Journal of Applied Analysis and Computation
Volume 11, Number 1, February 2021, 59–80

Website:http://www.jaac-online.com
DOI:10.11948/20190089

EXISTENCE AND EXPONENTIAL STABILITY OF MILD SOLUTIONS FOR SECOND-ORDER NEUTRAL STOCHASTIC FUNCTIONAL DIFFERENTIAL EQUATION WITH RANDOM IMPULSES*

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Abstract In this paper, we consider the existence and exponential stability in mean square of mild solutions to second-order neutral stochastic functional differential equations with random impulses in Hilbert space. Firstly, the existence of mild solutions to the equations is proved by using the noncompact measurement strategy and the Mönch fixed point theorem. Then, the mean square exponential stability for the mild solution of the considered equations is obtained by establishing an integral inequality. Finally, an example is given to illustrate our results.

Keywords Existence of mild solution, exponential stability, second-order neutral stochastic functional differential equation, random impulsive, Itô integrals, measurement of noncompact.

MSC(2010) 34K50, 34K45, 34F05.

1. Introduction

Impulsive differential equations give a model to describe the system with occurrence of an abrupt change in the state at some time points. Many authors have studied the various kinds of differential equations with fixed time impulses [5, 9, 19, 20, 31]. But in the real world, impulses often exist at random time points. To better reflect this phenomena in reality, Wu and Meng [30] brought forward the general differential equations with random impulses, where the impulsive moments are random variables and any solution of the equations is a stochastic process. The properties of mild solutions to some integer-order differential equations with random impulses have been obtained, for example [1, 6, 13, 14, 16, 24, 25, 27, 28]. The random impulsive differential equations involving fractional derivative also have been discussed in [26, 32] and so on.

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*The authors were supported by the National Natural Science Foundation of China (61773217, 61374080) and the Innovation Platforms Open Foundation of Hunan Educational Committee (541109100002).

As we known, impulsive stochastic differential equations played an important role in modeling many practical processes. However, in some actual cases, random effects not only arising from Gaussian white noise but also from other factors. Therefore, it is significant to integrate random impulsive effects into systems. As far as we known, the earliest research on stochastic differential equation with random impulses can been seen in [29]. The existence and uniqueness of solutions to the following stochastic differential equation with random impulses have been investigated by Zhou and Wu [33],

$$\begin{cases} dx(t) = \mu(t, x(t))dt + \sigma(t, x(t))dw_t \quad a.e., t > t_0,\ t \neq \xi_k, \\ x(\xi_k) = b_k(\tau_k)x(\xi_k^-) \quad a.e., \qquad\qquad\qquad k = 1, 2, \cdots, \\ x_{t_0} = z, \end{cases}$$

where τ_k denotes the waiting time and ξ_k denotes the impulsive moment. Both are random variables and satisfy $\xi_0 = t_0, \xi_k = \xi_{k-1} + \tau_k$ for $k = 1, 2, \cdots$; w_t is an m-dimensional Wiener process; z is a random variable. Li et al. [11] have discussed a class of random impulsive neutral stochastic functional evolution inclusions with the same type impulsive condition as in [33]. Authors have proved the existence of mild solutions for their considered equations by using multi-valued mapping fixed point theorem of kakutani type and theory of evolution systems under the assumption that the semigroup is compact. Recently, the existence and Hyers-Ulam stability of mild solutions for random impulsive stochastic functional ordinary differential equations have considered in [10] by using Krasnoselskii's fixed point theorem.

As one of the differential equations with important applications, second-order differential equations have also drawn attentions of more and more scholars. The study of the second-order stochastic differential equations have been considered by many researchers, such as [2, 3, 7, 18, 23] and the references therein.

Motivated by the above discussion, in this paper, we consider the following second-order neutral stochastic functional differential equations with random impulses:

$$\begin{cases} d[x'(t) - g(t, x_t)] = [Ax(t) + f(t, x_t)]dt + \sigma(t, x_t)dW(t),\ t > t_0,\ t \neq \xi_k, \\ x(\xi_k) = b_k(\tau_k)x(\xi_k^-),\ x'(\xi_k) = b_k(\tau_k)x'(\xi_k^-),\ k = 1, 2, \ldots, \\ x_{t_0} = \phi,\ \ x'(t_0) = \psi, \end{cases} \tag{1.1}$$

where $A : D(A) \subset H \to H$ is the infinitesimal generator of a strongly continuous cosine family $\{C(t), t \geq 0\}$. $W(t)$ is a given Q-Winer process with a finite trace nuclear covariance operator $Q > 0$. τ_k is a random variable defined from Ω to $D_k \equiv (0, d_k)$ for $k = 1, 2, \cdots$. Suppose that τ_i and τ_j are independent of each other as $i \neq j$, $(i, j = 1, 2, \cdots)$. The impulsive moments ξ_k are random variables and satisfy $\xi_k = \xi_{k-1} + \tau_k$, $k = 1, 2, \cdots$, Obviously, $\{\xi_k\}$ is a processes with independent increments. $0 < t_0 = \xi_0 < \xi_1 < \xi_2 < \cdots < \xi_k < \cdots < \lim_{k\to\infty} \xi_k = \infty$, and $x(\xi_k^-) = \lim_{t\to\xi_{k-0}} x(t)$. $b_k : D_k \to H$, for each $k = 1, 2, \cdots$. The time history $x_t(\theta) = \{x(t + \theta) : -\tau \leq \theta \leq 0\}$ with some given $\tau > 0$. Moreover, g, f, σ,and ϕ, ψ will be specified later.

The main purpose of this paper is to study the existence and exponential stability of system (1.1). Firstly, basing on the properties of sine and cosine operators and taking the random impulsive efference into account, we give the definition of mild solutions of (1.1). Without the need to assume that the system (1.1) generates a compact semigroup, we deal with the existence problems of mild solution to this system by using Mönch fixed point via measure of noncompactness. In particular, to overcome the difficulty caused by the presence of random term, we apply a new noncompact measure criterion for Itô integrals. Then, we discuss the exponential stability of mild solution for considered system by establishing an integral inequality that applied to second-order functional differential equations with random impulse. Our work may generalize some existing results of second-order impulsive stochastic differential equations to more general random impulses cases.

The rest of our paper contains the following five sections. Section 2 provides some basic definitions, notations and lemmas. Section 3 is devoted to the existence of mild solutions of (1.1) under weakly compactness conditions combining with the strategies of noncompact measurement and Mönch fixed point theorem. In Section 4, the mean square exponential stability for the mild solution of (1.1) is studied in Hilbert spaces. In Section 5, an example is given to show our exponential stability result. At last, we conclude the paper and give the future research direction in Section 6.

2. Preliminaries

Let H and K be two real Hilbert spaces. For convenience, we denote their norm by $\|\cdot\|$ and denote their inner product by $<\cdot,\cdot>$. $L(K,H)$ represent the space of all bounded linear operators from K into H. Let $(\Omega,\mathscr{F},P)$ be a complete probability space equipped with a normal filtration $\{\mathscr{F}_t\}_{t\geq t_0}$. $\mathscr{F}_{t_0}$ containing all P-null sets. We suppose $\{\xi_k, k\geq 0\}$ generate a counting process $\{N(t), t\geq t_0\}$ and $\mathscr{F}_t^{(1)}$ denote the minimal σ-algebra generated by $\{N(r), r\leq t\}$. We suppose $\{W(t), t\geq t_0\}$ is a K-valued Winer process and denote the $\mathscr{F}_t^{(2)}=\sigma\{W(r), r\leq t\}$. Referring to [29], we assume that $\mathscr{F}_\infty^{(1)}$, $\mathscr{F}_\infty^{(2)}$, and $\mathscr{F}_{t_0}$-adapt random variables ϕ, φ are mutually independent, and $\mathscr{F}_t=\mathscr{F}_t^{(1)}\vee\mathscr{F}_t^{(2)}$.

We assume that there exists a complete orthonormal system $\{e_n\}_{n=1}^\infty$ in K, a bounded sequence of non-negative real numbers λ_n such that $Qe_n=\lambda_n e_n, n=1,2,\ldots$. Let $\{\beta_n(t)\}(n=1,2,\ldots)$ be a sequence of real valued one-dimensional standard Brownian motions mutually independent over $(\Omega,\mathscr{F},P)$. A Q-Wiener process can be defined by $W(t)=\Sigma_{n=1}^\infty\sqrt{\lambda_n}\beta_n(t)e_n,\ (t\geq 0)$. Set $\psi\in L(K,H)$, we define

$$\|\psi\|_Q^2=Tr(\psi Q\psi^*)=\sum_{n=1}^{\infty}\|\sqrt{\lambda_n}\psi e_n\|^2.$$

If $\|\psi\|_Q^2<\infty$ then ψ is called a Q-Hilbert-Schmidt operator. Let $L_Q(K,H)$ denote the space of all Q-Hilbert-Schmidt operator $\psi: K\to H$. The completion $L_Q(K,H)$ of $L(K,H)$ with respect to the topology induced by the norm $\|\cdot\|_Q$, where $\|\psi\|_Q^2=<\psi,\psi>$ is a Hilbert space.

