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ASYMPTOTIC SPREADING IN A COMPETITION SYSTEM WITH NONLOCAL DISPERSAL*

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Abstract This paper is concerned with the long time behavior of a competition system with nonlocal dispersal. When the initial conditions of both unknown functions satisfy proper decay behavior, we obtain the rough spreading speed of one unknown function and show the upper and lower bounds of spreading speed of another unknown function. Moreover, a numerical example is given to illustrate our analytic results. Our conclusions imply that both the linear part and nonlinear part in reaction terms may affect the spreading speeds. Moreover, in such a competitive system with constant coefficients, we may observe the propagation terraces in some component.

Keywords Noncooperative system, upper and lower solutions, auxiliary system.

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1. Introduction

Competition phenomena are universal in real world, and many competition models were proposed in population dynamics, see some examples in Cantrell and Cosner [6] and Murray [19, 20]. When both spatial and temporal variables are involved in some competition models, their traveling wave solutions have been widely studied to model the competition-coexistence process, competition-exclusion process [7, 14, 20, 22–24]. Moreover, the corresponding initial value problems were also applied to explore the invasion process between the aboriginal and the invader [24], two competition invaders [11, 17].

In population dynamics, besides the Fick diffusion formulated by Laplacian operator [19, Section 11.1], there are also some other diffusion recipes to model different questions. For example, the following nonlocal dispersal operator has been utilized to model the long range effect and random walk of individuals [19, 20]

$$u_t(t,x) = \int_{\mathbb{R}^N} J(x-y)[u(t,y)-u(t,x)]dy, x \in \mathbb{R}^N, t>0, N \in \mathbb{N},$$

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in which J is a probability function formulating spatial dispersal of individuals and u is the unknown function. In particular, such a diffusion mode was also utilized in many models arising from other fields [4, 10]. Recently, the nonlocal dispersal has been considered in competition system of two species [2, 3, 9, 12, 15, 21, 25, 26]

$$\begin{cases} \dfrac{\partial u_1(t,x)}{\partial t} = d_1[J_1 * u_1](t,x) + r_1 u_1(t,x)[1 - u_1(t,x) - a_1 u_2(t,x)], \\ \dfrac{\partial u_2(t,x)}{\partial t} = d_2[J_2 * u_2](t,x) + r_2 u_2(t,x)[1 - a_2 u_1(t,x) - u_2(t,x)], \end{cases} \tag{1.1}$$

in which u_1, u_2 are unknown functions denoting population densities in population dynamics, $x \in \mathbb{R}, t > 0$, all the parameters are positive and

$$[J_i * u_i](t,x) = \int_{\mathbb{R}} J_i(x-y)[u_i(t,y) - u_i(t,x)]dy,\ i = 1,2,\ t > 0,\ x \in \mathbb{R}$$

with probability functions J_1, J_2. When the propagation dynamics is concerned, different traveling wave solutions of (1.1) have been studied in Bao et al. [2, 3], Fang and Zhao [9], Pan [21], Yu and Yuan [25], Zhang and Zhao [26]. Here, a traveling wave solution of (1.1) is a special entire solution $(u_1(t,x), u_2(t,x)) = (\phi(x+ct), \psi(x+ct))$ satisfying proper boundary conditions, where (ϕ, ψ) is the wave profile that propagates through the one-dimensional spatial domain at a constant velocity $c \in \mathbb{R}$.

Although the traveling wave solutions could formulate some important evolution processes in population dynamics [14, 20, 22], they are special solutions that could not formulate many phenomena. For example, when the coinvasion-coexistence process is described by traveling wave solutions [21, 25], two species have the same invasion speed. In the corresponding initial value problem of nonlocal dispersal system (1.1), do two competitive species have different invasion speeds if two species are invaders? For the classical diffusion-competition systems, the question has been studied in [11, 17]. To further formulate the invasion process of two competitive species in nonlocal dispersal model (1.1), we investigate the following initial value problem

$$\begin{cases} \dfrac{\partial u_1(t,x)}{\partial t} = d_1[J_1 * u_1](t,x) + r_1 u_1(t,x)[1 - u_1(t,x) - a_1 u_2(t,x)], \\ \dfrac{\partial u_2(t,x)}{\partial t} = d_2[J_2 * u_2](t,x) + r_2 u_2(t,x)[1 - a_2 u_1(t,x) - u_2(t,x)], \\ u_1(0,x) = u_1(x),\ \ u_2(0,x) = u_2(x), x \in \mathbb{R}, t > 0, \end{cases} \tag{1.2}$$

in which $u_1(x), u_2(x)$ are initial conditions that satisfy proper decay behavior clarified later, and the probability functions satisfy the following assumptions:

(J) J_i is nonnegative, even, Lebesgue integrable such that $\int_{\mathbb{R}} J_i(y)dy = 1$, and for some $\lambda > 0$, $\int_{\mathbb{R}} J_i(y)e^{\lambda y}dy < \infty, i = 1,2$.

When the invasion process of two competitors is involved similar to that in [11,17], it is difficult to directly apply the propagation theory of monotone semiflows [9,16,18,24]. In this paper, based on comparison principle appealing to competitive systems, we try to construct some auxiliary equations and functions to estimate the long time behavior of both unknown functions. Our results can show the nontrivial effect of a_1, a_2 that reflects the interspecific competition, and two competitors may have distinct spreading speeds. Moreover, we also give some numerical examples to illustrate our conclusions.

The rest of this paper is organized as follows. In Section 2, we present some results on existence, invariance of initial value problem. The main conclusions are proved by estimating the spreading speeds of two species in Section 3. We then give some numerical simulations in Section 4.

2. Preliminaries

Let X be the Banach space of uniformly continuous and bounded functions from $\mathbb{R}$ to $\mathbb{R}$ equipped with supremum norm, and $u \in X^+$ implies that $u \in X, u(x) \geq 0, x \in \mathbb{R}$. Moreover, if $a < b$, then

$$X_{[a,b]} = \{u : u \in X \text{ and } a \leq u(x) \leq b, x \in \mathbb{R}\}.$$

Consider the following initial value problem

$$\begin{cases} \dfrac{\partial u(t,x)}{\partial t} = d[J * u](t,x) + u(t,x)\left[r - u(t,x)\right], x \in \mathbb{R}, t > 0, \\ u(0,x) = \chi(x) \in X^+, x \in \mathbb{R}, \end{cases} \tag{2.1}$$

where J satisfies (J), $d > 0$ and $r > 0$ are constants. Evidently, (2.1) satisfies comparison principle even if r is replaced by a function. Also define

$$c' = \inf_{\lambda > 0} \frac{d\left[\int_{\mathbb{R}} J(y)e^{\lambda y}dy - 1\right] + r}{\lambda}, \tag{2.2}$$

then c' is bounded. By Jin and Zhao [13], we have the following conclusion.

Lemma 2.1. *Assume that $\chi(x) \in X_{[0,r]}$. Then (2.1) admits a solution $u(t,\cdot) \in X_{[0,r]}$ for all $t > 0$. If $\chi(x)$ has nonempty support, then for any $c < c'$, we have*

$$\liminf_{t\to\infty} \inf_{|x|<ct} u(t,x) = \limsup_{t\to\infty} \sup_{|x|<ct} u(t,x) = r.$$

If $\chi(x)$ has compact support, then

$$\lim_{t\to\infty} \sup_{|x|>ct} u(t,x) = 0 \textit{ for any given } c > c'.$$

Remark 2.1. If $\chi(x)$ has nonempty compact support in Lemma 2.1, then c' is the spreading speed of $u(t,x)$ (see [1]).

Due to the competitive nonlinearity, we introduce the following definition of upper and lower solutions and show the comparison principle of (1.2).

