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SOLITON SOLUTIONS FOR ANTI-CUBIC NONLINEARITY USING THREE ANALYTICAL APPROACHES

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Abstract In this article, three constructive techniques namely, $\mathrm{Exp_a}$-function method, the modified Kudryashov method and the generalized tanh-method are adopted to analyze the nonlinear Schrödinger equation having anti-cubic nonlinearity. Nonlinear Schrödinger equation is a comprehensive model that governs wave behavior in optical fiber. Cubic-quintic nonlinear Schrödinger equation, additionally having anti-cubic nonlinear term is investigated to construct bright, dark, kink and singular soliton solutions. The graphical representations of the soliton propagation are also demonstrated by the solutions obtained using these three techniques.

Keywords Nonlinear schrödinger equation, anti-cubic nonliearity, modified kudryashov method, $\mathrm{exp_a}$-function method, generalized tanh-method.

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1. Introduction

The importance of nonlinear partial differential equations (NLPDEs) is acceptable due to their immense use in almost all branches of science like physics, chemistry, biology, mechanics, fiber optics, electronics and atmospheric science. Many physical phenomena are described by the models, based upon these NLPDEs. So obtaining various exact and accurate solutions of these NLPDEs is a fascinating subject in mathematical research. During last few years, many important developments [1–4, 7, 21, 22, 24, 25, 27, 30, 32–39, 41–48, 51, 53] were made for searching the exact solutions for NLPDEs.

Nonlinear Schrödinger equation (NLSE) is a special class of NLPDEs. The NLSEs are applicable in different fields of biological, physical and engineering sciences. NLSE is often used in various processes of plasma physics, nuclear physics, fiber optics and fluid dynamics. The importance of NLSE is evident from the fact that it describes the modeling of many important phenomena, such as wave dynamics in optical fibers, modeling of structure of DNA and wave pattern producing in

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semiconductor material [14]. NLSE with different forms of nonlinearities has immense applications in modeling of many physical phenomena of theoretical physics. Such applications are noticed in water waves, nonlinear optics, Bose-Einstein condensates and nonlinear quantum field theory.

Soliton is a useful feature discussed with analysis of NLSE. Optical soliton is very eminent area of research in field of nonlinear models during past two decades. Optical solitons are used in communication systems through optical fibers. Optical solitons were first studied by Zakharov and Shabat [52] in 1971. These solitons provides very accurate signal transmission over extremely long distances. This provides scope for innovation and development in future communication technology.

In last two decades, various methods were adopted to solve NLSEs with different forms of nonlinearity, such as the trial solution approach [17], the ansatz approach [40], the semi-inverse variational principle [11, 12], the $\tan(\varphi/2)$-expansion method [29], the extended (G'/G)-expansion scheme [16], the NLSE- based constructive method [13], elliptic function method [8], the extended trial equation method [15], simplest equation approach [18] and the first integral method [19].

Nonlinear Schrödinger equation with anti-cubic nonlinear term was firrst observed by Fedele *et al*. [20] during 2003. Afterward, the models with anti-cubic nonlinearity were worked out by many researchers [6, 9, 10, 12, 15, 28, 49]. As Exp_a-function method [5], generalized tanh-method [26, 50] and modified-Kudryashov method [23, 31] r are also very useful and efficient methods to acquire traveling wave solutions of many NLPDEs. So these reliable approaches are applied to acquire soliton solutions of NLSE having anti-cubic nonlinearity. We extract optical soliton solutions and interpret them graphically.

2. The Governing Model

The Nonlinear Schrödinger equation (NLSE) having anti-cubic nonlinearity of form

$$\iota q_t + \delta_1 q_{xx} + \left(\delta_2 |q|^{-4} + \delta_3 |q|^2 + \delta_4 |q|^4\right) q = 0. \tag{2.1}$$

Here, q(x,t) is the function that describes wave profile, x and t are spatial and temporal coordinates respectively. The first term on left side is the temporal evolution of the pulse while δ_1 is coefficient of group velocity dispersion (GVD). The coefficients δ_2, δ_3 and δ_4 introduce anti-cubic, cubic and quintic nonlinearities respectively. If δ_2=0, then Eq. (2.1) reduces to cubic-quintic NLSE.

