forces and frequency dispersion [36].

Recently, both mathematician and physicist make much important efforts in

[†]The corresponding authors. Email addresses: sftian@cumt.edu.cn, shoufu2006@126.com (S. Tian), ttzhang@cumt.edu.cn (T. Zhang)
[1]School of Mathematics and Institute of Mathematical Physics, China University of Mining and Technology, Xuzhou 221116, China

this area and demonstrate various useful techniques [5]. These results are used to find solition solutions and rogue waves solutions [1, 6, 31, 32, 37]. At present, high-dimensional NLEEs are getting more and more attention. Especially, the (3+1)-dimensional KP-type equations not only demonstrate the inelastic interactions but also admit localized coherent structures. The KdV-type and KP-tye equations are some very important nonlinear equations in nonlinear mathematical physics [3, 13, 25, 26]. There are also recent systematical studies on lump solutions and interaction solutions to integrable equations by Ma and his collaborators [15–17, 19].

As we know, the most classic KP equation [11] reads

$$(u_t + 6uu_x + u_{3x})_x + 3u_{yy} = 0, \tag{1.1}$$

which can characterize the growth of shallow water waves in quasi one-dimension, with the weak influence. It also describes the evolution of quasi-one-dimensional shallow water waves when effects of the surface tension and the viscosity are negligible. A variety of modified and extended KP equations has been examined in the literature [4, 18, 24]. For example, the (3+1)-dimensional B-type KP equation (BKP) is presented by

$$u_{ty} + \alpha u_{xxxy} + 3\beta(u_x u_y)_x + \gamma u_{xz} = 0, \tag{1.2}$$

it can be viewed as a shifted (2+1)-dimensional BKP equation for $z = x$. In this work, we will study a (3+1)-dimensional KP-type equation given by

$$5\frac{\partial}{\partial z}u_x - 6u_y\frac{\partial}{\partial x}u_x - 6u_x\frac{\partial}{\partial x}u_y + \frac{\partial}{\partial y}(u_t - 2u_{xxx}) = 0, \tag{1.3}$$

where u is a complex function about x, y, z, and t. It is a second member in the entire Kadomtsev-Petviashvili (KP) hierarchy [10]. Some researchers use the simplified Hereman-Nuseir form to establish one and two soliton solutions for each extended equation, and develop specific constraints that guarantee the existence of multiple soliton solutions for each equation in [35]. The rogue wave and a pair of resonance stripe solitons of the KP equation have been considered in [39]. Eq. (1.3) is a special case in [20, 21] where the authors used a generalized Hirota disturbance mechanism to construct multiple rouge wave solutions of differential equations. Here we are inspired to further generalize them to generate some new phenomena for the equation (1.3). In [22, 23], there are recent innovative studies on lump solutions with higher-order dispersion relations and for linear partial differential equations (PDEs). The main purpose of this paper is to study the travelling wave solutions, lump solutions, rogue wave solutions and interaction solutions of the (3+1)-dimensional KP-type equation (1.3) by using the symbolic calculation methods [7, 27, 28, 33]. In [38], authors studied abundant mixed lump-soliton solutions of the BKP equation.

The paper is arranged as follows: In Sect. 2, we obtain its bilinear representation by using Hirota's bilinear method and Bell's polynomial theory. By using the resulting bilinear form, in Sect. 3, we derive Bäcklund transformation of the equation. Then, we also get the traveling wave solutions via the Bäcklund transformation. In Sect. 4, the lump solutions are constructed by the using the bilinear form. In Sect. 5, we construct interaction solution for the lump solution with one stripe soliton by combining quadratic function. In Sect. 6, we obtain rogue waves and rational breather waves by using some ansätz functions. In Sect. 7, we present its mult-kink solitary wave solutions via a very natural way. Finally, some conclusions are presented in the last section.

2. Bilinear formalism

First of all, we introduce a potential field transformation

$$u = q_x. \tag{2.1}$$

Substituting (2.1) into (1.3), integrating the equation with respect to x twice, and employing the results provided in [29,30,34], we can obtain

$$E(q) = P_{ty} + 5P_{xz} - 2P_{3xy} = 0, \tag{2.2}$$

under the following variable transformation

$$q = 2\ln f \Leftrightarrow u = 2(\ln f)_x, \tag{2.3}$$

where the Hirota D-operator is determined as follows:

$$\begin{aligned} &D_x^{n_1} D_y^{n_2} D_t^{n_3} g \cdot f \\ =&(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'})^{n_1}(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'})^{n_2}(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'})^{n_3} g(x,y,t)f(x',y',t')|_{x=x',y=y',t=t'}, \end{aligned} \tag{2.4}$$

with $g(x,y,t)$ and $f(x',y',t')$ being functions of x, y, t and x', y', t'. Here n_1, n_2 and n_3 are all the non-negative integers. Then we obtain

$$\begin{aligned} &D_x D_t f \cdot f = 2(f_{xt} f - f_x f_t), \\ &D_x^2 f \cdot f = 2(f_{xx} f - f_{xx}^2), \\ &D_x D_y f \cdot f = 2(f_{xy} f - f_x f_y). \end{aligned} \tag{2.5}$$

Therefore, the Hirota's bilinear equation of (3+1)-dimensional KP-type equation (1.3) reads

$$(D_t D_y + 5D_x D_z - 2D_x^3 D_y) f \cdot f = 0, \tag{2.6}$$

under the transformation

$$u = 2(\ln f)_x, \tag{2.7}$$

where f is a real function about x, y, z and t, and $D_t D_y$, $D_x D_z$ and $D_x^3 D_y$ are the Hirota's bilinear operates.

