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EXISTENCE RESULTS OF A SCHRÖDINGER EQUATION INVOLVING A NONLINEAR OPERATOR*

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Abstract In this article, by making use of a completely different approach, i. e. monotone operator theory and analysis techniques, we present the existence of positive solutions for a nonlinear Schrödinger equations in a unique way under reasonable conditions. Moreover, illustrative examples are also presented.

Keywords Nonlinear Schrödinger equation, nondecreasing operator, positive solution, existence, monotonicity.

MSC(2010) 34B16, 34B15, 34B25, 34B18.

1. Introduction

In this article, we are concerned with the existence results for the following nonlinear Schrödinger equation

$$div(\mathcal{G}(|\nabla u|)\nabla u) = \lambda a(|v|)f(v,u),\ u,\ v \in R^N \tag{1.1}$$

where $\lambda > 0$ is a constant, $a \in L^1(0,+\infty)$, $f(u,v) \in L^1((0,+\infty)\times(0,+\infty),(0,+\infty))$, $N \geq 2$, and $\mathcal{G}$ is a nonlinear operator and satisfies

$$\mathcal{W} = \{\mathcal{G} \in C^2([0,\ +\infty),\ (0,\ +\infty)) |\ \exists\ \ \alpha > 0 \text{ such that } \ \mathcal{G}(cs) \leq c^\alpha \mathcal{G}(s),\ \ \forall\ 0 < c < 1\}.$$

Differential equation (1.1) appears to various fields, such as engineering, electromagnetism, control theory, viscoelasticity material, porous media and electrochemistry and so on, see [1–3, 14–17, 19, 20, 25–30] and the references therein. Recently, more and more researchers are pushing back the frontiers, and many methods have been developed. During the last decades, nonlinear partial differential equations and fractional differential equations have been successfully applied to diversified domains, and many nice fruits have been achieved [4–13, 15–33]. We refer the reader to [1–3, 14–21, 23–31] for some applications and details. When $\mathcal{G}(y) = 1$, Eq. (1.1) leads to the Poisson equation. If $\mathcal{G}(y) = |y|^{p-2}$, $p \geq 2$, Eq. (1.1) induces to the $p-$Poisson equation.

For some special cases of equation (1.1) have been developed by [3, 14–17, 19, 20, 25–29]. In particular, nonlinear functional analysis methods, such as variational method, upper and lower solutions together with weak comparison principle ([3, 14,

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19, 25–29]), and fixed point theorems together with the symmetry of the associated Green's function [1, 2] play a key role for researching nonlinear partial differential equations and ordinary differential equations.

Recently, by making use of the same technique as in ([1, 2, 28]), Sun et al. [25] considered the existence and uniqueness of positive monotone solution for the following nonlinear Schrödinger equation $\Delta z + g(|r|)a(z) = 0$, where $|r| \in E_L,\ g(|r|) \in C^{\lambda}_{loc}(E_L, R)$, $\lambda \in (0, 1)$, $a(z) \in C^{\lambda}_{loc}(R, R)$ locally Hölder continuous and $E_L = \{z \in R : |z| > L\}$, $S_L = \{r \in R^2 : |r| = L\}$. Li et al. [20] studied the existence of positive solutions for the following fractional $q-$difference Schrödinger equation

$$\begin{cases} (D_q^{\beta} y)(t) + \dfrac{b}{\overline{p}}(E - w(t))y(t) = 0, \quad 0 < t < 1, \\ y(0) = D_q y(0) = D_q y(1) = 0, \end{cases} \tag{1.2}$$

where $0 < q < 1$, $2 < \beta < 3$, $w(t)$ is the trapping potential, b expresses the mass of a particle, $\overline{p}$ is the planck constant and E represents the energy of a particle. The main tool in [20] is a fixed point theorem in a cone. In [28], by utilizing the monotone iterative approach, Wang developed the problem (1.2) and obtained twin iterative positive solutions for the problem (1.2).

The primary work of the paper is to investigate bounded positive solutions of a nonlinear Schrödinger equation appeared in [30] Zhang et al.(2018). By applying the monotone operator theory and analysis techniques, we not only establish excellent sufficient conditions to ensure the existence of bounded positive solutions, but also construct a sharper sufficient condition to confirm monotone positive solution. The results are well illustrated with the aid of examples.

The rest of this paper is organized as follows. In Section 2, we present some lemmas and new propositions. In Section 3, we propose the main results with new approach. In Section 4, we give out the examples to demonstrate the main result. Finally in Section 5, we conclude.