3. Algorithm of applied methods

In this section, we briefly explain the constructive steps of Exp_a-function method, modified Kudryashov method and generalized tanh-method.

Let PDE as follows

$$G(u, u_x, u_y, u_z, u_t, u_{xt}, u_{yt}, u_{xx}, \cdots) = 0, \tag{3.1}$$

where G is a polynomial involving u and its various partial derivatives, including the highest order derivative term and all nonlinear terms.

Step 1 (Conversion of PDE into ODE)

We use the wave transformation as

$$q(x,t) = u(\xi).e^{\iota(-\kappa x+\omega t)}, \quad \text{where } \xi = \alpha(x-ct), \tag{3.2}$$

into Eq. (3.1), to convert that into ordinary differential equation as

$$H(u, u', u'', u''', \cdots) = 0. \tag{3.3}$$

Here, u is a function of ξ and $u' = du/d\xi$; and c is the wave-speed.
Note: First step, conversion of PDE into ODE, is common in all three methods, so we will start description of each method from step 2.

3.1. Exp$_\mathbf{a}$-function method

The further steps for Exp_a-function method are as follows:-
Step 2. Let

$$u(\xi) = \frac{\sum_{i=0}^{N} a_i a^{i\xi}}{\sum_{i=0}^{N} b_i a^{i\xi}}, \tag{3.4}$$

is supposed to be the solution of Eq. (3.3), where a_i $(i = 1, 2, 3, \cdots, N)$ and b_i $(i = 1, 2, 3, \cdots, N)$ are constants to be determined afterward.
Step 3. Now balancing the highest-order derivative and the nonlinear term of highest order which occur in Eq. (3.3), we determine the values of N.
Step 4. Then use Eq. (3.4) into Equation (3.3), we get an expression in $a^{i\xi}$, where $(i = 1, 2, 3, 4, \cdots, \theta)$ as

$$P(a^{\xi} = t_0 + t_1 a^{\xi} + \cdots + t_\theta a^{\theta\xi} = 0. \tag{3.5}$$

Now accumulating all the terms in the coefficients of $a^{i\xi}$ where power(i) is same and then putting these equal to zero, we get a system of equations in all constant terms. Solving these equations using Maple, the values of involved parameters are find out. Then putting all values into Equation (3.4), we find amplitude component u. We ultimately attain the required solutions by putting these values in (3.2).

3.2. The modified Kudryashov method

The other principal steps carried out in the modified Kudryashov method to find the solution of given a NLEE are as:
Step 2. Let

$$u(\xi) = \sum_{i=0}^{N} a_i p^i, \tag{3.6}$$

is supposed to be the solution of Eq. (3.3), where a_i $(i = 1, 2, 3, \cdots, N)$ are constants to be determined afterward. Here, $p(\xi)$ is the following function

$$p(\xi) = \frac{1}{1+\lambda a^{\xi}}, \tag{3.7}$$

where, λ is arbitrary constant.

This function $p(\xi)$ clearly satisfies the following first order ordinary differential equation

$$\frac{dp}{d\xi} = (p^2 - p)\ln a. \tag{3.8}$$

Step 3. Now by homogeneous balancing of the highest-order derivative and the nonlinear term of highest order which occur in Eq. (3.3), we determine the values of N.

Step 4. Then use Eq. (3.6) and Eq. (3.8) into Eq. (3.3), we get an expression in p^i, where $(i = 1, 2, 3, 4, \cdots)$. Now accumulating the terms in coefficients of p^i for each i where power(i) is same and then putting these equal to zero, we get a system of equations in all constant terms. Solving these equations by using Maple, the values of all involved parameters are determined. Then putting all values into Equation (3.6) and using Eq. (3.7), we ultimately attain the required solutions.