Definition 2.1. Assume that $\overline{u}_1(t,x), \overline{u}_2(t,x), \underline{u}_1(t,x), \underline{u}_2(t,x)$ are nonnegative and continuous in $x \in \mathbb{R}, t \geq 0$. If they satisfy

$$\begin{cases} \dfrac{\partial \overline{u}_1(t,x)}{\partial t} \geq d_1[J_1 * \overline{u}_1](t,x) + r_1\overline{u}_1(t,x)[1 - \overline{u}_1(t,x) - a_1\underline{u}_2(t,x)], \\ \dfrac{\partial \overline{u}_2(t,x)}{\partial t} \geq d_2[J_2 * \overline{u}_2](t,x) + r_2\overline{u}_2(t,x)[1 - a_2\underline{u}_1(t,x) - \overline{u}_2(t,x)], \\ \dfrac{\partial \underline{u}_1(t,x)}{\partial t} \leq d_1[J_1 * \underline{u}_1](t,x) + r_1\underline{u}_1(t,x)[1 - \underline{u}_1(t,x) - a_1\overline{u}_2(t,x)], \\ \dfrac{\partial \underline{u}_2(t,x)}{\partial t} \leq d_2[J_2 * \underline{u}_2](t,x) + r_2\underline{u}_2(t,x)[1 - a_2\overline{u}_1(t,x) - \underline{u}_2(t,x)], \\ (\underline{u}_1(0,x), \underline{u}_2(0,x)) \leq (u_1(x), u_2(x)) \leq (\overline{u}_1(0,x), \overline{u}_2(0,x)) \end{cases} \tag{2.3}$$

for $t \geq 0, x \in \mathbb{R}$, then $(\overline{u}_1(t,x), \overline{u}_2(t,x)), (\underline{u}_1(t,x), \underline{u}_2(t,x))$ are a pair of upper and lower solutions of (1.2).

Lemma 2.2. *Assume that $(\overline{u}_1(t,x), \overline{u}_2(t,x)), (\underline{u}_1(t,x), \underline{u}_2(t,x)), t \geqslant 0, x \in \mathbb{R}$ are a pair of upper and lower solutions of* (1.2)*. Then the following conclusions hold.*

(i) $(\overline{u}_1(t,x), \overline{u}_2(t,x)) \geqslant (\underline{u}_1(t,x), \underline{u}_2(t,x)), t \geq 0, x \in \mathbb{R}$.

(ii) (1.2) *admits a unique solution satisfying*

$$(\overline{u}_1(t,x), \overline{u}_2(t,x)) \geqslant (u_1(t,x), u_2(t,x)) \geqslant (\underline{u}_1(t,x), \underline{u}_2(t,x)), t > 0, x \in \mathbb{R}.$$

3. Main Results

To state our main conclusion, we first introduce some notations. Define

$$\Theta_i(\lambda, c) = d_i \left(\int_{\mathbb{R}} J_i(y) e^{\lambda y} dy - 1 \right) - c\lambda + r_i, \ \ i = 1, 2,$$

and

$$c_i^* = \inf_{\lambda > 0} \frac{d_i \left[\int_{\mathbb{R}} J_i(y) e^{\lambda y} dy - 1 \right] + r_i}{\lambda}, \ \ i = 1, 2.$$

Lemma 3.1. *$c_1^* > 0, c_2^* > 0$ satisfy the following facts.*

(B1) *If $c < c_i^*$, then for any $\lambda > 0$ such that $\int_{\mathbb{R}} J_i(y) e^{\lambda y} dy < \infty$, $\Theta_i(\lambda, c) > 0$, $i = 1, 2$.*

(B2) *If $c = c_i^*$, then there exists $\lambda_i > 0$ such that $\Theta_i(\lambda_i, c) = 0$, $i = 1, 2$, and for any $\lambda > 0$ such that $\int_{\mathbb{R}} J_i(y) e^{\lambda y} dy < \infty$, $\Theta_i(\lambda, c) \geq 0$, $i = 1, 2$.*

In this section, we estimate the long time behavior of (1.2) when

$$u_1(x), u_2(x) \in X_{[0,1]}$$

such that

$$0 < \sup_{x \in \mathbb{R}} \left[u_i(x) e^{\lambda_i |x|} \right] < \infty, i = 1, 2. \tag{3.1}$$

Firstly, the global existence of solutions to (1.2) is evident since $(1, 1), (0, 0)$ are a pair of upper and lower solutions of (1.2), which was also investigated in [2, 15].