Let $T\in(t_0,+\infty), J:=[t_0,T], J_k=[\xi_k,\xi_{k+1}), k=0,1,\ldots,\widetilde{J}=\{t: t\in J, t\neq \xi_k, k=1,2\ldots\}$. We denote $L_2(\Omega,H)$ the collection of all square integrable, $\mathscr{F}_t$-measurable, H-valued random variables, with the norm $\|x\|_{L_2}=(E\|x\|^2)^{\frac{1}{2}}$, where the expectation E is defined by $E\|x\|^2=\int_\Omega\|x\|^2dP$. $L_2^0(\Omega,H)$ denotes the family of all square integrable $\mathscr{F}_{t_0}$-measurable, H-valued random variable in $L_2(\Omega,H)$. Let piecewise continuous space $PC(J, L_2(\Omega, H)) = \{x : J \to L_2(\Omega, H) : x$ is continuous on every J_k, and the left limits $x(\xi_k^-)$, $x'(\xi_k^-)$ exist, $k = 1, 2, \dots\}$

In this paper, we denote the space $\mathcal{C} = \mathcal{C}([-\tau, 0], H)$ consisting of all piecewise continuous functions mapping from $[-\tau, 0]$ to H with the norm $\|x\|_t = \sup\limits_{t-\tau \le s \le t} \|x(s)\|$ for each $t \ge t_0$. We denote by $\mathcal{B}$ the Banach space $\mathcal{B}([t_0 - \tau, T], L_2(\Omega, H))$, consisting of piecewise continuous, $\mathscr{F}_t$-measurable, $\mathcal{C}$-valued processes. The norm defined by

$$\|x\|_{\mathcal{B}} = (\sup_{t \in J} E\|x\|_t^2)^{\frac{1}{2}}.$$

Next we introduce some definitions and properties of sine and cosine operators. More details can refer to [22].

A bounded linear operators family $\{C(t), t \in \mathbb{R}\}$ is called a strongly continuous cosine family if and only if

(i) $C(0) = I$ (I is the identity operator in H);

(ii) $C(t)x$ is continuous in t, for all $x \in H$;

(iii) $C(t + s) + C(t - s) = 2C(t)C(s)$ for all $t, s \in \mathbb{R}$.

The corresponding strongly continuous sine family $\{S(t), t \in \mathbb{R}\}$ is defined by

$$S(t)x = \int_0^t C(s)x ds, \ \ x \in H, \ t \in \mathbb{R}. \tag{2.1}$$

Then the following property holds:

$$A \int_{t_0}^t S(s)x ds = [C(t) - C(t_0)]x. \tag{2.2}$$

Lemma 2.1 ([22]). *Let $\{C(t), t \in \mathbb{R}\}$ be a strongly continuous cosine family in H, then for all $s, t \in \mathbb{R}$ the following results are true:*

(i) C(t)=C(-t);

(ii) S(s+t)+S(s-t)=2S(s)C(t);

(iii) S(s+t)=S(s)C(t)+S(t)C(s);

(iv) S(t)=-S(-t);

(v) C(t+s)+C(s-t)=2C(s)C(t);

(vi) C(t+s)-C(t-s)=2AS(t)S(s).

Before investigating the mild solution of (1.1), we consider the second-order neutral functional differential equation, which is given by

$$\begin{cases} d[u'(t) - g(t, u_t)] = Au(t)dt + f(t, u_t)dt, \ t \ge 0, \\ u_0 = \phi \in \mathcal{C}, u'(0) = \varphi \in H, \ t \in (-\tau, 0], \end{cases} \tag{2.3}$$

where A is the infinitesimal generator of a strongly continuous cosine family $\{C(t), t \in \mathbb{R}^+\}$ and the functions $g, f \in L^1(0, T; H)$.

Lemma 2.2. *A continuously differentiable function $u(t) : [0, T] \to H$ is called the mild solution for Cauchy problem* (2.3)*, if it satisfies*

$$u(t) = C(t)\phi(0) + S(t)[\varphi - g(0, \phi)] + \int_0^t C(t-s)g(s, u_s)ds + \int_0^t S(t-s)f(s, u_s)ds \ \ t \ge 0,$$

where

$$S(t) = \frac{1}{2\pi i}\int_{\Gamma} e^{\lambda t}R(\lambda^2;A)d\lambda;$$

$$C(t) = \frac{1}{2\pi i}\int_{\Gamma} e^{\lambda t}\lambda R(\lambda^2;A)d\lambda,$$

and Γ *is a suitable path.*

Proof. Using the Laplace transform,

$$\begin{aligned}\mathcal{L}(d[u'(t)-g(t,u_t)])(\lambda) &= \lambda\mathcal{L}\big(u'(t)-g(t,u_t)\big)(\lambda)-[u'(0)-g(0,u_0)]\\ &= \lambda^2\mathcal{L}(u(t))(\lambda)-\lambda\phi(0)-\lambda\mathcal{L}(g(t,u_t))(\lambda)-[\varphi-g(0,\phi)].\end{aligned}$$

It follows from (2.3) that

$$\lambda^2\mathcal{L}(u(t))(\lambda)-\lambda\phi(0)-\lambda\mathcal{L}(g(t,u_t))(\lambda)-[\varphi-g(0,\phi)] = A\mathcal{L}(u(t))(\lambda)+\mathcal{L}(f(t,u_t))(\lambda).$$

Thus, we have

$$\begin{aligned}\mathcal{L}(u(t))(\lambda) &= (\lambda^2 I-A)^{-1}\big[\lambda\phi(0)+\varphi-g(0,\phi)+\lambda\mathcal{L}(g(t,u_t))(\lambda)+\mathcal{L}(f(t,u_t))(\lambda)\big]\\ &= \mathcal{L}(C(t))(\lambda)\phi(0)+\mathcal{L}(S(t)[\varphi-g(0,\phi)])(\lambda)+\mathcal{L}(C(t)*(g(t,u_t))(\lambda)\\ &\quad+\mathcal{L}(S(t)*(f(t,u_t))(\lambda).\end{aligned}$$

Then taking the inverse Laplace transform, we get

$$u(t) = C(t)\phi(0)+S(t)[\varphi-g(0,\phi)]+\int_0^t C(t-s)g(s,u_s)ds+\int_0^t S(t-s)f(s,u_s)ds.$$

The proof is completed. □

Take any value t_k of the random variables ξ_k, then the differential equation with random impulses can be regarded as that with general impulses. Consider the linear second-order differential equation with impulses conditions as shown below:

$$\begin{cases} u''(t) = Au(t)+f(t), t\geq 0,\ t\neq t_k,\\ u(0)=u_0, u'(0)=v_0,\\ u(t_k)=b_k u(t_k^-), u'(t_k)=b_k u'(t_k^-),\ k=1,2,\dots, \end{cases} \tag{2.4}$$

where $0=t_0<t_1<t_2<\cdots<t_k<\cdots$, $\{t_k, k\geq 1\}$ is a sequence of fixed impulsive points, $f(t):[0,T)\to H$ is an integrable function.

Lemma 2.3. *The piecewise continuous differentiable function* $u(t):[0,T]\to H$ *is a mild solution of* (2.4), *if and only if* $x(t)$ *satisfies the integral equation*

$$\begin{aligned}u(t) = &\prod_{i=1}^{k} b_i C(t)u_0+\prod_{i=1}^{k} b_i S(t)v_0+\sum_{i=1}^{k}\prod_{j=i}^{k} b_j\int_{t_{i-1}}^{t_i} S(t-s)f(s)ds\\ &+\int_{t_k}^{t} S(t-s)f(s)ds, \qquad t\in[t_k,t_{k+1}), k=0,1,\cdots.\end{aligned} \tag{2.5}$$

Proof. (i) For $t \in [0, t_1)$, the mild solution has been studied in [21],

$$u(t) = C(t)u_0 + S(t)v_0 + \int_0^t S(t-s)f(s)ds, \qquad t \in [0, t_1).$$

(ii) For $t \in [t_1, t_2)$, we set

$$u(t) = C(t-t_1)u(t_1) + S(t-t_1)u'(t_1) + \int_{t_1}^t S(t-s)f(s)ds, \ t \in [t_1, t_2). \tag{2.6}$$

Since

$$u(t_1) = b_1 u(t_1^-), \ u'(t_1) = b_1 u'(t_1^-),$$

and from (i) we know

$$u(t_1^-) = C(t_1)u_0 + S(t_1)v_0 + \int_0^{t_1} S(t_1-s)f(s)ds; \tag{2.7}$$

$$u'(t_1^-) = AS(t_1)u_0 + C(t_1)v_0 + \int_0^{t_1} C(t_1-s)f(s)ds. \tag{2.8}$$

Thus,

$$\begin{aligned} u(t) =& b_1 C(t-t_1)C(t_1)u_0 + b_1 S(t-t_1)AS(t_1)u_0 \\ &+ b_1 C(t-t_1)S(t_1)v_0 + b_1 S(t-t_1)C(t_1)v_0 \\ &+ b_1 C(t-t_1)\int_0^{t_1} S(t_1-s)f(s)ds + b_1 S(t-t_1)\int_0^{t_1} S(t_1-s)f(s)ds \\ &+ \int_{t_1}^t S(t-s)f(s)ds, \ \ t \in [t_1, t_2). \end{aligned}$$

Applying Lemma 2.1, we get

$$\begin{aligned} u(t) =& b_1 C(t)u_0 + b_1 S(t)v_0 + b_1 \int_0^{t_1} S(t_1-s)f(s)ds \\ &+ \int_{t_1}^t S(t-s)f(s)ds, \ \ t \in [t_1, t_2). \end{aligned}$$

(iii) For $t \in [t_2, t_3)$,

$$\begin{aligned} u(t) &= C(t-t_2)u(t_2) + S(t-t_2)u'(t_2) + \int_{t_2}^t S(t-s)f(s)ds \\ &= C(t-t_2)b_2 u(t_2^-) + S(t-t_2)b_2 u'(t_2^-) + \int_{t_2}^t S(t-s)f(s)ds. \end{aligned} \tag{2.9}$$

From the conclusions of (ii), we know

$$u(t_2^-) = b_1 C(t_2)u_0 + b_1 S(t_2)v_0 + b_1 \int_0^{t_2} S(t_2-s)f(s)ds + \int_{t_2}^t S(t_2-s)f(s)ds; \tag{2.10}$$

$$u'(t_2^-) = b_1 AS(t_2)u_0 + b_1 C(t_2)v_0 + b_1 \int_0^{t_1} C(t_2-s)f(s)ds + \int_{t_1}^{t_2} C(t_2-s)f(s)ds. \tag{2.11}$$

Taking these into (2.9) and using Lemma 2.1, we have

$$u(t) = b_2 b_1 C(t)u_0 + b_2 b_1 S(t)v_0 + b_2 b_1 \int_0^{t_1} S(t-s)f(s)ds + b_2 \int_{t_1}^{t_2} S(t-S)f(s)ds \\ + \int_{t_2}^{t} S(t-S)f(s)ds,\ t \in [t_2, t_3).$$

Similarly, for all $t \in [t_k, t_{k+1})$,

$$x(t) = \prod_{i=1}^{k} b_i C(t)u_0 + \prod_{i}^{k} b_i S(t)v_0 \\ + \sum_{i=1}^{k} \prod_{j=i}^{k} b_j \int_{t_{i-1}}^{t_i} S(t-s)f(s)ds + \int_{t_k}^{t} S(t-s)f(s)ds, \qquad t \in [t_k, t_{k+1}).$$

□

If the impulses exist in random, that is to say, we do not know when or how many impulses occur in a period of time. Hence, according to Lemmas 2.2, 2.3, we define the mild solution of system (1.1), applying index function, for $t \in J$.