3.3. The generalized tanh-method

The further main steps applied in the generalized tanh-method to find the solution of given a NLEE are as:

Step 2. Let

$$u(\xi) = \sum_{i=0}^{N} a_i \eta^i, \tag{3.9}$$

is supposed to be the solution of Eq. (3.3), where a_i $(i = 1, 2, 3, \cdots, N)$ are constants to be determined afterward. Here, $\eta(\xi)$ is the solution of following Ricatti equation

$$\eta' = m + \eta^2. \tag{3.10}$$

Step 3. Now by homogeneous balancing of the highest-order derivative and the nonlinear term of highest order which occur in Eq. (3.3), we determine the values of N.

Step 4. Then use Eq. (3.9) and Eq. (3.10) into Equation (3.3), we get an expression in η^i, where $(i = 1, 2, 3, 4, \cdots)$. Now accumulating the terms in coefficients of η^i for each i where power(i) is same and then putting these equal to zero, we get a system of equations in all constant terms. Solving these equations by using Maple, the values of all involved parameters are determined. Then putting all values into Equation (3.9) and considering solutions in the next step, we ultimately attain the required solutions.

Step 5. The well known solutions for Eq. (3.10) are described [50] as follows:

Case 1 For $m < 0$, solution to Eq. (3.10) are

$$\eta(\xi) = -\sqrt{-m}\tanh(\sqrt{-m}\xi), \tag{3.11}$$

and

$$\eta(\xi) = -\sqrt{-m}\coth(\sqrt{-m}\xi). \tag{3.12}$$

Case 2 For $m = 0$, solution to Eq. (3.10) is

$$\eta(\xi) = -\frac{1}{\xi}. \tag{3.13}$$

Case 3 For $m > 0$, solution to Eq. (3.10) are

$$\eta(\xi) = \sqrt{m}\tan(\sqrt{m}\xi), \tag{3.14}$$

and

$$\eta(\xi) = -\sqrt{m}\cot(\sqrt{m}\xi). \tag{3.15}$$

4. Mathematical Analysis

Using the following wave-transformation,

$$q(x,t) = h(\xi)e^{\iota\phi} \quad \text{where} \quad \xi = \alpha(x - ct), \;\; \phi(x,t) = -\kappa x + \omega t. \tag{4.1}$$

The function $h(\xi)$ is amplitude component and $\phi(x,t)$ is phase component of soliton, c is speed and α is inverse width of traveling wave, κ and ω are frequency and wave number of soliton respectively.

Using Eq. (4.1) into Eq. (2.1), we get the following ODE

$$\left(-\iota\alpha ch' - \omega h + \delta_1\alpha^2 h'' - 2\iota\delta_1\alpha\kappa h' - \delta_1\kappa^2 h + \delta_2 h^{-3} + \delta_3 h^3 + \delta_4 h^5\right) e^{\iota\phi} = 0. \tag{4.2}$$

As $e^{\iota\phi} \neq 0$, so Eq. (4.2) becomes

$$-\omega h + \delta_1\alpha^2 h'' - \delta_1\kappa^2 h + \delta_2 h^{-3} + \delta_3 h^3 + \delta_4 h^5 + \iota\left(-\alpha ch' - 2\delta_1\alpha\kappa h'\right) = 0. \tag{4.3}$$

The imaginary part of Eq. (4.3) yields

$$-\alpha ch' - 2\delta_1\alpha\kappa h' = 0 \Rightarrow c = -2\delta_1\kappa. \tag{4.4}$$

Now considering the real part of Eq. (4.3), we have

$$\delta_1\alpha^2 h'' - (\omega + \delta_1\kappa^2)h + \delta_2 h^{-3} + \delta_3 h^3 + \delta_4 h^5 = 0. \tag{4.5}$$

Balancing between highest order derivative term h'' and nonlinear term of highest order h^5 in Eq. (4.5), we have $N = \frac{1}{2}$.