Lemma 3.2. (1.2) *admits a unique solution satisfying*

$$(0,0) \leq (u_1(t,x), u_2(t,x)) \leq (1,1), t > 0, x \in \mathbb{R}.$$

Moreover, we have

$$u_1(t,x) > 0, u_2(t,x) > 0, t > 0, x \in \mathbb{R}.$$

By these thresholds, we formulate the outer expansion of both species as follows.

Theorem 3.1. *For any given $\epsilon > 0$, we have*

$$\lim_{t\to+\infty} \sup_{|x|>\left(c_1^*+\epsilon\right)t} u_1(x,t) = \lim_{t\to+\infty} \sup_{|x|>\left(c_2^*+\epsilon\right)t} u_2(x,t) = 0.$$

Proof. By Lemma 2.1, we see that

$$\frac{\partial u_1(t,x)}{\partial t} \leq d_1[J_1 * u_1](t,x) + r_1 u_1(t,x)[1 - u_1(t,x)], x \in \mathbb{R}, t > 0,$$

and

$$\frac{\partial u_2(t,x)}{\partial t} \leq d_2[J_2 * u_2](t,x) + r_2 u_2(t,x)[1 - u_2(t,x)], x \in \mathbb{R}, t > 0,$$

then Lemma 2.1 implies what we wanted. In particular, we can verify that

$$u_i(t,x) \leq \min\{1, e^{\lambda_i(\pm x + c_i^* t + t_0)}\}, t \geq 0, x \in \mathbb{R}, i = 1, 2,$$

in which $t_0 > 0$ such that

$$u_i(0,x) \leq \min\{1, e^{\lambda_i(\pm x + t_0)}\}, x \in \mathbb{R}, i = 1, 2.$$

The proof is complete. □

Further define constant

$$c_1 = \inf_{\lambda > 0} \frac{d_1 \left[\int_{\mathbb{R}} J_1(y) e^{\lambda y} dy - 1\right] + r_1(1 - a_1)}{\lambda}.$$

We can present the inner expansion of u_1 as follows.

Theorem 3.2. *Assume that $c_1 > c_2^*$. For any given small $\epsilon > 0$, we have*

$$\liminf_{t\to+\infty} \inf_{|x|<(c_1-\epsilon)t} u_1(x,t) \geqslant 1 - a_1$$

and

$$\liminf_{t\to+\infty} \inf_{|x|<\left(c_1^*-\epsilon\right)t} u_1(x,t) > 0.$$

Proof. We show the result if

$$2\epsilon < c_1 - c_2^*.$$

By Lemma 2.1, we have

$$\frac{\partial u_1(t,x)}{\partial t} \geq d_1[J_1 * u_1](t,x) + r_1 u_1(1 - a_1 - u_1(t,x))$$

for any $(t,x) \in (0,+\infty) \times \mathbb{R}$, then

$$\liminf_{t\to+\infty} \inf_{|x|<(c_1-\epsilon)t} u_1(t,x) \geqslant 1 - a_1.$$

By Theorem 3.1, for any $\epsilon' > 0$, there exists $T_1 > 0$ such that $t \geqslant T_1$ implies

(i) $\sup_{|x|\geq(c_2^*+\epsilon)t} u_2(t,x) < \epsilon'$

(ii) $\sup_{|x|<(c_1-\epsilon)t} (u_2(t,x)/u_1(t,x)) \leq 2/(1-a_1)$.

So we obtain

$$\frac{\partial u_1(t,x)}{\partial t} \geq d_1[J_1 * u_1](t,x) + r_1 u_1(t,x)\left(1 - a_1\epsilon' - \left(\frac{2}{1-a_1}+1\right)u_1(t,x)\right), x\in\mathbb{R}, t\geq T_1.$$

Note that $u_1(T_1,x)>0, x\in\mathbb{R}$, Lemma 2.1 implies

$$\liminf_{t\to+\infty} \inf_{|x|<(c-\epsilon/2)t} u_1(t,x) \geq \frac{1-a_1\epsilon'}{\frac{2}{1-a_1}+1}$$

with

$$c = \inf_{\lambda>0} \frac{d_1\left[\int_{\mathbb{R}} J_1(y)e^{\lambda y}dy - 1\right] + r_1(1-a_1\epsilon')}{\lambda}.$$