2. Preliminaries

Proposition 2.1. *Let $\mathcal{P}(x) = (a_0 + a_1 x + a_2 x^2)\mathcal{G}(x)$, $x \in (0, +\infty)$, where a_0, a_1 and a_2 are nonnegative constants with $a_2 \neq 0$, $\mathcal{G} \in \mathcal{W}$, then $\mathcal{P}$ has a nonnegative increasing inverse mapping $\mathcal{P}^{-1}(x)$ with*

$$\mathcal{P}^{-1}(\nu x) \geq \nu^{\frac{1}{\alpha}} \mathcal{P}^{-1}(x), \quad \textit{for} \quad 0 < \nu < 1, \quad \alpha > 0.$$

Proof. We claim that $\mathcal{G}(\in \mathcal{W})$ is a nonnegative increasing operator. Obviously, $\mathcal{G}$ is a nonnegative operator for $\mathcal{G} \in \mathcal{W}$. Further we prove that $\mathcal{G}$ is an increasing operator.

Without loss of generality, we assume that x, $y \in (0, +\infty)$ and $0 \leq x < y$, for any $\mathcal{G} \in \mathcal{W}$ and x, $y \in (0, +\infty)$. Then there are the following two possibilities:

Case 1 If $x = 0$, then we easily know that $\mathcal{G}(x) \leq \mathcal{G}(y)$ holds.

Case 2 If $x \neq 0$, then $x > 0$. Let $\varepsilon_0 = \frac{x}{y}$, thus $0 < \varepsilon_0 < 1$. By the property of $\mathcal{G}$ that

$$\mathcal{G}(x) = \mathcal{G}(\varepsilon_0 y) \leq \varepsilon_0^{\alpha} \mathcal{G}(y) < \mathcal{G}(y)$$

which implies that $\mathcal{G}$ is a strictly increasing operator. Thus $\mathcal{G}$ is a bijection on $(0, +\infty)$. Consequently, it follows $x \neq 0$ that $\mathcal{P}'(x) = [(a_0 + a_1 x + a_2 x^2)\mathcal{G}(x)]' > 0$, where a_0, a_1 and a_2 are nonnegative constants with $a_2 \neq 0$, which implies that $\mathcal{P}$ is a bijection on $(0, +\infty)$ and has a nonnegative increasing inverse mapping $\mathcal{P}^{-1}(x)$.

Now we consider two maps $\mathcal{P} : (0, +\infty) \longrightarrow (0, +\infty)$ and $\mathcal{P}^{-1} : (0, +\infty) \longrightarrow (0, +\infty)$ defined as $\mathcal{P}(x) \equiv (a_0 + a_1 x + a_2 x^2)\mathcal{G}(x)$, $x \in (0, +\infty)$, where a_0, a_1 and a_2 are nonnegative constants with $a_2 \neq 0$, $\mathcal{G} \in \mathcal{W}$. Let $\varepsilon = \nu^{\frac{1}{\alpha}}$ for $0 < \nu < 1$, then $0 < \varepsilon < 1$. Thus

$$\begin{aligned}\mathcal{P}(\varepsilon y) &= (a_0 + a_1\varepsilon y + a_2\varepsilon^2 y^2)\mathcal{G}(\varepsilon y) \le (a_0 + a_1\varepsilon y + a_2\varepsilon^2 y^2)\varepsilon^\alpha \mathcal{G}(y)\\ &= a_0\varepsilon^\alpha \mathcal{G}(y) + a_1\varepsilon^{\alpha+1} y\mathcal{G}(y) + a_2\varepsilon^{\alpha+2} y^2\mathcal{G}(y)\\ &= a_0\varepsilon^\alpha \mathcal{G}(y) + a_1\varepsilon^{\alpha} y\mathcal{G}(y) + a_2\varepsilon^{\alpha} y^2\mathcal{G}(y) = \varepsilon^\alpha(a_0 + a_1 y + a_2 y^2)\mathcal{G}(y) = \varepsilon^\alpha \mathcal{P}(y).\end{aligned}$$