So we take

$$h = u^{\frac{1}{2}}. \tag{4.6}$$

Using Eq. (4.6) into Eq. (4.5), we get

$$2\delta_1\alpha^2 u^{-\frac{1}{2}}u'' - \delta_1\alpha^2 u^{-\frac{3}{2}}(u')^2 - 4(\omega + \delta_1\kappa^2)u^{\frac{1}{2}} + 4\delta_2 u^{-\frac{3}{2}} + 4\delta_3 u^{\frac{3}{2}} + 4\delta_4 u^{\frac{5}{2}} = 0. \tag{4.7}$$

Multiplying Eq. (4.7) by $u^{\frac{3}{2}}$, we finally have the following ODE

$$2\delta_1\alpha^2 uu'' - \delta_1\alpha^2(u')^2 - 4(\omega + \delta_1\kappa^2)u^2 + 4\delta_2 + 4\delta_3 u^3 + 4\delta_4 u^4 = 0. \tag{4.8}$$

This can be written as

$$2\delta_1\alpha^2 uu'' - \delta_1\alpha^2(u')^2 - 4\delta_1(\sigma\omega + \kappa^2)u^2 + 4\delta_2 + 4\delta_3 u^3 + 4\delta_4 u^4 = 0, \tag{4.9}$$

where $\sigma = \dfrac{1}{\delta_1}$.

5. Solitons with anti-cubic nonlinearity

Now, we use prescribed methods to obtain the soliton solutions of NLSE having anti-cubic nonlinearity. We proceed according to steps mentioned for each method.

5.1. Applying $\mathrm{Exp_a}$-function method

Through balancing between the highest order nonlinear term u^4 and derivative term of the highest order uu'' in Eq. (4.9), we attain N=1.

Hence, solution form by Exp_a-function method as given in Eq. (3.4) is reduced into the following form

$$u(\xi) = \frac{a_0 + a_1 a^{\xi}}{b_0 + b_1 a^{\xi}}. \tag{5.1}$$

Here, a_0, a_1, b_0 and b_1 are constants which have to be determined.

Putting u, u' and u'' using Eq. (5.1) into Eq. (4.9), a polynomial is obtained in a^{ξ}. Then accumulating all the coefficient of $a^{i\xi}$ with same power i=0 ,1,2,3,4 and then putting these equal to zero, we get a system of algebraic equations as:

$$\begin{cases} -4\delta_1\sigma\omega b_1^2a_1^2 + 4\delta_3 b_1 a_1^3 - 4\delta_1\kappa^2 b_1^2 a_1^2 + 4\delta_4 a_1^4 + 4\delta_2 b_1^4 = 0, \\ 2\alpha^2\delta_1 \ln(a)^2 a_0 b_1^2 a_1 - 8\delta_1\kappa^2 b_1^2 a_0 a_1 - 2\alpha^2\delta_1 \ln(a)^2 a_1^2 b_0 b_1 - 8\delta_1\sigma\omega b_0 b_1 a_1^2 \\ +12\delta_3 b_1 a_0 a_1^2 - 8\delta_1\sigma\omega b_1^2 a_0 a_1 + 4\delta_3 b_0 a_1^3 + 16\delta_4 a_0 a_1^3 \\ +16\delta_2 b_0 b_1^3 - 8\delta_1\kappa^2 b_0 b_1 a_1^2 = 0, \\ -2\alpha^2\delta_1(\ln(a))^2 a_1 b_0 a_0 b_1 + 24\delta_4 a_0^2 a_1^2 - 16\delta_1\kappa^2 b_0 b_1 a_0 a_1 - 16\delta_1\sigma\omega b_0 b_1 a_0 a_1 \\ -4\delta_1\sigma\omega b_0^2 a_1^2 + \alpha^2\delta_1(\ln(a))^2 a_1^2 b_0^2 + \alpha^2\delta_1(\ln(a))^2 a_0^2 b_1^2 - 4\delta_1\kappa^2 b_0^2 a_1^2 - 4\delta_1\sigma\omega b_1^2 a_0^2 \\ +24\delta_2 b_0^2 b_1^2 + 12\delta_3 b_0 a_0 a_1^2 - 4\delta_1\kappa^2 b_1^2 a_0^2 + 12\delta_3 b_1 a_0^2 a_1 == 0, \\ 2\alpha^2\delta_1 \ln(a)^2 a_1 b_0^2 a_0 - 8\delta_1\sigma\omega b_0 b_1 a_0^2 - 2\alpha^2\delta_1 \ln(a)^2 a_0^2 b_1 b_0 - 8\delta_1\sigma\omega b_0^2 a_0 a_1 \\ -8\delta_1\kappa^2 b_0 b_1 a_0^2 + 12\delta_3 b_0 a_0^2 a_1 - 8\delta_1\kappa^2 b_0^2 a_0 a_1 + 16\delta_4 a_0^3 a_1 + 16\delta_2 b_0^3 b_1 + 4\delta_3 b_1 a_0^3 = 0, \\ 4\delta_3 b_0 a_0^3 - 4\delta_1\kappa^2 b_0^2 a_0^2 + 4\delta_2 b_0^4 + 4\delta_4 a_0^4 - 4\delta_1\sigma\omega b_0^2 a_0^2 = 0. \end{cases} \tag{5.2}$$