Definition 2.1. For a given $T \in (t_0, +\infty)$, a $\mathscr{F}_t$-adapted process function $\{x \in \mathcal{B}, t_0 - \tau \leq t \leq T\}$ is called a mild solution of system (1.1), if

(i) $x_{t_0}(s) = \phi(s) \in L_2^0(\Omega, \mathcal{B})$ for $-\tau \leq s \leq 0$;

(ii) $x'(t_0) = \varphi(t) \in L_2^0(\Omega, H)$ for $t \in J$;

(iii) the functions $g(s, x_t), f(s, x_t), \sigma(s, x_t)$ are integrable, and for a.e. $t \in J$, the following integral equation is satisfied:

$$x(t) = \sum_{k=0}^{+\infty} \Big[\prod_{i=1}^{k} b_i(\tau_i) C(t-t_0)\phi(0) + \prod_{i}^{k} b_i(\tau_i) S(t-t_0)[\varphi - g(0,\phi)] \\ + \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} C(t-s)g(s,x_s)ds + \int_{\xi_k}^{t} C(t-s)g(s,x_s)ds \\ + \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S(t-s)f(s,x_s)ds + \int_{\xi_k}^{t} S(t-s)f(s,x_s)ds \\ + \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S(t-s)\sigma(s,x_s)dW(s) \\ + \int_{\xi_k}^{t} S(t-s)\sigma(s,x_s)dW(s) \Big] I_{[\xi_k, \xi_{k+1})}(t), \qquad t \in [t_0, T], \tag{2.12}$$

where

$$\prod_{j=i}^{k} b_j(\tau_j) = b_k(\tau_k) b_{k-1}(\tau_{k-1}) \cdots b_i(\tau_i),$$

and $I_A(\cdot)$ is the index function, i.e.,

$$I_A(t) = \begin{cases} 1, \text{ if t} \in \text{A}, \\ 0, \text{ if t} \notin \text{A}. \end{cases}$$

Next, we recall the following relevant knowledges of non-compactness measure theory.

The definition of the Hausdorff non-compactness measure for a bounded set B in any Hilbert space H can be described as follows:

$$\alpha(B) = \inf\{\varepsilon > 0;\ B \text{ has a finite } \varepsilon-\text{net in } H\}.$$

Lemma 2.4 (see [4]). *Let H is a real Hilbert space, $B, D \subset H$ are bounded sets. Then, the following properties hold:*

(1) B is called a precompact set if and only if $\alpha(B) = 0$;

(2) $\alpha(B) = \alpha(\bar{B}) = \alpha(\bar{co}(B))$, where $\bar{B}$ and $\bar{co}$ B are the closure and the convex hull of B, respectively;

(3) If bounded subsets B, D in H, $B \subseteq D$, then $\alpha(B) \leq \alpha(D)$;

(4) $\alpha(\{x\} \cup B) = \alpha(B)$, for all $x \in H$, and all nonempty subset $B \subset H$;

(5) $\alpha(B + D) \leq \alpha(B) + \alpha(D)$, where $B + D = \{x + y; x \in B, y \in D\}$;

(6) $\alpha(B \cup D) \leq \max\{\alpha(B), \alpha(D)\}$;

(7) $\alpha(\lambda B) \leq |\lambda|\alpha(B)$ for any $\lambda \in R$;

(8) If $D \subset C(J; H)$ is bounded, then

$$\alpha(D(t)) \leq \alpha(D), \tag{2.13}$$

where

$$D(t) = \{u(t) :\ \textit{for all}\, u \in D, t \in J\}.$$

Furthermore if D is equicontinuous on J, then $D(t)$ is continuous for $t \in J$, and

$$\alpha(D) = \sup_{t \in J} \alpha(D(t)). \tag{2.14}$$

(9) If $D \subset C(J; H)$ is bounded and equicontinous, then $\alpha(D(t))$ is continuous on J, and

$$\alpha\left(\int_0^t D(s)ds\right) \leq \int_0^t \alpha(D(s))ds, \ \ \textit{for all}\ \ t \in J, \tag{2.15}$$

where

$$\int_0^t D(s)ds = \left\{\int_0^t u(s)ds :\ \ \textit{for all}\ \ u \in D, t \in J\right\};$$

(10) Let $\{u_n\}_{n=1}^{\infty}$ be a sequence of Bochner integrable functions from J to H, then $D = \{u_n\}_{n=1}^{\infty}$ is a bounded and countable set, and $\alpha(D(t))$ is the Lebesgue integral on H, satisfying

$$\alpha\left(\left\{\int_0^t u_n(s)ds : n \geq 1\right\}\right) \leq 2\int_0^t \alpha(D(s))ds. \tag{2.16}$$

(11) If D is bounded, then for any $\varepsilon > 0$, have sequence $\{u_n\}_{n=1}^{\infty} \subset D$, such that

$$\alpha(D) \leq 2\alpha(\{u_n\}_{n=1}^{\infty}) + \varepsilon. \tag{2.17}$$

Lemma 2.5. *For the case that $D \subset PC(J, H)$, it has been discussed in [8]. The following results can be obtained.*

(i) If $D \subset PC(J, H)$ is bounded, then $\alpha(D(t)) \leq \alpha(D)$ for all $t \in J$, where $D(t) = \{u(t) :\ \textit{for all}\, u \in D, t \in J\}$;

(ii) Furthermore, if D is equicontinuous on each $[t_k, t_{k+1})$, $k = 0, 1, \cdots$, and equicontinuous at $\{t_k^-, k = 1, 2, \cdots\}$, then $\alpha(D) = \sup_{t\in J} \alpha(D(t))$;

(iii) If $D \subset PC(J, H)$ satisfies both conditions in (i) and (ii), then $\alpha\left(\int_0^t D(s)ds\right) \le \int_0^t \alpha(D(s))ds$.

In order to deal with the measure of stochastic integral term, we need the following conclusions:

Lemma 2.6 (see [15])**.** *For any $p \ge 1$, and for an $L_Q(K, H)$-valued predictable process $u(\cdot)$ such that*

$$\sup_{s\in[0,t]} E\big\|\int_0^s u(\eta)dW(\eta)\big\|^{2p} \le (p(2p-1))^p (\int_0^t (E\big\|u(s)\big\|_Q^{2p})^{1/p} ds)^p, \quad t \in J.$$

Lemma 2.7. *If the set $D \subset L^p(J; L_Q(K, H))$, $W(t)$ is a Q-Wiener process,then for any $p \ge 2$, $t \in [t_0, T]$, Hausdorff non-compactness measure α satisfies*

$$\alpha\big(\int_{t_0}^t D(s)dW(s)\big) \le \sqrt{(T - t_0)\frac{p}{2}(p-1)}\alpha(D(t)),$$

where

$$\int_{t_0}^t D(s)dW(s) = \{\int_{t_0}^t u(s)dW(s) : for\ all\ u \in D, t \in [t_0, T]\}.$$

Details of the proof can refer to [17].

Combining with the Hausdorff's measure of noncompactness, we use Mönch fixed point thorem to prove the existence of mild solutions of systems (1.1).

Lemma 2.8 (Mönch's Fixed Point Theorem)**.** *Let D is a bounded convex subsets of H, and $0 \in D$. If a map $F : D \to H$ is continuous, and satisfies Mönch's conditions, i.e., there exist a countable set $M \subseteq D$, $M \subseteq \overline{co}(\{0\} \cup F(M))$, such that $\overline{M}$ is a compact set. Then F has a fixed point in D.*

Definition 2.2. The solution of equation (1.1) is said to be exponentially stable in mean square, if there exists positive constants $C > 0$ and $\lambda > 0$ such that

$$E\|x(t)\|^2 \le Ce^{-\lambda t},\ t \ge t_0.$$

3. Existence of Mild Solution

In this section, we prove the existence of the mild solutions of random impulsive stochastic differential equation (1.1). we need the following assumptions.

(H1) $S(t), C(t)$ $(t \in J)$ are equicontinuous and there exist positive constants $N, \tilde{N}$ such that

$$\sup_{t\in J} \|C(t)\| \le N,\ \sup_{t\in J} \|S(t)\| \le \tilde{N}. \tag{3.1}$$

(H2) The function $f : J \times \mathcal{C} \to H$ satisfies the following conditions:

(i) For each $t \in J$, the function $f(t, \cdot) : \mathcal{C} \to H$ is continuous; and for each $v \in \mathcal{C}$, the function $f(\cdot, v) : J \to H$ is measurable.