3. Bäcklund transformation and traveling wave solutions

In this section, we will construct an important function M to obtain bilinear operator, and get the Bäcklund transformation by using the bilinear operator. Then, we obtain the traveling wave solutions of Eq. (1.3).

3.1. Bäcklund transformation

Firstly, assuming that there exists another real function to derive the Bäcklund transformation, so we have a bilinear equation as

$$(D_t D_y + 5D_x D_z - 2D_x^3 D_y) f' \cdot f' = 0. \tag{3.1}$$

Now, we construct a key function M as follows:

$$M = [(D_tD_y+5D_xD_z - 2D_x^3D_y)f' \cdot f']f \cdot f - [(D_tD_y + 5D_xD_z - 2D_x^3D_y)f \cdot f]f' \cdot f', \tag{3.2}$$

and assuming $M = 0$, one obtains

$$[(D_tD_y+5D_xD_z-2D_x^3D_y)f' \cdot f']f \cdot f = [(D_tD_y+5D_xD_z-2D_x^3D_y)f \cdot f]f' \cdot f', \tag{3.3}$$

where we have several types of exchange formulas of bilinear operator as:

$$(D_iD_jf'f')ff - (D_iD_jff)f'f' = 2D_j(D_if'f)ff', \quad (D_iD_jf'f)ff' = (D_jD_if'f)ff', \tag{3.4}$$

and

$$\begin{aligned} &2(D_i^3D_jf'f')ff - 2(D_i^3D_jff)f'f' \\ =&D_i[(3D_i^2D_jf'f)ff' + (3D_i^2f'f)(D_jff') + (6D_iD_jf'f)(D_iff')] \\ &+ D_j[(D_i^3f'f)ff' + (3D_i^2f'f)(D_jff')], \end{aligned} \tag{3.5}$$

with $i, j = x, y, z, t$. It shows that f' can be used to solve (2.6) and also denotes a solution of (3.1). Now, we obtain the detailed calculation as:

$$\begin{aligned} M =& [(D_tD_yf'f')ff - (D_tD_yff)f'f'] - 2[(D_x^3f'f')ff - (D_x^3ff)f'f'] \\ &+ 5[(D_xD_zf'f')ff - (D_xD_zff)f'f'] \\ =& 2D_y(D_tf'f)ff' - D_x[(3D_x^2D_yf'f)ff' + (3D_x^2f'f)(D_yff') + 6(D_xD_yf'f)(D_xff')] \\ &- D_y[(D_x^3f'f)ff' + (3D_x^2f'f)(D_xff')] + 10D_z(D_xf'f)ff' \\ =& 2D_y(D_tf'f)ff' + 10D_z(D_xf'f)ff' \\ &- D_x[(3D_x^2D_yf'f + a_1D_yf'f + a_2f'f)ff' + (3D_x^2f'f + a_3D_yf'f + a_4f'f)(D_yff') \\ &+ (6D_xD_yf'f + 6a_5D_xf'f)(D_xff')] \\ &- D_y[(D_x^3f'f - a_1D_xf'f + a_6f'f)ff' + (3D_x^2f'f + a_7D_xf'f - a_4f'f)(D_xff')] \\ =& D_x(A_1f'f)ff' + D_x(A_2f'f)(D_yff') + D_x(A_3f'f)(D_xff') \\ &+ D_y(A_4f'f)ff' + D_y(A_5f'f)(D_xff'). \end{aligned}$$

By using the exchange (3.4)-(3.6), the Bäcklund transformations of (1.3) are given by

$$\begin{aligned} A_1f'f &= -(3D_x^2D_y + a_1D_y + a_2 - 10D_z)f'f = 0, \\ A_2f'f &= -(3D_x^2 + a_3D_y + a_4)f'f = 0, \\ A_3f'f &= -(6D_xD_y + 6a_5D_x)f'f = 0, \\ A_4f'f &= (2D_t - D_x^3 + a_1D_x - a_6)f'f = 0, \\ A_5f'f &= -(3D_x^2 + a_7D_x - a_4)f'f = 0. \end{aligned} \tag{3.6}$$

3.2. Traveling wave solutions

Next, we substitute a solution $f = 1$ into the (1.3), we has

$$D_s^ngf = D_s^ng = \frac{\partial^n}{\partial s^n}g, \quad n \geq 1, \tag{3.7}$$

which is reduced to the initial variable u. The Bäcklund transformation (3.6) can be a group of linear equations given by

$$
\begin{aligned}
&3f'_{xxy} + a_1 f'_y + a_2 f' - 10 f'_z = 0,\\
&3f'_{xx} + a_3 f'_y + a_4 f' = 0,\\
&f'_{xy} + a_5 f'_x = 0,\\
&2f'_t - f'_{xxx} + a_1 f'_x - a_6 f' = 0,\\
&3f'_{xx} + a_7 f'_x - a_4 f' = 0.
\end{aligned} \tag{3.8}
$$