Let $x = \mathcal{P}(y)$, then $\varepsilon\mathcal{P}^{-1}(x) = \varepsilon y \le \mathcal{P}^{-1}(\varepsilon^\alpha \mathcal{P}(y)) = \mathcal{P}^{-1}(\varepsilon^\alpha x)$. Let $\nu = \varepsilon^\alpha$, then $\nu^{\frac{1}{\alpha}}\mathcal{P}^{-1}(x) \le \mathcal{P}^{-1}(\nu x)$, which leads to $\mathcal{P}^{-1}(\nu x) \ge \nu^{\frac{1}{\alpha}}\mathcal{P}^{-1}(x)$. □

Remark 2.1. Obviously, if $r \ge 1$, we have

$$L^{-1}(rs) \le r^{\frac{1}{\alpha}} L^{-1}(s), \quad \alpha > 0. \tag{2.1}$$

Remark 2.2. The operator set $\mathcal{W}$ includes a large class of operators and the standard type of operator is $\mathcal{G}(x) = \sum_{i=1}^{n} x^{\zeta_i}, \zeta_i > 0$. In fact, taking $\zeta = \min\{\zeta_1, \cdots, \zeta_n\}$, then $\zeta > 0$. Thus, we have $\mathcal{G}(cx) \le c^\alpha \mathcal{G}(x)$ for $0 < c < 1$.

Remark 2.3. Lemma 2.1 improves the corresponding results of Zhang [30] from many aspects.

By the method of change of variables and simple computation, the Schrödinger equation (1.1) can be transformed to a more convenient form. Thus we could get the following new lemmas.

Lemma 2.1. *The Schrödinger equation (1.1) has a solution u if and only if it solves the following third order ordinary differential equation*

$$\begin{cases}(\mathcal{G}(|u'|u'))' + \dfrac{N-1}{v}\mathcal{G}(|u'|)u' = \lambda a(|v|)f(v,u), \; u, \, v \in R^N,\\ u'(v) \ge 0, \; v \in [0, +\infty), \; u(0) = u_0 > 0.\end{cases} \tag{2.2}$$

It is easy to know that Lemma 2.1 is a special case of the following Lemma.

Lemma 2.2. *If the following ordinary differential equation*

$$\begin{cases}(\mathcal{G}(|y'|(y')^q))' + \dfrac{N-1}{v}\mathcal{G}(|y'|)(y')^q = \lambda a(v)f(v,y(v)),\\ y'(v) \ge 0, \; v \in [0, +\infty), \; y(0) = y_0 > 0\end{cases} \tag{2.3}$$

has a solution, then the equation (1.1) has a solution.

Proof. For $q = 1$ and $\mathcal{G}(|y'|) = 1$, then the problem (2.3) becomes

$$\begin{cases}y'' + \dfrac{N-1}{v}y' = \lambda a(v)f(v,y(v)),\\ y'(v) \ge 0, \; v \in [0, +\infty), \; y(0) = y_0 > 0.\end{cases} \tag{2.4}$$

By the technique of change of variables and simple computation, the equation

$$\triangle\, y = \lambda a(v)f(v,y(v)) \tag{2.5}$$

can be transformed to the equation (2.4).

For more general and convenient form could be transformed by the equation (1.1). Therefore, if $y(v)$ is a solution of the problem (2.3), then $y(v)$ is a solution of Eq. (1.1). The proof of Lemma 2.2 is completed. $\square$

It follows from Proposition 2.1. we know that if $\mathcal{P}(v) = v^q\mathcal{G}(v)$, $q > 0$ is a constant, then

$$y(v) - y(0) = \int_0^v \mathcal{P}^{-1}\left(\frac{1}{t^{N-1}}\int_0^t s^{N-1}\lambda a(s)f(s,y(s))ds\right)dt$$

Thus

$$|y'(v)| = \left|\mathcal{P}^{-1}\left(\frac{1}{v^{N-1}}\int_0^v s^{N-1}\lambda a(s)f(s,y(s))ds\right)\right|.$$

Consequently

$$\begin{aligned}\mathcal{G}(|y'(v)|)(y')^q &= \mathcal{P}\left(\mathcal{P}^{-1}\left(\left(\int_0^v s^{N-1}\lambda a(s)f(s,y(s))ds\right)\right)\right.\\ &= \frac{1}{v^{N-1}}\int_0^v s^{N-1}\lambda a(s)f(s,y(s))ds\end{aligned}$$

Therefore

$$(\mathcal{G}(|y'|)(y')^q)' + \frac{N-1}{v^N}\int_0^v s^{N-1}\lambda a(s)f(s,y(s))ds = \lambda a(v)f(v,y(v))$$

Thus we get

$$(\mathcal{G}(|y'|)(y')^q)' + \frac{N-1}{v}\mathcal{G}(|y'|)(y')^q = \lambda a(v)f(v,y(v)).$$

Remark 2.4. For $q = 1$, by making use of the similar methods, we easily get

$$(\mathcal{G}(|y'|)y')' + \frac{N-1}{v}\mathcal{G}(|y'|)y' = \lambda a(v)f(v,y(v)).$$

Thus Lemma 2.2 holds.