(ii) There exist a continuous function $m(t) \in L^1(J, \mathbb{R}^+)$, and a continuous positive nondecreasing function $\Theta_f : \mathbb{R}^+ \to \mathbb{R}^+$, such that

$$E\|f(t, v)\|^2 \le m(t)\Theta_f(E\|v\|_t^2),$$

for arbitrary $(t, v) \in J \times \mathcal{C}$, and the function Θ_f satisfying

$$\lim_{r \to \infty} \inf \frac{\Theta_f(r)}{r} = 0.$$

(iii) There exists a positive function $K_f(t) \in L^1(J, \mathbb{R}^+)$, for arbitrary bounded subset $Q \subset \mathcal{C}$, the Hausdorff non-compact measure β satisfies

$$\beta(f(t, Q)) \leq K_f(t) \sup_{-\tau \leq \theta \leq 0} \beta(Q(\theta)).$$

(H3) The function $g : J \times \mathcal{C} \to H$ satisfies that:

(i) For each $t \in J$, the function $g(t, \cdot) : \mathcal{C} \to H$ is continuous; and for each $v \in \mathcal{C}$, the function $g(\cdot, v) : J \to H$ is measurable.

(ii) There exist a continuous function $n(t) \in L^1(J, \mathbb{R}^+)$, and a continuous positive nondecreasing function $\Theta_g : \mathbb{R}^+ \to \mathbb{R}^+$, such that

$$E\|g(t, v)\|^2 \leq n(t)\Theta_g(E\|v\|_t^2),$$

for arbitrary $(t, v) \in J \times \mathcal{C}$, and the function Θ_g satisfying

$$\lim_{r \to \infty} \inf \frac{\Theta_g(r)}{r} = 0.$$

(iii) There exists a positive function $K_g(t) \in L^1(J, \mathbb{R}^+)$, for arbitrary bounded subset $Q \subset \mathcal{C}$, the Hausdorff non-compact measure β satisfies

$$\beta(g(t, Q)) \leq K_g(t) \sup_{-\tau \leq \theta \leq 0} \beta(Q(\theta)).$$

(H4) The function $\sigma : J \times \mathcal{C} \to L_Q(K, H)$ satisfies the following:

(i) For each $t \in J$, the function $\sigma(t, \cdot) : \mathcal{C} \to L_Q(K, H)$ is continuous, and for each $v \in \mathcal{C}$, the function $\sigma(\cdot, v) : J \to L_Q(K, H)$ is measurable.

(ii) There exist a continuous function $\mu(t) \in L^1(J, \mathbb{R}^+)$, and a continuous positive nondecreasing function $\Theta_\sigma : \mathbb{R}^+ \to \mathbb{R}^+$, such that

$$E\|\sigma(t, v)\|^2 \leq \mu(t)\Theta_\sigma(E\|v\|_t^2),$$

for arbitrary $(t, v) \in J \times \mathcal{C}$, and the function Θ_σ satisfying

$$\lim_{r \to \infty} \inf \frac{\Theta_\sigma(r)}{r} = 0.$$

(iii) There exists a positive function $K_\sigma(t) \in C(J, \mathbb{R}^+)$, for arbitrary bounded subset $Q \subset \mathcal{C}$, the Hausdorff non-compact measure β satisfies

$$\beta(\sigma(t, Q)) \leq K_\sigma(t) \sup_{-\tau \leq \theta \leq 0} \beta(Q(\theta)), \ K_\sigma^* = \sup_{t \in J} K_\sigma(t).$$

(H5) $E\Big\{ \max\limits_{i,k}\{\prod_{j=i}^k \|b_j(\tau_j)\|\}\Big\} < \infty$, i.e., there exist constants M, for all $\tau_j \in D_j$ $(j = 1, 2, \ldots)$, such that

$$E\Big\{ \max_{i,k}\{\prod_{j=i}^{k} \|b_j(\tau_j)\|\}\Big\} \leq M.$$

(H6) Let

$$
\begin{aligned}
H :=& 2N\max\{1,M\}\|K_g\|_{L^1(J,R^+)} + 2\widetilde{N}\max\{1,\widetilde{M}\}\|K_f\|_{L^1(J,R^+)}\\
&+ 2\widetilde{N}\max\{1,M\}K_\sigma^*\sqrt{(T-t_0)Tr(Q)} < 1.
\end{aligned}
$$

Theorem 3.1. *If assumptions (H1)-(H6) are satisfied, then there exists at least one mild solution of the system* (1.1).

Proof. We define the operator $\Phi : \mathcal{B} \to \mathcal{B}$ by Φx such that

$$
\begin{aligned}
(\Phi x)(t) =& \phi(t),\ t \in [t_0-\tau, t_0],\\
(\Phi x)(t) =& \sum_{k=0}^{+\infty}\Big[\prod_{i=1}^{k} b_i(\tau_i)C(t-t_0)\phi(0) + \prod_{i}^{k} b_i(\tau_i)S(t-t_0)[\varphi - g(0,\phi)]\\
&+ \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} C(t-s)g(s,x_s)ds + \int_{\xi_k}^{t} C(t-s)g(s,x_s)ds\\
&+ \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} S(t-s)f(s,x_s)ds + \int_{\xi_k}^{t} S(t-s)f(s,x_s)ds\\
&+ \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} S(t-s)\sigma(s,x_s)dW(s)\\
&+ \int_{\xi_k}^{t} S(t-s)\sigma(s,x_s)dW(s)\Big]I_{[\xi_k,\xi_{k+1})}(t), \qquad t \in [t_0, T].
\end{aligned}
$$

Then the problem of finding mild solutions for problem (1.1) is reduced to finding the fixed point of Φ. Let $B_r = \{x \in \mathcal{B} : \|x\|_{\mathcal{B}}^2 \le r\}$ stands for the closed ball with center at x and radius $r > 0$ in $\mathcal{B}$. We divide the proof into several steps:

Step I. We prove that there exits r such that Φ maps B_r into B_r.

For $t \in J$ we have

$$
\begin{aligned}
E\|(\Phi x)(t)\|^2 \le& 5E\Big[\sum_{i=1}^{\infty}\prod_{i=1}^{k}\|b_i(\tau_i)\|\|C(t-t_0)\|\|\phi(0)\|I_{[\xi_k,\xi_{k+1})}(t)\Big]^2\\
&+ 5E\Big[\sum_{i=1}^{\infty}\prod_{i=1}^{k}\|b_i(\tau_j)\|\|\varphi - g(0,\phi)\|I_{[\xi_k,\xi_{k+1})}(t)\Big]^2\\
&+ 5E\Big[\sum_{i=1}^{\infty}[\sum_{i=1}^{k}\prod_{j=i}^{k}\|b_j(\tau_j)\|\int_{\xi_{i-1}}^{\xi_i}\|C(t-s)\|\|g(s,x_s)\|ds\\
&+ \int_{\xi_k}^{t}\|C(t-s)\|\|g(s,x_s)\|ds]I_{[\xi_k,\xi_{k+1})}(t)\Big]^2\\
&+ 5E\Big[\sum_{i=1}^{\infty}[\sum_{i=1}^{k}\prod_{j=i}^{k}\|b_j(\tau_j)\|\int_{\xi_{i-1}}^{\xi_i}\|S(t-s)\|\|f(s,x_s)\|ds\\
&+ \int_{\xi_k}^{t}\|S(t-s)\|\|f(s,x_s)\|ds]\Big]I_{[\xi_k,\xi_{k+1})}(t)\Big]^2\\
&+ 5E\Big[\sum_{i=1}^{\infty}[\sum_{i=1}^{k}\prod_{j=i}^{k}\|b_j(\tau_j)\|\int_{\xi_{i-1}}^{\xi_i}\|S(t-s)\|\|\sigma(s,x_s)\|dW(s)
\end{aligned}
$$

$$+\int_{\xi_k}^{t}\|S(t-s)\|\|\sigma(s,x_s)\|dW(s)\big]I_{[\xi_k,\xi_{k+1})}(t)\Big]^2 := 5\sum_{i=1}^{5}R_i,$$

where

$$\begin{aligned}
R_1 &\le N^2E\big\{\max_k\{\prod_{i=1}^{k}\|b_i(\tau_i)\|\}\big\}^2E\|\phi(0)\|^2 \le N^2M^2E\|\phi(0)\|^2,\\
R_2 &\le \widetilde{N}^2E\Big\{\max_k\{\prod_{i=1}^{k}\|b_i(\tau_i)\|\}\Big\}^2E\|\varphi-g(0,\phi)\|^2\\
&\le M^2\widetilde{N}^2E\|\varphi-g(0,\phi)\|^2,\\
R_3 &\le N^2E\Big\{\max_{i,k}\{1,\prod_{j=i}^{k}\|b_j(\tau_j)\|\}\Big\}^2(T-t_0)\int_{t_0}^{t}E\|g(s,x_s)\|^2ds\\
&\le N^2\max\{1,M^2\}(T-t_0)\int_{t_0}^{t}n(t)\Theta_g(E\|x\|_s^2)ds,\\
R_4 &\le \widetilde{N}^2E\Big\{\max_{i,k}\{1,\prod_{j=i}^{k}\|b_j(\tau_j)\|\}\Big\}^2\big[\int_{t_0}^{t}E\|f(s,x_s)\|ds\big]^2\\
&\le \widetilde{N}^2\max\{1,M^2\}(T-t_0)\int_{t_0}^{t}m(t)\Theta_f(E\|x\|_s^2)ds,\\
R_5 &\le \widetilde{N}^2E\Big\{\max_{i,k}\{1,\prod_{j=i}^{k}\|b_j(\tau_j)\|\}\Big\}^2E\big\|\int_{t_0}^{t}\sigma(s,x_s)dW(s)\big\|^2\\
&\le \widetilde{N}^2\max\{1,M^2\}Tr(Q)\int_{t_0}^{t}E\|\sigma(s,x_s)\|^2ds\\
&\le \widetilde{N}^2\max\{1,M^2\}Tr(Q)\int_{t_0}^{t}\mu(s)\Theta_\sigma(E\|x\|_s^2)ds.
\end{aligned}$$