Then, we assume a function f as follows:

$$f' = 1 + \eta e^{\psi_1 x + \psi_2 y + \psi_3 z - \psi_4 t}, \quad \psi_1 \neq 0, \tag{3.9}$$

where ψ_1, ψ_2, ψ_3 and ψ_4 are some constants, and η is a real parament. Taking a_2, a_4 and a_6 are equal to zero, and η is real paraments, we can obtain

$$
\begin{aligned}
&a_1 = \frac{2\psi_4 + \psi_1^3}{\psi_1}, \quad a_3 = \frac{-3\psi_1^2}{\psi_2}, \quad a_5 = -\psi_2 \quad a_7 = -3\psi_1,\\
&\psi_3 = \frac{3\psi_1^2\psi_2 + a_1\psi_2}{10}, \quad \psi_4 = \frac{\psi_1^3 - a_1\psi_1}{-2}.
\end{aligned} \tag{3.10}
$$

Then, we can get the following exponential wave solution

$$u = 2[\ln f']_x, \tag{3.11}$$

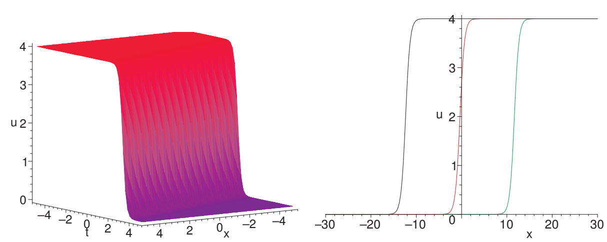
with $f' = 1 + \eta e^{-\frac{a_7}{3}x - a_5 y + \frac{3\psi_1^2\psi_2 + a_1\psi_2}{10}z + \frac{\psi_1^3 - a_1\psi_1}{2}t}$. We introduce a one-order function as follows

$$f' = \psi_1 x + \psi_2 y + \psi_3 z - \psi_4 t. \tag{3.12}$$

Substituting (3.12) into (3.8), and taking $a_i = 0 (2 \leq i \leq 7)$, we obtain the following rational solution

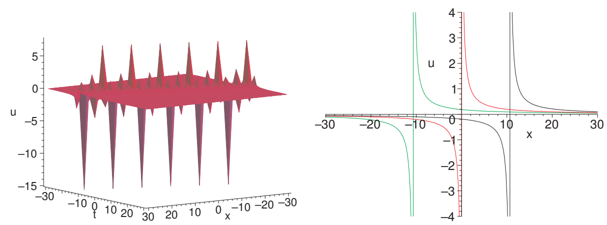
$$u = \frac{2\psi_1}{\psi_1 x + \psi_2 y + \psi_3 z - \psi_4 t}. \tag{3.13}$$

The solution of u is plotted in Fig. 1. We know that the amplitude, velocity and width of the traveling wave solution keep invariable during the propagation. It can show that the amplitudes of the excited state are limited and almost same in different spaces.



(a) (b)

Figure 1. (Color online) The traveling wave solution (3.11) by choosing suitable parameters: $\eta = 2, \psi_1 = 2, \psi_2 = 4, \psi_3 = \frac{44}{5}, \psi_5 = -6$. (a) Plot3d perspective view of the real part of the wave($z = 0, y = 2$). (b)The wave propagation pattern of the wave along the x axis ($t = -4, t = 0, t = 4$).



(a) (b)

Figure 2. (Color online) The rational one-soliton solution (3.13) by choosing suitable parameters: $\psi_1 = -8, \psi_2 = -3, \psi_3 = -5, \psi_5 = -\frac{71}{9}$. (a) Plot3d perspective view of the real part of the wave($y = 2$). (b)The wave propagation pattern of the wave along the x axis ($t = -4, t = 0, t = 4$).

4. Lump solutions

We have obtained the Hirota bilinear equation (2.6) from the second section via the Bell polynomial. When $z = x$, the equation is the following formula

$$\begin{aligned} B(f\cdot f) &= (D_tD_y + 5D_xD_x - 2D_x^3D_y)f\cdot f \\ &= 2[f_{ty}f - f_tf_y - 2(f_{3xy}f - 3f_{2xy}f_x + 3f_{xx}f_{xy} - f_{3x}f_y) + 5f_{xx}f - 5f_xf_x]. \end{aligned} \tag{4.1}$$

Now, we make the following assumption to seek lump solution of (1.3):

$$\begin{aligned} &f = g^2 + h^2 + b_1, \\ &g = b_2x + b_3y + b_4t + b_5, \quad h = b_6x + b_7y + b_8t + b_9, \end{aligned} \tag{4.2}$$

where $b_i (1 \le i \le 9)$ are real paraments to be determined. Substituting (4.2) into (4.1), we can obtain a polynomial of the variables x, y and t. Eliminating the coefficients of the polynomial via the symbolic computation, we can get b_1, b_4, b_8 as follows:

$$b_1 = \frac{6b_3^2(b_3^2+b_7^2)}{5b_7^2}, b_4 = -\frac{5b_3^2}{b_3^2+b_7^2}, b_8 = \frac{5b_3^2b_7}{b_3^2+b_7^2}. \tag{4.3}$$

Then, f can be determined by

$$\begin{aligned} f = {}&[b_2x + b_3y - \frac{5b_3^2}{b_3^2+b_7^2}t + b_5]^2 \\ &+ [b_6x + b_7y + \frac{5b_3^2b_7}{b_3^2+b_7^2}t + b_9]^2 \\ &+ \frac{6b_3^2(b_3^2+b_7^2)}{5b_7^2}. \end{aligned} \tag{4.4}$$