Let $\epsilon'>0$ be small, then $c > c_1^* - \epsilon/2$ such that

$$\liminf_{t\to+\infty} \inf_{|x|<(c_1^*-\epsilon)t} u_1(t,x) > 0$$

holds. The proof is complete. □

Theorems 3.1 and 3.2 imply that the interspecific competition does not change some threshold of propagation dynamics under proper conditions. We now show the effect of interspecific competition by investigating the property of u_2. Let

$$\begin{aligned} c_3^* &= \inf_{\lambda>0} \frac{d_2\left[\int_{\mathbb{R}} J_2(y)e^{\lambda y}dy - 1\right] + r_2(1-a_2(1-a_1))}{\lambda} \\ &= \frac{d_2\left[\int_{\mathbb{R}} J_2(y)e^{\lambda_3 y}dy - 1\right] + r_2(1-a_2(1-a_1))}{\lambda_3} \end{aligned}$$

and

$$c_4^* = \inf_{\lambda>0} \frac{d_2\left[\int_{\mathbb{R}} J_2(y)e^{\lambda y}dy - 1\right] + r_2(1-a_2)}{\lambda}.$$

Corollary 3.1. *Assume that $\int_{\mathbb{R}} J_2(y)e^{\lambda y}dy$ is twice differentiable in $\lambda>0$ if $\int_{\mathbb{R}} J_2(y)e^{\lambda y}dy<\infty$ and $a_2(1-a_1)>0$. Then $\lambda_2>\lambda_3$.*

Proof. By the definitions of c_2^*, c_3^*, we have

$$d_2\int_{\mathbb{R}} J_2(y)ye^{\lambda_2 y}dy = c_2^*, d_2\int_{\mathbb{R}} J_2(y)ye^{\lambda_3 y}dy = c_3^*.$$

By the smoothness, we see that $d_2\int_{\mathbb{R}} J_2(y)ye^{\lambda y}dy$ is strictly increasing in λ, then $c_2^*>c_3^*$ implies that $\lambda_2>\lambda_3$. The proof is complete. □

Theorem 3.3. *Assume that $a_2>0, c_1>c_2^*$. Then for any $\epsilon>0$, we have*

$$\liminf_{t\to+\infty} \inf_{|x|<(c_4^*-\epsilon)t} u_2(t,x) \geq 1-a_2.$$

Moreover, if $\lambda_2>\lambda_3$ such that

$$(\lambda_2 c_2^* - \lambda_3 c_3^*)/(\lambda_2-\lambda_3) < c_1,$$

then for any $\epsilon>0, \limsup_{t\to+\infty} \sup_{|x|>(c_3^*+\epsilon)t} u_2(t,x) = 0$.

Proof. Note that

$$\frac{\partial u_2(t,x)}{\partial t} \geq d_2[J_2 * u_2](t,x) + r_2 u_2(t,x)[1 - a_2 - u_2(t,x)], x \in \mathbb{R}, t > 0,$$

then

$$\liminf_{t\to+\infty} \inf_{|x|<(c_4^*-\epsilon)t} u_2(t,x) \geq 1 - a_2$$

is evident by Lemma 2.1.

We regard u_1 as a given function, and estimate u_2 by the known properties of u_1 (that is, we regard the equation of u_2 as a nonautonomous equation). Now, we construct a proper upper solution of u_2 to show the conclusion. Let $\varepsilon > 0$ be small such that

$$\begin{aligned} c_3^* + \epsilon &= \inf_{\lambda>0} \frac{d_2\left(\int_{\mathbb{R}} J_2(y)e^{\lambda y}dy - 1\right) + r_2(1 - a_2(1 - a_1 - \varepsilon))}{\lambda} \\ &= \frac{d_2\left(\int_{\mathbb{R}} J_2(y)e^{\lambda_3' y}dy - 1\right) + r_2(1 - a_2(1 - a_1 - \varepsilon))}{\lambda_3'}. \end{aligned}$$

Then $\lambda_3' \to \lambda_3$ as $\varepsilon, \epsilon \to 0$. Without loss of generality, we assume that $\varepsilon, \epsilon > 0$ such that $\lambda_2 > \lambda_3'$ and $(\lambda_2 c_2^* - \lambda_3'(c_3^* + \epsilon))/(\lambda_2 - \lambda_3') < c_1$.