If we assume that $\Phi(B_r)\nsubseteq B_r$, then for every positive constant $r>0$, there exists a $x^r\in B_r$, such that $E\|(\Phi x^r)\|_{\mathcal{B}}^2>r$. Therefore

$$\begin{aligned}
r<\sup_{t_0\le t\le T}E\|(\Phi x^r)\|_t^2 \le 5\Big[&N^2M^2E\|\phi(0)\|^2+M^2\widetilde{N}^2E\|\varphi-g(0,\phi)\|^2\\
&+N^2\max\{1,M^2\}(T-t_0)\|n\|_{L^1(J,\mathbb{R})}\Theta_g(r)\\
&+\widetilde{N}^2\max\{1,M^2\}(T-t_0)\|m\|_{L^1(J,\mathbb{R})}\Theta_f(r)\\
&+\widetilde{N}^2\max\{1,M^2\}Tr(Q)\|\mu\|_{L^1(J,\mathbb{R})}\Theta_\sigma(r)\Big].
\end{aligned}\tag{3.2}$$

Divide both side of the above inequality by r, and taking $r\to\infty$.
Since

$$\lim_{r\to\infty}\inf\frac{\Theta_g(r)}{r}=0,\ \lim_{r\to\infty}\inf\frac{\Theta_f(r)}{r}=0,\ \lim_{r\to\infty}\inf\frac{\Theta_\sigma(r)}{r}=0,$$

taking these into (3.2), it implies that $1\le 0$, which is a contradiction. Thus there exists a $r>0$, $\Phi(B_r)\subseteq B_r$.

Step II. We prove that Φ is continuous in B_r. Let $\{x_n\} \to x$ in B_r (as $n \to \infty$), then

$$\begin{aligned}
&E\|(\Phi x^n)(t) - (\Phi x)(t)\|^2 \\
\leq & 3E\Big\| \sum_{k=0}^{+\infty} \big[\sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} C(t-s)[g(s,x_s^n) - g(s,x_s)]ds \\
&+ \int_{\xi_k}^{t} C(t-s)[g(s,x_s^n) - g(s,x_s)]ds \big] I_{[\xi_k,\xi_{k+1})}(t) \Big\|^2 \\
&+ 3E\Big\| \sum_{k=0}^{+\infty} \big[\sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S(t-s)[f(s,x_s^n) - f(s,x_s)]ds \\
&+ \int_{\xi_k}^{t} S(t-s)[f(s,x_s^n) - f(s,x_s)]ds \big] I_{[\xi_k,\xi_{k+1})}(t) \Big\|^2 \\
&+ 3E\Big\| \sum_{k=0}^{+\infty} \big[\sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S(t-s)[\sigma(s,x_s^n) - \sigma(s,x_s)]dW(s) \\
&+ \int_{\xi_k}^{t} S(t-s)[\sigma(s,x_s^n) - \sigma(s,x_s)]dW(s) \big] I_{[\xi_k,\xi_{k+1})}(t) \Big\|^2.
\end{aligned}$$

By the continuity of functions g, f, σ in the assumptions of (H2)-(H4), and by Lebesgue dominated theorem, for $t \in J$ we have

$$\begin{aligned}
E\|(\Phi x^n)(t) - (\Phi x)(t)\|^2 \leq & N^2 \max\{1, M^2\}(T-t_0)^2 \sup_{t_0 \leq t \leq T} E\|g(s,x_t^n) - g(s,x_t)\|^2 \\
&+ \widetilde{N}^2 \max\{1, M^2\}(T-t_0)^2 \sup_{t_0 \leq t \leq T} E\|f(s,x_s^n) - f(s,x_s)\|^2 \\
&+ \widetilde{N}^2 \max\{1, M^2\}(T-t_0) \sup_{t_0 \leq t \leq T} E\|\sigma(s,x_s^n) - \sigma(s,x_s)\|^2 \\
&\to 0, \ (as\ n \to \infty).
\end{aligned}$$

Therefore, $\|(\Phi x^n) - (\Phi x)\|_{\mathcal{B}}^2 \to 0\ (as\ n \to \infty)$, which implies that Φ is continuous in B_r.

Step III. We show that the operator $\Phi(B_r)$ is equicontinuous on every J_k. Let $\xi_k \leq t_1 < t_2 < \xi_{k+1}, k = 0, 1, 2, \ldots$, and $x \in B_r$, then for any fixed $x \in B_r$, we have

$$\begin{aligned}
&E\|(\Phi x)(t_2) - (\Phi x)(t_1)\|^2 \\
\leq & 5E\Big\| \prod_{i=1}^{k} b_i(\tau_i)[C(t_2-t_0) - C(t_1-t_0)]\phi(0) \Big\|^2 \\
&+ 5E\Big\| \prod_{i=1}^{k} b_i(\tau_i)[S(t_2-t_0) - S(t_1-t_0)][\varphi - g(0,\phi)] \Big\|^2 \\
&+ 5E\Big\| \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} [C(t_2-s) - C(t_1-s)]g(s,x_s)ds \\
&+ \int_{\xi_k}^{t_1} [C(t_2-s) - C(t_1-s)]g(s,x_s)ds + \int_{t_1}^{t_2} C(t_2-s)g(s,x_s)ds \Big\|^2
\end{aligned}$$

$$
\begin{aligned}
&+5E\Big\|\sum_{i=1}^{k}\prod_{j=i}^{k}b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}[S(t_2-s)-S(t_1-s)]f(s,x_s)ds\\
&+\int_{\xi_k}^{t_1}[S(t_2-s)-S(t_1-s)]f(s,x_s)ds+\int_{t_1}^{t_2}S(t_2-s)f(s,x_s)ds\Big\|^2\\
&+5E\Big\|\sum_{i=1}^{k}\prod_{j=i}^{k}b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}[S(t_2-s)-S(t_1-s)]\sigma(s,x_s)dW(s)\\
&+\int_{\xi_k}^{t}[S(t_2-s)-S(t_1-s)]\sigma(s,x_s)dW(s)+\int_{t_1}^{t_2}S(t_2-s)\sigma(s,x_s)dW(s)\Big\|^2.
\end{aligned}
$$

By the equicontinuity of $C(t), S(t)$ of the assumption (H1), the assumptions of (H2)-(H5), and Lebesgue dominated theorem, as $t_2 \to t_1$, on every J_k we have

$$E\|(\Phi x)(t_2)-(\Phi x)(t_1)\|^2 \to 0.$$

This proves that $(\Phi(B_r))$ is equicontinuous on J.

Step IV. We show that the $M\ddot{o}nch's$ condition holds.

Let $B = \bar{co}(\{0\}\cup\Phi(B_r))$. For any $D \subset B$, without loss of generality, we assume that $D = \{x^n\}_{n=1}^{\infty}$. Then it is easy to verify that Φ maps D into itself and $D \subset \bar{co}(\{0\}\cup\Phi(B_r))$ is equicontinuous on J_k. Now, we show that $\beta(D) = 0$, where β is the Hausdorff Measure of noncompactness.

Here, for convenience, we denote $\Phi = \Phi_1 + \Phi_2 + \Phi_3$ where

$$
\begin{aligned}
(\Phi_1 x)(t) =& \sum_{k=0}^{+\infty}\Big[\prod_{i=1}^{k}b_i(\tau_i)C(t-t_0)\phi(0)+\prod_{i=1}^{k}b_i(\tau_i)S(t-t_0)[\varphi-g(0,\phi)]\\
&+\sum_{i=1}^{k}\prod_{j=i}^{k}b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}C(t-s)g(s,x_s)ds+\int_{\xi_k}^{t}C(t-s)g(s,x_s)ds\Big]I_{[\xi_k,\xi_{k+1})}(t),\\
(\Phi_2 x)(t) =& \sum_{k=0}^{+\infty}\Big[\sum_{i=1}^{k}\prod_{j=i}^{k}b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}S(t-s)f(s,x_s)ds\\
&+\int_{\xi_k}^{t}S(t-s)f(s,x_s)ds\Big]I_{[\xi_k,\xi_{k+1})}(t),\\
(\Phi_3 x)(t) =& \sum_{k=0}^{+\infty}\Big[\sum_{i=1}^{k}\prod_{j=i}^{k}b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}S(t-s)\sigma(s,x_s)dW(s)\\
&+\int_{\xi_k}^{t}S(t-s)\sigma(s,x_s)dW(s)\Big]I_{[\xi_k,\xi_{k+1})}(t).
\end{aligned}
$$