3. Main Results

For convenience, we list the following assumptions.

(A_1) $f \in L^1([0, +\infty) \times [0, +\infty); (0, +\infty))$. There exists a nondecreasing positive functional $\psi \in C([0, +\infty); (0, +\infty))$, and a positive constant $R > 0$ such that $f(v, R) > 0$ with

$$\lim_{u\to+\infty}\frac{u^2}{\psi(u)e^u} = +\infty \text{ and } f(v, u) \le \psi(u) \quad \text{for} \quad |u| \le R. \tag{3.1}$$

Further, there exists a positive constant $\delta > 0$ with $0 < \delta < \alpha$ such that

$$\psi(mu) \ge m^\delta\psi(u) \text{ for } u \in [0,+\infty),\ m \in (0,1). \tag{3.2}$$

Remark 3.1. By making use of (3.1), we have $\int_{[0,\,+\infty)}\psi(u)du < +\infty$, that is $\psi \in L^1[0,\,+\infty)$.

It follows from (3.2) that

$$\psi(mu) \le m^{\delta}\psi(u) \quad \text{for} \quad u \in [0,+\infty),\, m > 1. \tag{3.3}$$

Theorem 3.1. *Suppose that (A_1) holds, $a \in L^1([0,\,+\infty);[0,\,+\infty))$, and satisfies the fast decay condition*

$$\int_0^{+\infty} \mathcal{P}^{-1}\left[\frac{1}{t^{N-1}}\int_0^t s^{N-1}a(s)ds\right]dt < +\infty. \tag{3.4}$$

Then the problem (1.1) has at least one bounded positive solution.

Proof. It follows from (A_1) that f is a. e. finite and bounded on $[0,\,+\infty)\times[0,\,+\infty)$.

In fact, let $\{E_n\}_{n=1}^{\infty} \subset E \subset [0,+\infty)\times[0,+\infty), E_n \subset E_{n+1}, n = 1,2,\cdots, E_+ = E[f = +\infty], E_- = E[f = -\infty], f^+ = \max\{f,0\}, [f(v,u)]_n = \min\{f(v,u),n\}$, $(v,u) \in E, n = 1,2,\cdots$. Then $E = \bigcup\limits_{n=1}^{\infty} E_n$ and for any $n \ge N$, we have

$$\begin{aligned}+\infty > \int_E f^+(v,u)d\sigma &\ge \int_{E_n}[f^+(v,u)]_n d\sigma\\ &\ge \int_{E_N\cap E_+}[f^+(v,u)]_n d\sigma = n\cdot m(E_N\cap E_+).\end{aligned}$$

Let $n \to \infty$, then $m(E_N\cap E_+) = 0, N = 1,2,\cdots$. Thus $mE_+ = 0$.

By the same way, we easily get $mE_- = 0$. Let $E_0 = E_+\cup E_-$, then $mE_0 = 0$ and f is finite and bounded on $E\backslash E_0$. Denote $M_0 = \sup\limits_{(v,u)\in E\backslash E_0} f(v,\,u)$, then $f(v,u) \le M_0$, for $(v,u) \in E\backslash E_0$.

It follows from (3.1)–(3.3) that there exists a large enough real number $\widetilde{\rho} > 0$ such that

$$R_0 = [(\lambda+1)(\psi(1)+1)]^{\frac{1}{\alpha}}\int_0^{\widetilde{\rho}} \mathcal{P}^{-1}\left[\frac{1}{t^{N-1}}\int_0^t s^{N-1}a(s)ds\right]dt < +\infty. \tag{3.5}$$