Fix $T_1 > 0$ such that

$$\inf_{|x|<(c_1-\epsilon)t} u_1(t,x) \geqslant 1 - a_1 - \varepsilon, t > T_1.$$

We now define

$$\overline{u}_2(t,x) = \min\left\{e^{\lambda_2(\pm x + c_2^* t)+T}, e^{\lambda_3'(\pm x+(c_3^*+\epsilon)t)+T}, 1\right\},$$

then we obtain an upper solution of u_2 if T is large. We now verify the upper solution. Firstly, we consider the initial condition. Since $\lambda_2 > \lambda_3'$, we have

$$\overline{u}_2(t,x) = \min\left\{e^{\lambda_2(-x+c_2^*t)+T}, e^{\lambda_2(x+c_2^*t)+T}, 1\right\}$$

if $|x|$ is large, so the initial condition is true by fixing initial time $T > 0$, which is evident from the proof of Theorem 3.1. When $\overline{u}_2(t,x) = e^{\lambda_2(\pm x+c_2^*t)+T}$ or $\overline{u}_2 = 1$, the verification is evident by Theorem 3.1. When $\overline{u}_2(t,x) = e^{\lambda_3'(\pm x+(c_3^*+\epsilon)t)+T}$, then

$$\lambda_3'(-|x| + (c_3^* + \epsilon)t) < \lambda_2(-|x| + c_2^* t)$$

or

$$|x| < \frac{\lambda_2 c_2^* - \lambda_3'(c_3^* + \epsilon)}{\lambda_2 - \lambda_3'} t < c_1 t$$

implies $|x| < (c_1 - \epsilon)t$, so the inequality is true by the definitions of c_3^*, λ_3' and the property of $u_1(t,x) \geq 1 - a_1 - \varepsilon$. We complete the proof by Lemma 2.1 and the positivity of semigroup generated by $u_{2,t} = d_2[J_2 * u_2](t,x)$. □

Remark 3.1. In Theorem 3.3, we show the effect of interspecific competition when $c_1 > c_2^*$. It is a pity that we can not show further properties on the spreading speed of u_2 if it exists.

4. Numerical Simulation

In this section, we give a numerical example to illustrate our results. Let $u(t,x)$ be a given function, we define

$$L_u^t(\alpha) = \inf\{x : u(t,x) = \alpha \text{ with any given } t\}, \alpha \in \mathbb{R},$$
$$\mathcal{L}_u^t(\alpha) = \sup\{x < 0 : u(t,x) = \alpha \text{ with any given } t\}, \alpha \in \mathbb{R}.$$

Evidently, they are level sets of $u(t,x)$ and for proper $\alpha \in \mathbb{R}$, the movement speed of $L_u^t(\alpha)$ could reflect the spreading speed of u. We consider the following initial value problem

$$\begin{cases} \dfrac{\partial u_1(t,x)}{\partial t} = 2[J * u_1](t,x) + 3u_1(t,x)[1 - u_1(t,x) - au_2(t,x)], \\ \dfrac{\partial u_2(t,x)}{\partial t} = [J * u_2](t,x) + u_2(t,x)[1 - bu_1(t,x) - u_2(t,x)], \\ u_1(0,x) = \ u_2(0,x) = \cos x, x \in [-\pi/2, \pi/2], \\ u_1(0,x) = \ u_2(0,x) = 0, |x| > \pi/2, \end{cases} \tag{4.1}$$

in which $(a,b) = (0,0)$ or $(a,b) = (0.1, 0.4)$ and

$$[J * u_i](t,x) = \frac{1}{2}\int_{-1}^{1} [u_i(t,y) - u_i(t,x)]dy, i = 1,2.$$

If $(a,b) = (0,0)$, then (4.1) becomes two independent equations and the spreading speeds of u_1, u_2 can be obtained by Lemma 2.1. That is, when the interspecific competition vanishes, then u_1 admits the rough spreading speed determined by the following minimal value (see the left of Figure 1)