By Lemma 2.4, Lemma 2.7 and the assumptions of (H1)-(H5), we have

$$
\begin{aligned}
\beta(\{(\Phi_1 x^n)(t)\}_{n=1}^{\infty}) &\le 2\max\{1,M\}N\int_{t_0}^{t}\beta(\{g(s,x_s^n)\}_{n=1}^{\infty})ds\\
&\le 2\max\{1,M\}N\int_{t_0}^{t}K_g(s)\sup_{-\tau\le\theta\le 0}\beta(\{x_s^n(\theta)\}_{n=1}^{\infty})ds\\
&\le 2N\max\{1,M\}\|K_g\|_{L^1(J,R^+)}\sup_{t\in J}\beta(\{x^n(t)\}_{n=1}^{\infty}),
\end{aligned}
$$

$$\begin{aligned}
\beta(\{(\Phi_2 x^n)(t)\}_{n=1}^{\infty}) &\leq 2\max\{1,M\}\widetilde{N}\int_{t_0}^{t}\beta(\{f(s,x_s^n)\}_{n=1}^{\infty})ds \\
&\leq 2\widetilde{N}\max\{1,M\}\|K_f\|_{L^1(J,R^+)}\sup_{t\in J}\beta(\{x^n(t)\}_{n=1}^{\infty}),\\
\beta(\{(\Phi_3 x^n)(t)\}_{n=1}^{\infty}) &\leq \max\{1,M\}\widetilde{N}\beta(\{\int_{t_0}^{t}\sigma(s,x_s^n)dW(s)\}_{n=1}^{\infty})\\
&\leq 2\max\{1,M\}\widetilde{N}\sqrt{(T-t_0)Tr(Q)}K_\sigma(t)\sup_{-\tau\leq\theta\leq 0}\beta(\{x_t^n(\theta)\}_{n=1}^{\infty})\\
&\leq 2\widetilde{N}\max\{1,M\}\sqrt{(T-t_0)Tr(Q)}K_\sigma^*\sup_{t\in J}\beta(\{x^n(t)\}_{n=1}^{\infty}).
\end{aligned}$$

Thus,

$$\begin{aligned}
\beta(\{(\Phi x^n)(t)\}_{n=1}^{\infty}) \leq& \beta(\{(\Phi_1 x^n)(t)\}_{n=1}^{\infty}) + \beta(\{(\Phi_2 x^n)(t)\}_{n=1}^{\infty}) + \beta(\{(\Phi_3 x^n)(t)\}_{n=1}^{\infty})\\
\leq&\Big[2N\max\{1,M\}\|K_g\|_{L^1(J,R^+)} + 2\widetilde{N}\max\{1,\widetilde{M}\}\|K_f\|_{L^1(J,R^+)}\\
&+2\widetilde{N}\max\{1,M\}K_\sigma^*\sqrt{(T-t_0)Tr(Q)}\Big]\sup_{t\in J}\beta(\{x^n(t)\}_{n=1}^{\infty})\\
\leq& H\sup_{t\in J}\beta(D(t)).
\end{aligned}$$

By Lemma 2.4 and assumption (H6), we know

$$\beta(D)\leq\beta(\bar{co}(\{0\}\cup\Phi(D)))=\beta(\Phi(D))\leq H\sup_{t\in J}\beta(D(t))=H\beta(D)<\beta(D),$$

which implies $\beta(D)=0$, the set D is a relatively compact set. By Lemma 2.8, Φ has at least one fixed point x in B_r. That is to say, the system (1.1) has at least a mild solution. This completes the proof. □

4. Exponential Stability

In this section, we study the exponential stability of system (1.1). In order to obtain our exponential stability result, firstly, we establish the following delay integral inequality.

Lemma 4.1. *Assume that a function $y:[t_0-\tau,+\infty)\to[0,+\infty)$ and there exist some positive constants α,β and $\eta_i(i=1,2,3,4)>0$ such that*

$$y(t)\leq\begin{cases}\eta_1 e^{-\alpha(t-t_0)}+\eta_2 e^{-\beta(t-t_0)}+\eta_3\int_{t_0}^{t}e^{-\alpha(t-s)}\sup\limits_{\theta\in[-\tau,0]}y(s+\theta)ds\\ +\eta_4\int_{t_0}^{t}e^{-\beta(t-s)}\sup\limits_{\theta\in[-\tau,0]}y(s+\theta)ds,\quad t\geq t_0,\\ \eta_1 e^{-\alpha(t-t_0)}+\eta_2 e^{-\beta(t-t_0)},\quad t\in[t_0-\tau,t_0],\end{cases}\tag{4.1}$$

holds. If $\frac{\eta_3}{\alpha}+\frac{\eta_4}{\beta}<1$, then

$$y(t)\leq Ce^{-\lambda(t-t_0)},\ t\in[t_0,+\infty),\tag{4.2}$$

where $\lambda\in(0,\alpha\wedge\beta)$ is a positive root of the algebra equation:

$$\frac{\eta_3}{\alpha-\lambda}e^{\lambda\tau}+\frac{\eta_4}{\beta-\lambda}e^{\lambda\tau}=1,\tag{4.3}$$

and $C=\max\{\eta_1+\eta_2,\frac{\eta_1(\alpha-\lambda)}{\eta_3 e^{\lambda\tau}e^{t_0(\alpha-\lambda)}},\frac{\eta_2(\beta-\lambda)}{\eta_4 e^{\lambda\tau}e^{t_0(\beta-\lambda)}}\}$.

Proof. Let $F(\mu) = \frac{\eta_3}{\alpha-\lambda}e^{\lambda\tau} + \frac{\eta_4}{\beta-\lambda}e^{\lambda\tau} - 1$, and then from (4.3) and existence theorem of the root, there exist a positive constant $\lambda \in (0, \alpha\wedge\beta)$, such that $F(\lambda) = 0$. For any $\epsilon > 0$, let

$$C_\epsilon = \max\{\eta_1 + \eta_2 + \epsilon, \frac{(\eta_1+\epsilon)(\alpha-\lambda)}{\eta_3 e^{\lambda\tau} e^{t_0(\alpha-\lambda)}}, \frac{(\eta_1+\epsilon)(\beta-\lambda)}{\eta_4 e^{\lambda\tau} e^{t_0(\beta-\lambda)}}\}. \tag{4.4}$$

In order to prove this lemma, we claim that (4.2) implies:

$$y(t) \leq C_\epsilon e^{-\lambda(t-t_0)}, \ \ t \in [t_0 - \tau, +\infty). \tag{4.5}$$

Obviously, for any $t \in (-\tau + t_0, t_0]$, (4.5) holds. By contradiction, we assume that there exists a $t_1 > t_0$ such that

$$y(t) < C_\epsilon e^{-\lambda(t-t_0)}, \ t \in [t_0 - \tau, t_1) \text{ and } \ y(t_1) = C_\epsilon e^{-\lambda(t_1-t_0)}, \tag{4.6}$$

then

$$\begin{aligned}
y(t_1) \leq & \eta_1 e^{-\alpha(t_1-t_0)} + \eta_2 e^{-\beta(t_1-t_0)} \\
& + \eta_3 C_\epsilon \int_{t_0}^{t_1} e^{-\alpha(t_1-s)} \sup_{\theta\in[-\tau,0]} e^{-\lambda(s+\theta-t_0)} ds \\
& + \eta_4 C_\epsilon \int_{t_0}^{t_1} e^{-\beta(t_1-s)} \sup_{\theta\in[-\tau,0]} e^{-\lambda(s+\theta-t_0)} ds \\
\leq & \eta_1 e^{-\alpha(t_1-t_0)} + \eta_2 e^{-\beta(t_1-t_0)} + \eta_3 C_\epsilon \int_{t_0}^{t_1} e^{-\alpha(t_1-s)} e^{-\lambda(-\tau+s-t_0)} ds \\
& + \eta_4 C_\epsilon \int_{t_0}^{t_1} e^{-\beta(t_1-s)} e^{-\lambda(-\tau+s-t_0)} ds \\
\leq & [\eta_1 - \frac{\eta_3 C_\epsilon e^{\lambda\tau} e^{t_0(\alpha-\lambda)}}{\alpha-\lambda}] e^{-\alpha(t_1-t_0)} + [\eta_2 - \frac{\eta_4 C_\epsilon e^{\lambda\tau} e^{t_0(\beta-\lambda)}}{\beta-\lambda}] e^{-\beta(t_1-t_0)} \\
& + C_\epsilon [\frac{\eta_3}{\alpha-\lambda} e^{\lambda\tau} + \frac{\eta_4}{\beta-\lambda} e^{\lambda\tau}] e^{-\lambda(t_1-t_0)}.
\end{aligned} \tag{4.7}$$

From the definition of λ and C_ϵ, we have

$$\begin{aligned}
\eta_1 - \frac{\eta_3 e^{\lambda\tau} e^{t_0(\alpha-\lambda)}}{\alpha-\lambda} \times C_\epsilon & \leq \eta_1 - \frac{\eta_3 e^{\lambda\tau}}{\alpha-\lambda} \times \frac{(\eta_1+\epsilon)(\alpha-\lambda)}{\eta_3 e^{\lambda\tau} e^{t_0(\alpha-\lambda)}} \\
& \leq -\epsilon < 0, \\
\eta_2 - \frac{\eta_4 e^{\lambda\tau} e^{t_0(\beta-\lambda)}}{\beta-\lambda} \times C_\epsilon & \leq \eta_2 - \frac{\eta_4 e^{\lambda\tau}}{\beta-\lambda} \times \frac{(\eta_2+\epsilon)(\beta-\lambda)}{\eta_4 e^{\lambda\tau} e^{t_0(\beta-\lambda)}} \\
& \leq -\epsilon < 0,
\end{aligned}$$

and

$$\frac{\eta_3}{\alpha-\lambda} e^{\lambda\tau} + \frac{\eta_4}{\beta-\lambda} e^{\lambda\tau} = 1.$$

Thus, by (4.7), we see that $y(t_1) < C_\epsilon e^{-\lambda(t-t_0)}$, which is contrary to (4.6). Therefore (4.5) holds. Since ϵ is arbitrary small, (4.2) holds. □

For system (1.1), we need the following assumptions:

(A1) There exist positive constants $K > 0$ and $\mu_1, \mu_2 > 0$ such that $\|C(t)\| \leq Ke^{-\mu_1 t}$, $\|S(t)\| \leq Ke^{-\mu_2 t}$, $t \geq 0$.