Let $R = z_0 + R_0(\sup\limits_{z\in\widehat{G}}\|z\| + 1)$, where $\widehat{G} \subset (0,\widetilde{\rho})$ is a bounded subset on $(0,+\infty)$. By the symmetry of z and using the standard integrating procedure of (2.2), one has

$$\begin{aligned}z(\rho) &= z(0) + \int_0^{\rho}\mathcal{P}^{-1}[\frac{1}{t^{N-1}}\int_0^t v^{N-1}\lambda a(v)f(v,\,z(v))dv]dt\\ &\le z_0 + (z(\rho)+1)^{\frac{\delta}{\alpha}}[(\lambda+1)(\psi(1)+1)]^{\frac{1}{\alpha}}\int_0^{\rho}\mathcal{P}^{-1}(\frac{1}{t^{N-1}}\int_0^t v^{N-1}a(v)dv)dt\\ &\le z_0 + (\|z\|+1)^{\frac{\delta}{\alpha}}R_0 \le R, \quad \forall\ \rho \le \widetilde{\rho},\end{aligned} \tag{3.6}$$

which implies that the solution $z(\rho)$ of the equation (1.1) is bounded. Thus the equation (1.1) has at least one bounded positive solution, the proof is completed. $\square$

Remark 3.2. Under a sharper condition we propose a new result. Theorem 3.2. extends and improves the corresponding results of [25, 28, 30] from many aspects.

Theorem 3.2. *Suppose that (A_1) holds, and $a \in L^1([0, +\infty); [0, +\infty))$, $\mathcal{P}^{-1} \in L^1[0, +\infty)$. Then the problem (1.1) has at least one bounded monotone positive solution.*

Proof. By making use of the condition $a \in L^1([0, +\infty); [0, +\infty)), \mathcal{P}^{-1} \in L^1[0, +\infty)$, we easily know that

$$\int_0^{+\infty} \mathcal{P}^{-1} \left[\frac{1}{t^{N-1}} \int_0^t s^{N-1} a(s) ds \right] dt < +\infty. \tag{3.7}$$

It is well known that (1.1) has a positive radial solution if and only if the following integral equation

$$z(\rho) = z(0) + \int_0^\rho \mathcal{P}^{-1} \left[\frac{1}{t^{N-1}} \int_0^t v^{N-1} \lambda a(v) f(v,\, z(v)) dv \right] dt \tag{3.8}$$

has a positive radial solution. It follows from the problem (3.8) that there generates a positive increasing sequence $\{z_m(\rho)\}_{m=1}^{\infty}$, which is bounded on $[0,\, R^*] \subset (0, +\infty)$ for fixed large enough $R^* > 0$. To do this, we choose the initial values $z(0) = z_0 > 0,\ \forall\ z \geq 0$ for the problem (2.3). Then we define recursively a sequence $\{z_m(\rho)\}_{m \geq 1}$ on $[0, +\infty)$ by $z(0) = z_0$. Thus for any $\rho \geq 0$, we have

$$z_m(\rho) = z_0 + \int_0^\rho \mathcal{P}^{-1} \left[\frac{1}{t^{N-1}} \int_0^t v^{N-1} \lambda a(v) f(v,\, z_{m-1}(v)) dv \right] dt. \tag{3.9}$$

It is easy to see that $\{z_m(\rho)\}_{m=1}^{\infty}$ is an increasing sequence of nonnegative and increasing functions, and $z_m(\rho) \geq z_0$, for any $\rho \geq 0$ and $m = 1,\, 2,\, \cdots$.

We claim that the sequences $\{z_m(\rho)\}_{m=1}^{\infty}$ are bounded subsets on $[0,\, R^*]$. For fixed $\rho > 0$, we notice that ψ is increasing with respect to the independent variable on $[0, +\infty)$. Then by (A_1) and Remark 3.1, we have

$$\begin{aligned} z_m(\rho) &= z_0 + \int_0^\rho \mathcal{P}^{-1} [\frac{1}{t^{N-1}} \int_0^t v^{N-1} \lambda a(v) f(v,\, z_{m-1}(v)) dv] dt \\ &\leq z_0 + (z_{m-1}(\rho)+1)^{\frac{\delta}{\alpha}} [(\lambda+1)(\psi(1)+1)]^{\frac{1}{\alpha}} \int_0^\rho \mathcal{P}^{-1} (\frac{1}{t^{N-1}} \int_0^t v^{N-1} a(v) dv) dt \\ &\leq z_0 + (\|z\| + 1)^{\frac{\delta}{\alpha}} R_0 \leq R, \quad \forall\ \rho \leq R^*. \end{aligned} \tag{3.10}$$

Consequently, we see that the sequence of solution $\{z_m(\rho)\}_{m=1}^{\infty}$ is bounded and monotone. □