The solution of u can be written as

$$u = 4\frac{b_2g + b_6h}{f}, \tag{4.5}$$

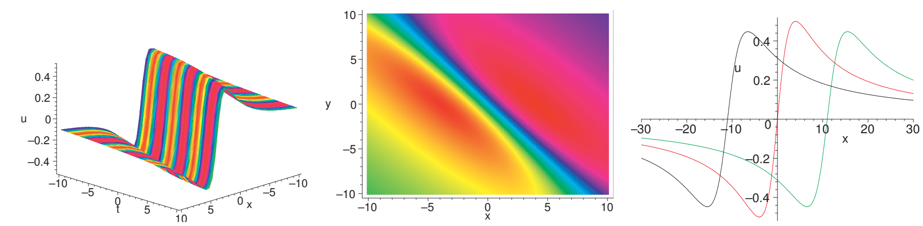
with

$$g = b_2x + b_3y - \frac{5b_3^2}{b_3^2+b_7^2}t + b_5,$$

$$h = b_6x + b_7y + \frac{5b_3^2b_7}{b_3^2 + b_7^2}t + b_9, \tag{4.6}$$

under the determinant conditions $b_3^2 + b_7^2 \neq 0$ and $b_1 > 0$.

As depicted in Fig. 3, we know that the amplitude, velocity and width of the lump solution keep invariable during the propagation. It can show that the amplitudes of the excited state are limited and almost same in different spaces.



(a) (b) (c)

Figure 3. (Color online) The lump solution (4.5) by choosing suitable parameters: $b_1 = 1, b_3 = 2, b_6 = 3, b_7 = 2$. (a) Plot3d perspective view of the real part of the wave($y = 0$). (b)The overhead view of the wave. (c) The wave propagation pattern of the wave along the x axis ($t = -4, t = 0, t = 4$).

5. Interaction solutions

Now, we discuss the interaction solution between lump solution and one stripe soliton by using quadratic function with exponential function. We also write f as a positive function

$$f = m^2 + n^2 + l + b_1, \tag{5.1}$$

with

$$\begin{aligned} m &= b_2x + b_3y + b_4t + b_5, \\ n &= b_6x + b_7y + b_8t + b_9, \\ l &= ke^{k_1x + k_2y + k_3t}. \end{aligned} \tag{5.2}$$

Substituting (5.2) into (4.1), we obtain

$$\begin{aligned} k_1 &= -\frac{4b_6^2}{k_2(b_2^2 + b_6^2)}, & k_3 &= \frac{8b_6^4(5b_2^2 - b_6^2)}{k_2^3(b_2^2 + b_6^2)^3}, \\ b_1 &= \frac{k_2^2(b_2^2 + b_6^2)^2(b_2^4 - b_6^4)}{16b_6^4b_2^2}, & b_3 &= -\frac{b_2k_2^2(b_2^2 + b_6^2)}{4b_6^2}, \\ b_4 &= -\frac{10b_6^2b_2(b_2^2 - 5b_6^2)}{(b_2^2 + b_6^2)^2k_2^2}, & b_7 &= \frac{k_2^2(b_2^2 + b_6^2)}{4b_6}, \\ b_8 &= -\frac{10b_6^3(5b_2^2 - b_6^2)}{k_2^2(b_2^2 + b_6^2)^2}, & b_5 &= 0, \quad b_9 = 0, \end{aligned} \tag{5.3}$$

under the conditions $k > 0, b_2^2 > b_6^2, k_2b_2b_6 \neq 0$. Then f can be determined by

$$f = [b_2x - \frac{b_2k_2^2(b_2^2 + b_6^2)}{4b_6^2}y - \frac{10b_6^2b_2(b_2^2 - 5b_6^2)}{(b_2^2 + b_6^2)^2k_2^2}t]^2$$

$$
\begin{aligned}
&+ [b_6x + \frac{k_2^2(b_2^2+b_6^2)}{4b_6}y - \frac{10b_6^3(5b_2^2-b_6^2)}{k_2^2(b_2^2+b_6^2)^2}t]^2 \\
&+ ke^{-\frac{4b_6^2}{k_2(b_2^2+b_6^2)}x+k_2y+\frac{8b_6^4(5b_2^2-b_6^2)}{k_2^3(b_2^2+b_6^2)^3}} + \frac{k_2^2(b_2^2+b_6^2)^2(b_2^4-b_6^4)}{16b_6^4b_2^2}.
\end{aligned} \tag{5.4}
$$

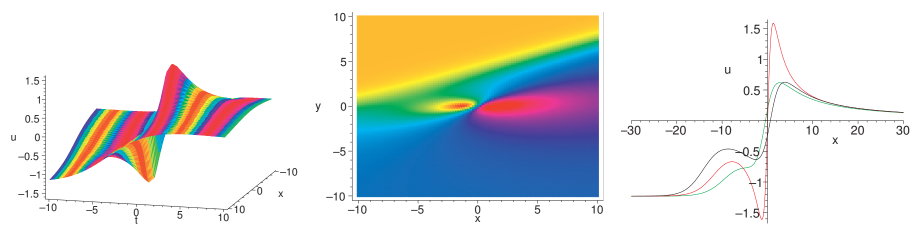
The solution of u can be written as

$$
u = 2\frac{2b_2m + 2b_6n + kk_1e^{k_1x+k_2y+k_3t}}{f}, \tag{5.5}
$$

with

$$
\begin{aligned}
m &= b_2x - \frac{b_2k_2^2(b_2^2+b_6^2)}{4b_6^2}y - \frac{10b_6^2b_2(b_2^2-5b_6^2)}{(b_2^2+b_6^2)^2k_2^2}t, \\
n &= b_6x + \frac{k_2^2(b_2^2+b_6^2)}{4b_6}y - \frac{10b_6^3(5b_2^2-b_6^2)}{k_2^2(b_2^2+b_6^2)^2}t.
\end{aligned} \tag{5.6}
$$