(A2) There exist constants $L_g, L_f, L_\sigma > 0$, such that for any $v_1, v_2 \in \mathcal{C}, t \geq t_0$,

$$E\|g(t,v_1) - g(t,v_2)\|^2 \leq L_g E\|v_1 - v_2\|_t^2,\ E\|g(t,0)\| = 0;$$
$$E\|f(t,v_1) - f(t,v_2)\|^2 \leq L_f E\|v_1 - v_2\|_t^2,\ E\|f(t,0)\| = 0;$$
$$E\|\sigma(t,v_1) - \sigma(t,v_2)\|^2 \leq L_\sigma E\|v_1 - v_2\|_t^2,\ E\|\sigma(t,0)\| = 0.$$

(A3) There exist constants $M > 0$, for all $\tau_j \in D_j$ $(j = 1, 2, \ldots)$, such that

$$E\Big\{\max_{i,k}\{\prod_{j=i}^{k}\|b_j(\tau_j)\|\}\Big\} \leq M.$$

Theorem 4.1. *Assume that (A1)-(A3) hold, then the mild solution of* (1.1) *is exponentially stable in mean square provided that*

$$5[\max\{1, M^2\}K^2\mu_1^{-2}L_g + \max\{1, M^2\}K^2\mu_2^{-1}(\mu_2^{-1}L_f + L_\sigma Tr(Q))] \leq 1. \tag{4.8}$$

Proof. From the definition of the mild solutions of system (1.1), we have

$$\begin{aligned} E\|x(t)\|^2 \leq & 5E\big[\sum_{k=0}^{+\infty}[\prod_{i=1}^{k}\|b_i(\tau_i)\|\|C(t-t_0)\|\|\phi(0)\|I_{(\xi_k,\xi_{k+1}]}(t)\big]^2 \\ & + 5E\big[\sum_{k=0}^{+\infty}[\prod_{i=1}^{k}\|b_i(\tau_i)\|S(t-t_0)\|\|\varphi - g(0,\phi)\|]I_{(\xi_k,\xi_{k+1}]}(t)\big]^2 \\ & + 5E\big[\sum_{k=0}^{+\infty}[\sum_{i=1}^{k}\prod_{j=i}^{k}\|b_j(\tau_j)\|\int_{\xi_{i-1}}^{\xi_i}\|C(t-s)\|\|g(s,x_s)\|ds \\ & + \int_{\xi_k}^{t}\|C(t-s)\|\|g(s,x_s)\|ds]I_{(\xi_k,\xi_{k+1}]}(t)\big]^2 \\ & + 5E\big[\sum_{k=0}^{+\infty}[\sum_{i=1}^{k}\prod_{j=i}^{k}\|b_j(\tau_j)\|\int_{\xi_{i-1}}^{\xi_i}\|S(t-s)\|\|f(s,x_s)\|ds \\ & + \int_{\xi_k}^{t}\|S(t-s)\|\|f(s,x_s)\|ds]I_{(\xi_k,\xi_{k+1}]}(t)\big]^2 \\ & + 5E\big[\sum_{k=0}^{+\infty}[\sum_{i=1}^{k}\prod_{j=i}^{k}\|b_j(\tau_j)\|\int_{\xi_{i-1}}^{\xi_i}\|S(t-s)\|\|\sigma(s,x_s)\|dW(s) \\ & + \int_{\xi_k}^{t}\|S(t-s)\|\|\sigma(s,x_s)\|dW(s)]I_{(\xi_k,\xi_{k+1}]}(t)\big]^2 := 5\sum_{i=1}^{5}U_i,\ (t \geq t_0), \end{aligned} \tag{4.9}$$

where

$$U_1 \leq M^2K^2E\|\phi(0)\|^2e^{-\mu_1(t-t_0)}, \tag{4.10}$$
$$U_2 \leq 2M^2K^2(E\|\varphi\|^2 + L_gE\|\phi\|_t^2)e^{-\mu_2(t-t_0)}, \tag{4.11}$$
$$\begin{aligned} U_3 & \leq \max\{1, M^2\}(\int_{t_0}^{t}Ke^{-\mu_1(t-s)}E\|g(s,x_s)\|ds)^2 \\ & \leq \max\{1, M^2\}K^2\mu_1^{-1}L_g\int_{t_0}^{t}e^{-\mu_1(t-s)}E\|x\|_s^2ds, \end{aligned} \tag{4.12}$$

$$\begin{aligned} U_4 &\leq \max\{1, M^2\}(\int_{t_0}^{t} K e^{-\mu_2(t-s)} E\|f(s, x_s)\| ds)^2 \\ &\leq \max\{1, M^2\} K^2 \mu_2^{-1} L_f \int_{t_0}^{t} e^{-\mu_2(t-s)} E\|x\|_s^2 ds, \end{aligned} \tag{4.13}$$

$$\begin{aligned} U_5 &\leq \max\{1, M^2\}(\int_{t_0}^{t} K^2 e^{-2\mu_2(t-s)} E\|\sigma(s, x_s)\|_Q^2 ds) \\ &\leq \max\{1, M^2\} K^2 L_\sigma Tr(Q) \int_{t_0}^{t} e^{-\mu_2(t-s)} E\|x\|_s^2 ds. \end{aligned} \tag{4.14}$$

Then putting (4.10)-(4.14) into (4.9), we obtained that for $t \in J$,

$$\begin{aligned} E\|x(t)\|^2 \leq\ & 5M^2K^2E\|\phi(0)\|^2 e^{-\mu_1(t-t_0)} + 10M^2K^2(E\|\varphi\|^2 + L_g E\|\phi\|_t^2) e^{-\mu_2(t-t_0)} \\ &+ 5\max\{1, M^2\} K^2 \mu_1^{-1} L_g \int_{t_0}^{t} e^{-\mu_1(t-s)} \sup_{\theta \in [-\tau, 0]} E\|x(s+\theta)\|^2 ds \\ &+ 5\max\{1, M^2\} K^2 (\mu_2^{-1} L_f + L_\sigma Tr(Q)) \int_{t_0}^{t} e^{-\mu_2(t-s)} \sup_{\theta \in [-\tau, 0]} E\|x(s+\theta)\|^2 ds. \end{aligned}$$

By Lemma 4.1 and (4.8), we obtain

$$E\|x\|^2 \leq C e^{-\lambda(t-t_0)}, t \in [t_0, +\infty),$$

where $\lambda \in (0, \mu_1 \wedge \mu_2)$, and

$$\begin{aligned} C = & \max\{5M^2K^2E\|\phi(0)\|^2 + 10M^2K^2(E\|\varphi\|^2 + L_g E\|\phi\|_t^2), \\ & 5M^2K^2E\|\phi(0)\|^2(\mu_1 - \lambda)[5e^{\lambda\tau} e^{t_0(\mu_1-\lambda)} \max\{1, M^2\} K^2 \mu_1^{-1} L_g]^{-1}, \\ & 10M^2K^2(E\|\varphi\|^2 + L_g E\|\phi\|_t^2)(\mu_2 - \lambda)[5e^{\lambda\tau} e^{t_0(\mu_2-\lambda)} \\ & \max\{1, M^2\} K^2 (\mu_2^{-1} L_f + L_\sigma Tr(Q))]^{-1}\}. \end{aligned}$$

This implies that the mild solution of system (1.1) is exponentially stable in mean square moment. This completes the proof. □

5. An Application

In this section, we apply our results to a stochastic partial differential equations with random impulses. We take the space $H = L^2([0, \pi])$, Define $A : D(A) \subset H \to H$ by $A = \frac{\partial^2}{\partial x^2}$, with domain

$$D(A) = \{z \in H | z \text{ and } \frac{\partial z}{\partial x} \text{ are absolutely continuous}, \frac{\partial^2 z}{\partial x^2} \in H, z(0) = z(\pi) = 0\}.$$

For $z \in D(A)$, $Az = -\sum_{n=1}^{\infty} n^2 < z, z_n > z_n$, where $\{z_n : n \in \mathbb{Z}\}$ is an orthonormal basis of H, $z_n(x) := \frac{1}{\sqrt{2\pi}} e^{inx}$ $n \in \mathbb{Z}^+, x \in [0, \pi]$. We know that A generate strong continuous operators $C(t)$ and $S(t)$ in a Hilbert space H, such that $C(t)z = \sum_{n=1}^{\infty} cos(nt) < z, z_n > z_n$, and $S(t)z = \sum_{n=1}^{\infty} sin(nt)/n < z, z_n > z_n$, for $t \in \mathbb{R}$. And we assume that $S(t)$ is not a compact semigroup and $\beta(S(t)D) \leq \beta(D)$, where $D \in H$ denotes a bounded set, β is the Hausdorff measure of noncompactness.

Consider the second-order neutral stochastic functional differential equation of the form:

$$\begin{cases} \dfrac{\partial}{\partial t}\big[\dfrac{\partial}{\partial t}z(t,x)-\dfrac{u_1}{5}\displaystyle\int_{-r}^{0}\mu_1(s)z(t+s,x)ds\big] \\ =\big[\dfrac{\partial^2}{\partial x^2}z(t,x)+\dfrac{u_2}{5}\displaystyle\int_{-r}^{0}\mu_2(s)z(t+s)ds)\big]dt+\dfrac{u_3}{5}\displaystyle\int_{-r}^{0}\mu_3(s)z(t+s)dW(t), \\ t\geq t_0,\ t\neq \xi_k,\ x\in[0,\pi], \\ z(\xi_k,x)=\rho(k)\tau_k z(\xi_k^-,x),\ k=1,2,\ldots, \\ \dfrac{\partial}{\partial t}z(\xi_k,x)=\rho(k)\tau_k\dfrac{\partial}{\partial t}z(\xi_k^-,x), \\ z(t_0,x)=\phi(\theta,x),\ \theta\in[-r,0],\ x\in[0,\pi],\ r>0, \\ \dfrac{\partial}{\partial t}z(t_0,x)=\varphi(x),\ x\in[0,\pi], \\ z(t,0)=z(t,\pi)=0. \end{cases} \tag{5.1}$$

Let τ_k be a random variable defined on $D_k\equiv(0,d_k)$ where $0<d_k<+\infty$, for $k=1,2,\cdots$. Suppose τ_i and τ_j are independent of each other as $i\neq j$ for $i,j=1,2,\cdots$. $\xi_0=t_0>0$ and $\xi_k=\xi_{k-1}+\tau_k$ for $k=1,2,\cdots$. $W(t)$ denotes a standard cylindrical Wiener process in H. Further more, let ρ be a function of k. $\mu_i:[-r,0]\to\mathbb{R}$ are positive functions and $u_i>0$ for $i=1,2,3$. $\|C(t)\|,\|S(t)\|$ are bounded on $\mathbb{R}$. $\|C(t)\|\leq e^{-\pi^2 t}$ and $\|S(t)\|\leq e^{-\pi^2 t}$ $(t\geq 0)$.