4. Further discussion

Example 4.1. Consider the existence of positive solution for the following nonlinear Schrödinger elliptic equation

$$div((|\nabla u|)^5 \nabla u) = \lambda a(|v|) e^{-2u} \sin u, \text{ for } \quad u,\, v \in R^N, \tag{4.1}$$

where $\mathcal{G}(v)=(a_0+a_1v+a_2v^2)^{-1}v^5$, a_0, a_1 and a_2 are nonnegative constants, with $a_2\neq 0$, $f(v,u)=e^{-2u}\sin u$. Choosing $\alpha=5,\delta=\frac{1}{5}$, then $\mathcal{G}\in\mathcal{W}$ and f satisfies (A_1). When $N\geq 11$ and $a(v)=\frac{(N-11)v^{10}+N-1}{v(v^{10}+1)^2}$, we have

$$\begin{aligned}
&\int_0^{+\infty}\mathcal{P}^{-1}\left[\frac{1}{t^{N-1}}\int_0^t v^{N-1}a(v)dv\right]dt\\
&=\int_0^{+\infty}\mathcal{P}^{-1}\left[\frac{1}{t^{N-1}}\int_0^t \frac{(N-11)v^{10}+N-1}{v^{2-N}(v^{10}+1)^2}dv\right]dt\\
&=\int_0^{+\infty}(\frac{1}{t^{N-1}}\int_0^t \frac{(N-11)v^{10}+N-1}{v^{2-N}(v^{10}+1)^2}dv)^{-\frac{1}{5}}dt<\int_0^{+\infty}\frac{dv}{(1+v^{10})^{\frac{1}{5}}}<+\infty.
\end{aligned}\tag{4.2}$$

Conditions of Theorem 3.2 are satisfied, then the equation (4.1) has a bounded positive solution.

Example 4.2. Consider the existence of positive solution for the following nonlinear Schrödinger elliptic equation

$$div((|\nabla u|)^2\nabla u)=\lambda a(|v|)e^{-u^3-6u-v}\cos(v+u),\ \text{for any}\ \ u,\,v\in R^N,\tag{4.3}$$

where $\mathcal{G}(v)=(a_0+a_1v+a_2v^2)^{-1}v^2$, a_0, a_1 and a_2 are nonnegative constants, with $a_2\neq 0$, $f(v,u)=e^{-u^3-6u-v}\cos(v+u)$. Choosing $\alpha=2,\delta=\frac{1}{2}$, then $\mathcal{G}\in\mathcal{W}$ and f satisfies (A_1). When $N\geq 17$ and $a(v)=\frac{(N-17)v^{16}+N-1}{v(v^{16}+1)^2}$, we have

$$\begin{aligned}
&\int_0^{+\infty}\mathcal{P}^{-1}\big(\frac{1}{t^{N-1}}\int_0^t v^{N-1}a(v)dv\big)dt\\
&=\int_0^{+\infty}\mathcal{P}^{-1}[\frac{1}{t^{N-1}}\int_0^t \frac{(N-17)v^{16}+N-1}{v^{2-N}(v^{16}+1)^2}dv]dt\\
&=\int_0^{+\infty}(\frac{1}{t^{N-1}}\int_0^t \frac{(N-17)v^{16}+N-1}{v^{2-N}(v^{16}+1)^2}dv)^{-\frac{1}{2}}dt<\int_0^{+\infty}\frac{dv}{(1+v^{16})^{\frac{1}{2}}}<+\infty.
\end{aligned}\tag{4.4}$$

Conditions of Theorem 3.2 are satisfied, then the equation (4.1) has at least one bounded monotone positive solution.

Remark 4.1. If $f(v,\,u)=f(u)$ is nonnegative, increasing with respect to u and not identically zero, we have the existence of positive solution without other conditions.

Remark 4.2. From above the examples, we not only present a novelty condition of the existence of at least one positive solution for the equation (4.1), but also establish a iterative sequence of the solution. We also note that the positive solution is monotone.

5. Conclusions

In this paper we have studied a sharper condition for the nonlinear Schrödinger equation (1.1) in the cases when $\lambda>0$ is a constant. From a stability of the positive steady state point of view, solving the equation (1.1), we adopt a change of variables. Then the Schrödinger equation (1.1) can be transformed to a more convenient form.

We plan to explore the idea of ideal monotone bounded solution and its connections to stability further by extending the ideas and results of this paper, including those described in [3, 14, 19, 25, 28–30], among other possibilities.

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