As depicted in Fig. 4, we know that the amplitude, velocity and width of the interaction solution keep invariable during the propagation. It can show that the amplitudes of the excited state are limited and almost same in different spaces.



(a) (b) (c)

Figure 4. (Color online) The interaction solution (5.5) by choosing suitable parameters: $k = 2, k_2 = 2, b_2 = 3, b_6 = 2$. (a) Plot3d perspective view of the real part of the wave($y = 0$). (b)The overhead view of the wave. (c)The wave propagation pattern of the wave along the x axis ($t = -4, t = 0, t = 4$).

6. Rogue wave solutions

In order to find the rogue wave solution of (1.3), let f admit

$$
f = 1 + (n_1x + n_2y + n_3z + n_4t)^2 + n_5x^2 + n_6(y+z)^2 + n_7t^2, \tag{6.1}
$$

where $n_i(i = 1, \cdots, 7)$ are free constants. Substituting (6.1) into (2.6), we can get the following results

$$
n_2 = -\frac{n7}{5n_1^3}, n_3 = \frac{n_7(36n_1^3+n_7)}{180n_1^5}, n_4 = \frac{n_7}{10n_1^3}, n_5 = n_6 = 0. \tag{6.2}
$$

Thus, we can obtain u as follows:

$$
u = \frac{4n_1(n_1x - \frac{n7}{5n_1^3}y + \frac{n_7(36n_1^3+n_7)}{180n_1^5}z + \frac{n_7}{10n_1^3}t)}{1 + (n_1x - \frac{n7}{5n_1^3}y + \frac{n_7(36n_1^3+n_7)}{180n_1^5}z + \frac{n_7}{10n_1^3}t)^2 + n_7t^2}. \tag{6.3}
$$

As depicted in Fig. 5, we know that the amplitude, velocity and width of the breather wave solution keep invariable during the propagation. It can show that the amplitudes of the excited state are limited and almost same in different spaces.

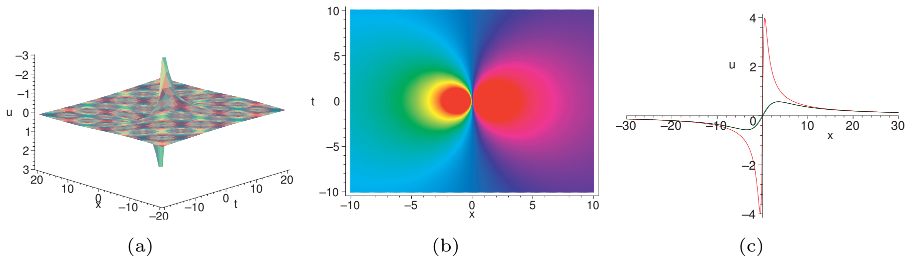


Figure 5. (Color online) The breather wave solution (6.3) by choosing suitable parameters: $n_1 = 2, n_7 = 3$. (a)Plot3d perspective view of the real part of the wave($y = 0, z = 0$). (b) The overhead view of the wave. (c) The wave propagation pattern of the wave along the x axis ($t = -4, t = 0, t = 4$).

In Fig. 5, we find it has a pair of peaks in the opposite direction, which is called the new breather wave solution. Its graph can be plotted by choosing appropriate parameters. And there have two similar wave shapes of the rogue wave, so it is also the two-dimensional rogue wave of (1.3). Thus, we have a rogue wave solution form as follows:

$$\tilde{u} = u_x = 2\frac{2n_1^2 f - 4n_1^2\left(n_1 x - \frac{n7}{5n_1^3}y + \frac{n_7(36n_1^3+n_7)}{180n_1^5}z + \frac{n_7}{10n_1^3}t\right)^2}{f^2}. \tag{6.4}$$

We can easily find that $\tilde{u}$ is also a solution of (1.3), and upper dominant peak and two holes exist in Fig. 6. We provide one group of graphs related to the solution by choosing proper parameters. From (6.4) and Fig. 6, we also find the symmetry of rogue wave will be influenced via some parameters.

As depicted in Fig. 6, we know that the amplitude, velocity and width of the rogue wave solution keep invariable during the propagation. It can show that the amplitudes of the excited state are limited and almost same in different spaces.