We assume that

(i) The function $\mu(\theta)\geq 0$ is continuous on $[-r,0]$, $\int_{-r}^{0}\mu_i^2(\theta)d\theta<\infty$ $(i=1,2,3)$.

(ii) $\max\limits_{i,k}\Big\{\prod_{j=i}^{k}E[\|\rho(j)\tau_j\|^2]\Big\}<M$.

Under the above assumptions, and by choosing some suitable functions μ_1,μ_2,μ_3,ρ, we can show that $L_g=\frac{ru_1}{25}\int_{-r}^{0}\mu_1^2(\theta)d\theta$, $L_f=\frac{ru_2}{25}\int_{-r}^{0}\mu_2^2(\theta)d\theta$, $L_\sigma=\frac{ru_3}{25}\int_{-r}^{0}\mu_3^2(\theta)d\theta$. Hence the hypothesis of Theorem 4.1 holds. From Theorem 4.1, we know the exponential stability in mean square for mild solution of system (5.1) are obtained, provided that

$$\frac{\max\{1,M^2\}ru_1}{\pi^4}\int_{-r}^{0}\mu_1^2(\theta)d\theta+\max\{1,M^2\}\big[\frac{ru_2}{\pi^4}\int_{-r}^{0}\mu_2^2(\theta)d\theta+\frac{ru_3}{\pi^2}\int_{-r}^{0}\mu_3^2(\theta)d\theta\big]<5.$$

6. Conclusion

In this work, the existence and exponential stability results to second-order neutral stochastic functional systems with random impulses has been presented. Using the noncompact measurement and the Mönch fixed point theorem, the existence of mild solutions has been proved. Here we applied a new noncompact measure criterion for Itô integrals to solve the calculation of Itô integrals. The exponential stability for the considered equations has been obtained by establishing integral inequality. The control and optimization theory based on random impulses is our next research topic. Further, we can also consider the case of random persistent impulses. For some results on persistent impulsive effect on stability of functional systems we refer to [12].

Acknowledgements

The authors would like to thank the editor and the reviewers for their helpful comments and suggestions. This work was jointly supported by the National Natural Science Foundation of China (61773217, 61374080) and the Innovation Platforms Open Foundation of Hunan Educational Committee (541109100002).

References

[1] A. Angural, S. Wu, and A. Vinodkumar, *Existence and exponential stabilty of semilinear functional differential equations with random impulses under non-uniqueness*, Nonlinear Anal., 2011, 74(2), 331–342.

[2] G. Arthi, H. Park, and H. Y. Jung, *Exponential stability for second order neutral stochastic differential equations with impulses*, Internat. J. Control, 2015, 88(6), 1300–1309.

[3] H. Chen, *The asymptotic behavior for second-order neutral stochastic partial differential equations with infinite delay*, Discrete Dyn. Nat. Soc., 2011, Art. ID 584510, 15 pp.

[4] H. Deinz, *On the behavious of measure of noncompactness with respect to differentiation and integration of vector-valued functions*, Nonliear Anal., 1983, 7, 1351–1371.

[5] S. Deng, X. Shu and J. Mao, *Existence and exponential stability for impulsive neutral stochastic functional differential equations driven by fBm with noncompact semigroup via Mönch fixed point*, J. Math. Anal. Appl., 2018, 467(1), 398–420.

[6] M. Gowrisankar, P. Mohankumar and A. Vinodkumar, *Stability results of random impulsive semilinear differential equations*, Acta Math. Sci., 2014, 34(4), 1055–1071.

[7] F. Jiang, H. Yang and Y. Shen, *A note on exponential stability for second-order neutral stochastic partial differential equations with infinite delays in the presence of impulses*, Appl. Math. Comput., 2016, 287/288, 125–133.

[8] S. Ji and G, Li, *A unified approach to nonlocal impulsive differential equations with the measure of noncompactness*, Adv. Difference Equ., 2012, 2012:182, 14 pp.

[9] V. Lakshmikantham, D. D. Bainov and P. S. Simeonov, *Theory of Impulsive Differential Equations*, World Scientific, Singapore, 1989.

[10] S. Li, L. Shu, X. Shu and F. Xu, *Existence and Hyers-Ulam stability of random impulsive stochastic functional differential equations with finite delays*, Stochastics, 2019, 91(6), 857–872.

[11] W. Li, Q. Zhou and H. an, *The existence of stochastic partial evolution conclusion with random impulses*, (Chinese) Acta. Math. appl. sin., 2015, 38(6).

[12] X. Li, J. Shen and R. Rakkiyappan, *Persistent impulsive effects on stability of functional differential equations with finite or infinite delay*, Appl. Math. Comput., 2018, 329, 14–22.

[13] C. Loganathan and S. Vijay, *Approximate controllability of random impulsive integro semilinear differential systems*, Nonlin. Dyn. Chaos, 2017, 5(1), 25–32.

[14] P. Niu, X. Shu and Y. Li, *The existence and Hyers-Ulam stability for second order random impulsive differential equations*, Dyn. Syst. Appl., 2019, 28(3), 673–690.

[15] G. Prato and J. Zabczyk, *Stochastic Equations in Infinite Dimensions*, Cambridge University Press, Cambridge, 1992.

[16] B. Radhakrishnan and M. Tanilarasi, *Existence of solutions for quasilinear random impulsive neutral differential evolution equation*, Arab. J. Math. Sci., 2018, 24(2), 235–246.

[17] L. Shu, X. Shu and J. Mao, *Approximate controllability and existence of mild solutions for Riemann-Liouville fractional stochastic evolution equations with nonlocal conditions of order* $1 < \alpha < 2$, Fract. Calc. Appl. Anal., 2019, 22(4), 1086–1112.

[18] R. Sakthivel, Y. Ren and H. Kim, *Asymptotic stability of second-order neutral stochastic differential equations*, J. Math. Phys., 2010, 51(5), 9 pp.

[19] X. Shu, Y. Lai and Y. Chen, *The existence of mild solutions for impulsive fractional partial differential equations*, Nonlinear Anal., 2011, 74(5), 2003–2011.

[20] X. Shu and Y. Shi, *A study on the mild solution of impulsive fractional evolution equations*, Appl. Math. Comput., 2016, 273, 465–476.

[21] C. Travis and G. Webb, *Compactness, regularity, and uniform continuity properties of strongly continuous cosine families*, Houst. J. Math., 1977, 3(4), 555–567.

[22] C. Travis and G. Webb, *Cosine families and abstract nonlinear second order diferential equations*, Acta. Math. Hung., 1978, 32(3–4), 75–96.

[23] V. Vijayakumar, R. Murugesu, R. Poongodi and S. Dhanalashmi, *Controllability of second-Order impulsive nonlocal Cauchy problem via measure of noncompactness*, Mediterr. J. Math., 2017, 14(1), 23 pp.

[24] A. Vinodkumar, M. Gowrisankar and P. Mohankumar, *Existence, uniqueness and stability of random impulsive neutral partial differential equations*, J. Egyptian Math. Soc., 2015, 23(1), 31–36.

[25] A. Vinodkumar and P. Indhumathi, *Global existence and stability results for mild solution of random impulsive partial integro-differential equations*, Filomat, 2018, 32(2), 439–455.

[26] A. Vinodkumar, K. Malar, M. Gowrisankar and P. Mohankumar, *Existence, uniqueness and stability of random impulsive fractional differential equations*, J. Acta Math. Sci., 2016, 36(2), 428–442.

[27] S. Wu and Y. Duan, *Oscillation, stability, and boundedness of second-order diferential systems with random impulses*, Comput. Math. Appl., 2005, 49, 1375–1386.

[28] S. Wu, X. Guo and Y. Zhou, *p-moment stability of functional differential equations with random impulses*, Comput. Math. Appl., 200, 52, 1683–1694.

[29] S. Wu, D. Han and H. Fu, *Existence and uniqueness of stochastic differential equations with random impulses*, Acta Math. Sin., 2008, 51(6), 1–12.

[30] S. Wu and X. Meng, *Boundedness of nonlinear differantial systems with impulsive effect on random moments*, Acta Math. Appl. Sin., Chinese Series, 2004, 20(1), 147–154.

[31] X. Yang, X. Li, Q. Xi and P. Duan, *Review of stability and stabilization for impulsive delayed systems*, Math. Biosci. Eng., 2018, 15(6), 1495–1515.

[32] S. Zhang and W. Jiang, *The existence and exponential stability of random impulsive fractional differential equations*, Adv. Difference Equ., 2018, (404), 17 pp.

[33] Y. Zhou and S. Wu, *Existence and uniqueness of solutions to stochastic differential equations with random impulses under Lipschitz conditions*, Chinese J. Appl. Probab. Statist., 2010, 26(4), 347–356.