$$\inf\left\{c : \int_{-1}^{1} e^{\lambda y}dy - c\lambda + 1 = 0 \text{ admits positive real root}\right\} \approx 2.296,$$

and u_2 spreads at (see the right of Figure 1)

$$\inf\left\{c : \frac{1}{2}\int_{-1}^{1} e^{\lambda y}dy - c\lambda = 0 \text{ admits positive real root}\right\} \approx 0.905.$$

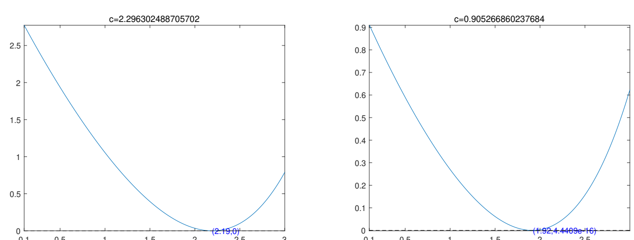

Figure 1. Simulation of eigenvalue equations.

Now, we show the spatial-temporal evolutionary of u_1, u_2 when $a = b = 0$ in Figure 2, from which we see that each species has almost constant spreading speed.

To further estimate the spreading speeds, we also show some level sets in Figure 3 and Table 1, from which we see that their spreading speeds are close to thresholds determined by Lemma 2.1.

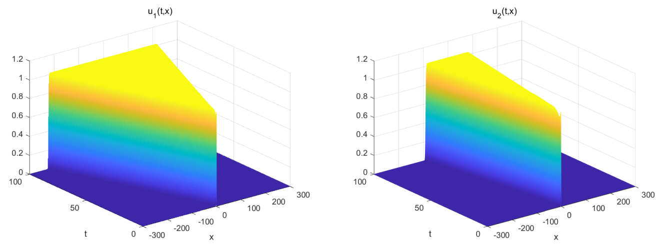

Figure 2. Spatial-temporal plots of u_1, u_2 if $a = b = 0$.

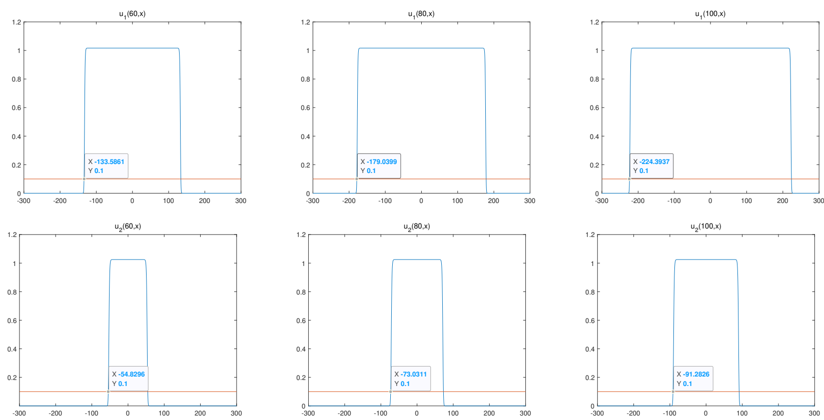

Figure 3. Level sets of u_1, u_2 if $a = b = 0$.

Table 1. Approximate level sets of u_1, u_2 if $a = b = 0$.

Time	60	80	100	Average moving speed for $t \in [60, 100]$
$L^t_{u_1}(0.1)$	-133.5861	-179.0399	-224.3937	2.270
$L^t_{u_2}(0.1)$	-54.8296	-73.0311	-91.2826	0.911

We now consider the effect of interspecific competition when $(a, b) = (0.1, 0.4)$, which is shown in Figure 4.