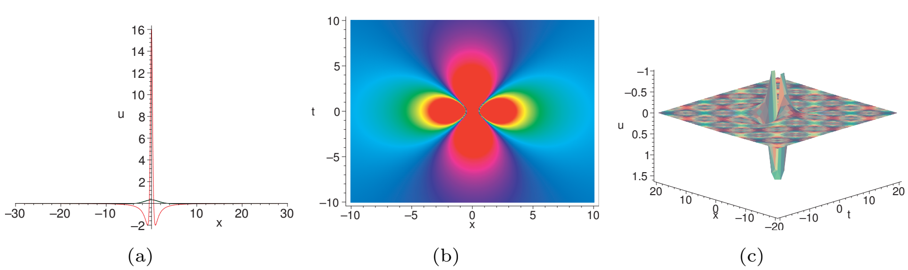


Figure 6. (Color online)The rogue wave solution (6.4) by choosing suitable parameters: $n_1 = 2, n_7 = 3$. (a)Plot3d perspective view of the real part of the wave($y = 0, z = 0$). (b)The overhead view of the wave. (c) The wave propagation pattern of the wave along the x axis ($t = -4, t = 0, t = 4$).

7. Multi-kink solitary wave solutions

Now, we construct the kink solitary wave solutions by assuming f expressed in the terms of the parameter ϵ

$$f = 1 + \epsilon f^1 + \epsilon^2 f^2 + \epsilon^3 f^3 + \cdots. \tag{7.1}$$

Substituting (1.3) into (2.6), and equating the coefficients of all power of ϵ^n, we obtain

$$f^1_{ty} + 5f^1_{xz} - 2f^1_{xxxy} = 0, \tag{7.2}$$

$$2(f^2_{ty} + 5f^2_{xz} - 2f^2_{xxxy}) = -(D_tD_y + 5D_xD_z - D_x^3D_y)f^1 \cdot f^1, \tag{7.3}$$

$$(f^3_{ty} + 5f^3_{xz} - 2f^3_{xxxy}) = -(D_tD_y + 5D_xD_z - D_x^3D_y)f^1 \cdot f^2. \tag{7.4}$$

We get a solution of f from the formula (7.2) as follows:

$$f = 1 + e^{\delta_1}, \tag{7.5}$$

with $\delta_1 = k_1x + l_1y + \alpha_1z + \omega_1t + \xi_1$. Substituting (7.5) into (2.6), we obtain $\omega_1 = \frac{-l_1 \pm \sqrt{l_1^2 + 4k_1^2l_1 - 12k_1\alpha_1}}{2}$. Taking $f^2 = f^3 = \cdots = 0$, so one kink soliton solutions is given by

$$u_1 = 2[\ln(1 + e^{\delta_1})]_x. \tag{7.6}$$

According to the above method, we also obtain the two kink soliton solutions as follows

$$u_2 = 2[\ln(1 + e^{\delta_1} + e^{\delta_2} + e^{\delta_1 + \delta_2 + A_{12}})]_x, \tag{7.7}$$

with $\delta_i = k_ix + l_iy + \alpha_iz + \omega_it + \xi_i$, $\omega_i = \frac{-l_i \pm \sqrt{l_i^2 + 4k_i^2l_i - 12k_i\alpha_i}}{2}$,

$$A_{12} = \frac{(\omega_1 - \omega_2)(l_1 - l_2) - 2(k_1 - k_2)^3(l_1 - l_2) + 5(k_1 - k_2)(\alpha_1 - \alpha_2)}{(\omega_1 + \omega_2)(l_1 + l_2) - 2(k_1 + k_2)^3(l_1 + l_2) + 5(k_1 + k_2)(\alpha_1 + \alpha_2)}, \tag{7.8}$$

where $k_i, l_i, \alpha_i, \omega_i$, and $\omega_i (i = 1, 2)$ are some constants.

As depicted in Fig. 7, we know that the amplitude, velocity and width of the double kink soliton solution keep invariable during the propagation. It can show that the amplitudes of the excited state are limited and almost same in different spaces.

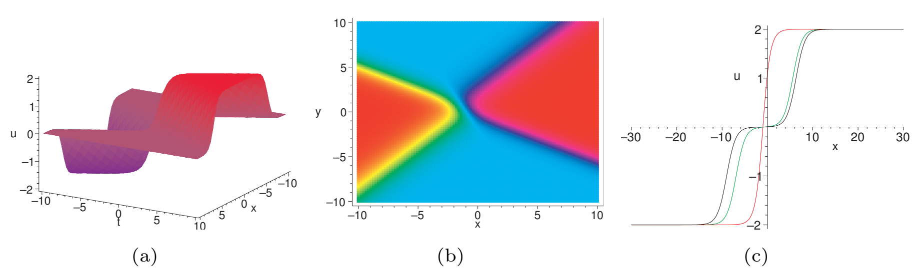


Figure 7. (Color online)The double kink soliton solution (7.7) by choosing suitable parameters: $k_1 = -1, k_2 = 1, l_1 = 1, l_2 = -1, \alpha_1 = 1, \alpha_2 = -1, \xi_1 = 1, \xi_2 = 1$. (a) Plot3d perspective view of the real part of the wave($y = 0$). (b) The overhead view of the wave. (c) The wave propagation pattern of the wave along the x axis ($t = -4, t = 0, t = 4$).