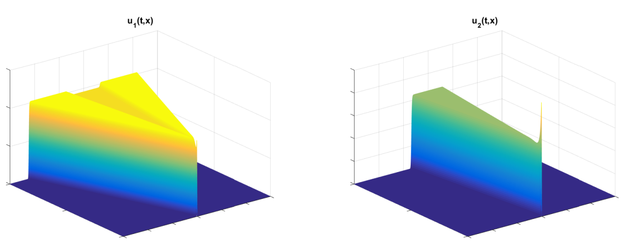

Figure 4. Spatial-temporal plots of u_1, u_2 if $a = 0.1, b = 0.4$.

We now estimate the invasion speed. According to our results in Section 3, u_1 admits the rough spreading speed that is close to 2.296, and the expansion speed of u_2 is not larger than 0.703 (see the left of Figure 5) and is not less than 0.678 (see the right of Figure 5), which are estimated by

$$\inf\left\{c : \frac{1}{2}\int_{-1}^{1} e^{\lambda y}dy - c\lambda - 0.36 = 0 \text{ admits positive real root}\right\} \approx 0.703,$$

$$\inf\left\{c : \frac{1}{2}\int_{-1}^{1} e^{\lambda y}dy - c\lambda - 0.40 = 0 \text{ admits positive real root}\right\} \approx 0.678.$$

Moreover, Figure 4 shows the role of interspecific comparing with Figure 2. To further estimate these speeds, we also present the level sets in Figure 6 and Table 2, from which we find the asymptotic spreading speeds that are close to our estimation.

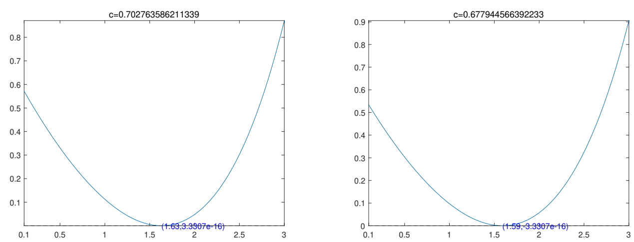

Figure 5. Simulation of eigenvalue equations.

Table 2. Approximate level sets of u_1, u_2 if $a = 0.1, b = 0.4$.

Time	60	80	100	Average moving speed for $t \in [60, 100]$
$L^t_{u_1}(0.1)$	-133.8862	-178.9399	-224.1937	2.258
$\mathcal{L}^t_{u_1}(0.9)$	-40.7284	-54.0795	-67.6306	0.673
$L^t_{u_2}(0.1)$	-41.1784	-54.7296	-68.1807	0.675

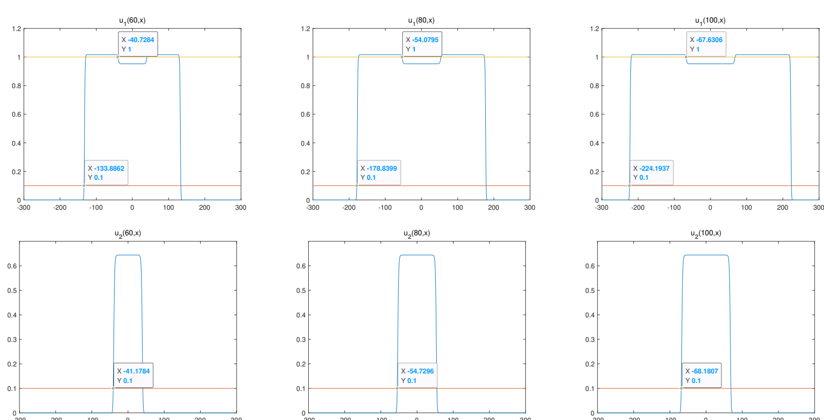

Figure 6. Level sets of u_1, u_2 if $a = 0.1, b = 0.4$.

Before ending this paper, we make the following remark.

Remark 4.1. From Figure 4, we observed that there are some different moving speeds of different level sets, which implies the existence of propagation terraces [8]. In this paper, we obtained some upper and lower bounds of moving speeds of propagation terraces, which implies that two functions may have different spreading speeds and one speed equals to the minimal wave speed in [21, 25]. To further estimate the different moving speeds of different terraces equals to show the precise spreading speeds of two competitive species, which needs further investigation.

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