8. Conclusions and discussions

We study the (3+1)-dimensional KP-type equation using the Hirota bilinear method and the graphical representations of the solutions. The traveling wave, lump, interaction solutions, rogue wave and multi-kink solitary wave solutions are obtained and the influence of the parameter choice is analysed. In order to find bilinear formalism and Bäcklund transformation, we need to try them several times, i.e., we need multiple integrals of x to find bilinear formalism and need to try the M or $2M$ multiple times. To construct the lump solutions, we need to satisfy one condition, i.e., $b_3^2 + b_7^2 \neq 0$ and $b_1 > 0$. When we construct the interaction solution between lump solution and one stripe soliton by using quadratic function with exponential function, we also have to satisfy the conditions $k > 0, b_2^2 > b_6^2, k_2 b_2 b_6 \neq 0$.

The traveling wave solution of (3.13) is presented in Fig. 1. We briefly analyse the effects of the free parameters on the amplitude and the widths of the traveling wave solution. Note that the amplitude, velocity and the width of the traveling wave stay the same during the propagation. It can show that the amplitude of the excited state is bounded. The dynamic behaviors of the lump solutions in different planes are demonstrated in Fig. 3. We observed that periodic line waves arise from the constant background by selecting the appropriate parameters $b_1 = 1, b_3 = 2, b_6 = 3, b_7 = 2$. Besides, the amplitude of the excited state is bounded.

From Figs. 1-4, we have shown the velocity, width and amplitude of these exact solutions keep invariable during the propagation. The amplitudes are same and almost limited in different spaces. Finally, we also have shown that a special function was provided to obtained its kink solitary solutions and rogue wave solutions. In Fig. 5, we find it has a pair of peaks in the opposite direction, which is called the new breather wave solution. Its graph can be plotted by choosing appropriate parameters $n_1 = 2, n_7 = 3$. And there have two similar wave shapes of the rogue wave, so it is also the two-dimensional rogue wave of (1.3). Therefore, we take the derivative of u with respect to x, we can derive Eq.(6.4) from Eq.(6.3). We can easily find the upper dominant peak and two holes in Fig. 6, and provide one group of graphs related to the solution by choosing proper parameters $n_1 = 2, n_7 = 3$. From Eq.(6.4) and Fig. 6, we also find the symmetry of rogue wave will be influenced via some parameters. As depicted in Figs. 5-7, it clearly shows this unique feature of these solutions. We also have studied the localization features by employing the contour line method. The method can also be extend to other types of nonlinear evolution equations in mathematical physics. It is hoped that our results can enrich the theories for the associated nonlinear evolution equations.

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References

[1] M. Boiti, L. Martina and O.K. Pashaev, *F. Pempineli, Dynamics of multidimensional solitons*, Phys. Lett. A, 1991, 160, 55–63.

[2] C. Dai and W. Huang, *Multi-rogue wave and multi-braather solutions in PT-symmertric coupled waveguides*, Appl. Math. Lett., 2014, 32, 35–40.

[3] M. Dong, S. Tian, X. Yan and L. Zou, *Solitary waves, homoclinic breather waves and rogue waves of the (3+1)-dimensional Hirota bilinear equation*, Comput. Math. Appl., 2018, 75(3), 957–964.

[4] G. Ebadi, N. Y. Fard, A. H. Bhrawy, S. Kumar, H. Triki, A. Yildirim and A. Biswas, *solitons and other solutions to the (3+1)-dimensional extended kadomtsev-petviashvili equation with power law nonlinearity*, Rom. Rep. Phys., 2013, 65, 27–62.

[5] L. Feng and T. Zhang, *Breather wave, rouge wave and solitary wave solutions of a coupled nonlinear Schödinger equation*, Appl. Math. Lett., 2018, 78, 133–140.

[6] D. Guo, S. Tian and T. Zhang, *Integrability, soliton solutions and modulation instability analysis of a (2+1)–dimensional nonlinear Heisenberg ferromagnetic spin chain equation*, Comput. Math. Appl., 2019, 77(3), 770–778.

[7] Y. Gurefe, E. Misirli, Y. Pandir, A. Sonmezoglu and M. Ekici, *New Exact Solutions of the Davey-Stewartson Equation with Power-Law Nonlinearity*, Bull. Malays. Math. Sci. Soc., 2015, 38, 1223–1234.

[8] C. Hamner, J. Chang, P. Engels and M. A. Hoefer, *Generation of Dark-Bright Soliton Trains in Superfluid-Superfluid Counterflow*, Phys. Rev. Lett., 2010, 106, 065302.

[9] H. Jiang, J. Xiang, C. Dai and Y. Wang, *Nonautonomous bright soliton solutions on continuous wave and cnoidal wave backgrounds in blood vessels*, Nonlinear Dyn., 2014, 75, 201–207.

[10] M. Jimbo and T. Miwa, *Solitons and infinite dimensional lie algebras*, Publ. Res. Inst. Mathe. Sci., 1983, 19, 943–1001.

[11] R. S. Johnson and S. Thompson, *A solution of the inverse scattering problem for the Kadomtsev-Petviashvili equation by the method of separation of variables*, Phys. Lett., 1978, 66(4), 279–281.

[12] P. G. Korteweg and G. Vries, *On the change of form of long waves advancing in a rectangular canal and on a new type of long stationary waves*, Philos Mag., 1895, 39, 422–443.

[13] J. Li and Z. Qiao, *Explicit soliton solutions of the Kaup-Kupershmidt equation through the dynamical system approach*, J. Appl. Anal. Comput., 2011, 1(2), 243–250.

[14] C. Li, J. He and K. Porseizan, *Rogue waves of the Hirota and the Maxwell-Bloch equations*, Phys. Rev. E, 2013, 87, 012913.

[15] W. Ma, *Abundant lumps and their interaction solutions of (3+1)-dimensional linear PDEs*, J. Geom. Phys., 2018, 133, 10–16.

[16] W. Ma and Y. Zhou, *Lump solutions to nonlinear partial differential equations via Hirota bilinear forms*, J. Differential Equations, 2018, 264, 2633–2659.

[17] W. Ma, *Lump solutions to the Kadomtsev-Petviashvili equation*, Phys. Lett. A, 2015, 379, 1975–1978.

[18] W. Ma, Z. Qin and X. La, *Lump soltions to dimensionally reduced p-gKP and p-gBKP equations*, Nonlinear Dyn., 2016, 84(2), 923–931.

[19] W. Ma and J. Yang, *Lump solutions to the BKP equation by symbolic computation*, Int. J. Mod. Phys. B, 2016, 30, 1640028.

[20] W. Ma and Z. Zhu, *Solving the (3+1)-dimensional generalized KP and BKP equations by the multiple exp-function algorithm*, Appl. Math. Comput., 2012, 218, 11871–11879.

[21] W. Ma, Y. Zhang, Y. Tang and J. Tu, *Hirota bilinear equations with linear subspaces of solutions*, Appl. Math. Comput., 2012, 218(13), 7174–7183.

[22] W. Ma and L. Zhang, *Lump solutions with higher-order rational dispersion relations*, Pramana. J. Phys., 2020, 94, ID0043.

[23] W. Ma, *Lump and interaction solutions to linear PDEs in (2+1) dimensions via symbolic computation*, Modern Phys. Lett. B, 2019, 33(36), 1950457.

[24] L. Na, *Bäcklund transformation and multi-soliton solutions for the (3+1)-dimensional BKP equation with Bell polynomials and symbolic computation*, Nonlinear Dyn., 2015, 82(1–2), 311–318.

[25] W. Peng, S. Tian and T. Zhang, *On the Breather Waves, Rogue Waves and Solitary Waves to a Generalized (2+1)-dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada Equation*, Filomat, 2018, 32(14), 4959–4969.

[26] W. Peng, S. Tian and T. Zhang, *Dynamics of breather waves and higher-order rogue waves in a coupled nonlinear Schrodinger equation*, EPL (Europhysics Letters), 2018, 123(5), 50005.

[27] C. Qin, S. Tian, L. Zou and T. Zhang, *Lie symmetry analysis, conservation laws and exact solutions of the generalized time fractional Burgers equation*, J. Appl. Anal. Comput., 2018, 8(6), 1727–1746.

[28] C. Qin, S. Tian, L. Zou and T. Zhang, *Lie symmetry analysis, conservation laws and exact solutions of fourth-order time fractional Burgers equation*, J. Appl. Anal. Comput., 2018, 8(6), 1727–1746.

[29] C. Qin, S. Tian, L. Zou and W. Ma, *Solitary Wave and Quasi-Periodic Wave Solutions to a (3+1)-Dimensional Generalized Calogero-Bogoyavlenskii-Schiff Equation*, Adv. Appl. Math. Mech., 2018, 10, 948–977.

[30] S. Tian and H. Zhang, *On the Integrability of a Generalized Variable-Coefficient Forced Korteweg-de Vries Equation in Fluids*, Stud. Appl. Math., 2014, 132, 212–246.

[31] X. Wang, S. Tian, C. Qin and T. Zhang, *Characteristics of the breather and rogue waves in a (2+1)-dimensional nonlinear Schrödinger equation*, Proc. Amer. Math. Soc., 2018, 146(8), 3353–3365.

[32] X. Wang, S. Tian, L. Zou and T. Zhang, *Dynamics of solitary waves and periodic waves in a (3+1)-dimensional nonlinear evolution equation*, East Asian J. Appl. Math., 2018, 8(3), 477–497.

[33] X. Wang and S. Tian, *Lie symmetry analysis, conservation laws and analytical solutions of the time-fractional thin-film equation*, Comput. Appl. Math., 2018, 37(5), 6270–6282.

[34] X. Wang, S. Tian, C. Qin and T. Zhang, *Characteristics of the breathers, rogue waves and solitary waves in a generalized (2+1)-dimensional Boussinesq equation*, EPL(Europhysics Letters), 2016, 115, 10002.

[35] A. M. Wazwaz, *Two forms of (3+1)dimensional B-type Kadomtsev-Petviashvili equation: multiple soliton solutions*, Phys. Scr., 2012, 86(3), 035007.

[36] A. M. Wazwaz and G. Xu, *Negative-order modified KdV equations: multiple soliton and multiple singular soliton solution*, Math. Methods Appl. Sci., 2016, 39, 661–667.

[37] X. Wen, *Construction of new exact rational form non-travelling wave solutions to the (2+1)-dimensional generalized BroerKaup system*, Appl. Math. Comput., 2010, 217, 1367–1375.

[38] J. Yang, W. Ma and Z. Qin, *Abundant mixed lump-soliton solutions of the BKP equation*, East Asian J. Appl. Math., 2018, 8(2), 224–232.

[39] X. Zhang and Y. Chen, *Rogue wave and a pair of resonance stripe solitons to a reduced (3+1)-dimensional Jimbo-Miwa equation*, Commun. Nonlinear Sci. Numer. Simul., 2017, 52, 24–31.