On solving this system of equations with help of Maple, we attain the values of involved parameters. We construct solution corresponding to these values as:

$$\begin{aligned} &\alpha = \sqrt{-\frac{1}{12\delta_4\delta_1}.\{\frac{3\delta_3 b_1 + 8\delta_4 a_1}{b_1 \ln(a)}\}}, \quad \kappa = \kappa, \\ &\omega = \frac{-48\delta_4\delta_1 b_1^2\kappa^2 + 24\delta_4 a_1\delta_3 b_1 + 32\delta_4^2 a_1^2 - 9\delta_3^2 b_1^2}{48\delta_4\delta_1\sigma b_1^2}, \\ &a_0 = \frac{-b_0(4\delta_4 a_1 + 3\delta_3 b_1)}{4\delta_4 b_1}, \quad a_1 = a_1, \quad b_0 = b_0, \quad b_1 = b_1, \\ &\delta_2 = \frac{-a_1^2(24\delta_4 a_1\delta_3 b_1 + 16\delta_4^2 a_1^2 + 9\delta_3^2 b_1^2)}{48\delta_4 b_1^4}. \end{aligned}$$

Putting these values in Eq. (5.1), we have

$$u(\xi) = \frac{-3\delta_3 b_0 b_1 - 4\delta_4 a_1(b_0 - b_1 a^{\xi}}{4\delta_4 b_1(b_0 + b_1 a^{\xi}} \tag{5.3}$$

where, $\xi = \alpha(x - ct) \Rightarrow \xi = \alpha(x + 2\delta_1\kappa t)$, using Eq. (4.4)

Now put this value of u in Eq. (4.6) and ultimately putting all these values in Eq. (4.1), we attain the exact traveling wave solution to Eq. (2.1) as follows

$$q(x,t) = \sqrt{\frac{-3\delta_3 b_0 b_1 - 4\delta_4 a_1(b_0 - b_1 a^{\alpha(x+2\delta_1\kappa t)})}{4\delta_4 b_1(b_0 + b_1 a^{\alpha(x+2\delta_1\kappa t)})}} \times e^{\iota(-\kappa x+\omega t)} \tag{5.4}$$

where, α and ω are as given above.

5.2. Applying modified Kudryashov method

Through balancing between the highest order nonlinear term u^4 and derivative term of the highest order uu'' in Eq. (4.9), we attain N=1.

Hence, solution form by modified Kudryashov method as given in Eq. (3.6) is reduced into the following form

$$u(\xi) = a_0 + a_1 p. \tag{5.5}$$

Here, a_0 and a_1 are constants which have to be determined.

Putting u, u' and u'' using Eq. (5.5) into Eq. (4.9) and also considering Eq. (3.8), a polynomial is obtained in $p(\xi)$. Then accumulating all the coefficient of p^i with same power $i = 0, 1, 2, \cdots, 4$ and then putting them equal to zero, we get a system of algebraic equations as:

$$\begin{cases} 4\delta_4 a_1^4 + 3\alpha^2\delta_1 a_1^2 \ln(a)^2 = 0, \\ 16\delta_4 a_0 a_1^3 + 4\alpha^2\delta_1 a_1 \ln(a)^2 a_0 - 4\alpha^2\delta_1 a_1^2 \ln(a)^2 + 4\delta_3 a_1^3 = 0, \\ -4\delta_1\kappa^2 a_1^2 - 6\alpha^2\delta_1 a_1 \ln(a)^2 a_0 - 4\delta_1\sigma\omega a_1^2 + 24\delta_4 a_0^2 a_1^2 + 12\delta_3 a_0 a_1^2 + \alpha^2\delta_1 a_1^2 \ln(a)^2 = 0, \\ 16\delta_4 a_0^3 a_1 - 8\delta_1\kappa^2 a_0 a_1 + 12\delta_3 a_0^2 a_1 - 8\delta_1\sigma\omega a_0 a_1 + 2\alpha^2\delta_1 a_1 \ln(a)^2 a_0 = 0, \\ -4\delta_1\sigma\omega a_0^2 + 4\delta_2 + 4\delta_3 a_0^3 + 4\delta_4 a_0^4 - 4\delta_1\kappa^2 a_0^2 = 0. \end{cases} \tag{5.6}$$

On solving this system of equations with help of Maple, we attain the values of involved parameters. We construct solution corresponding to these values.

$$\begin{aligned} &\kappa = \kappa,\ \alpha = \alpha,\ \sigma = \sigma, \\ &\omega = \frac{-16\delta_4^2 a_1^2\alpha^2 \ln(a)^2 + 27\alpha^2 \ln(a)^2\delta_3^2 - 128\delta_4^2 a_1^2\kappa^2}{128(\delta_4^2 a_1^2\sigma)}, \\ &a_0 = -\frac{4a_1\delta_4 + 3\delta_3}{8\delta_4},\ a_1 = a_1,\ \delta_1 = -\frac{4a_1^2\delta_4}{3\alpha^2 \ln(a)^2}, \\ &\delta_2 = -\frac{(4a_1\delta_4 + 3\delta_3)^2(16\delta_4^2 a_1^2 + 9\delta_3^2 - 24a_1\delta_4\delta_3)}{12288\delta_4^3}. \end{aligned}$$

Putting these values in Eq. (4.9) and also considering Eq. (3.7), we have

$$u(\xi) = -\frac{4a_1\delta_4 + 3\delta_3}{8\delta_4} + \frac{a_1}{1 + \lambda a^{\xi}}, \tag{5.7}$$

where, $\xi = \alpha(x - ct) \Rightarrow \xi = \alpha(x + 2\delta_1\kappa t)$, using Eq. (4.4).

Now put this value of u in Eq. (4.6) and ultimately putting all these values in Eq. (4.1), we attain the exact traveling wave solution to Eq. (2.1) as follows

$$q(x,t)=\sqrt{-\frac{4a_1\delta_4+3\delta_3}{8\delta_4}+\frac{a_1}{1+\lambda a^{\alpha(x+2\delta_1\kappa t)}}}\times e^{\iota(-\kappa x+\omega t)}. \tag{5.8}$$

where ω is as given above.

5.3. Applying generalized tanh-method

Adopting homogeneous balancing between the highest order nonlinear term u^4 and derivative term of the highest order uu'' in Eq. (4.9), we attain N=1.

Hence, solution form by generalized tanh-method as given in Eq. (3.9) is reduced into the following form

$$u(\xi)=a_0+a_1\eta. \tag{5.9}$$

Here, a_0 and a_1 are constants which have to be determined and $\eta(\xi)$ is such that satisfies Eq. (3.10). Putting u, u' and u'' using Eq. (5.9) into Eq. (4.9) and also considering Eq. (3.10), a polynomial is obtained in $\eta(\xi)$. Then accumulating all the coefficient of η^i with same power $i=0,1,2,\cdots,4$ and then putting them equal to zero, we get a system of algebraic equations as:

$$\begin{cases} 3\alpha^2\delta_1a_1^2+4\delta_4a_1^4=0,\\ 4\delta_3a_1^3+4\alpha^2\delta_1a_0a_1+16\delta_4a_0a_1^3=0,\\ 2\alpha^2\delta_1a_1^2m-4\delta_1\kappa^2a_1^2-4\delta_1\sigma\omega a_1^2+24\delta_4a_0^2a_1^2+12\delta_3a_0a_1^2=0,\\ -8\delta_1\sigma\omega a_0a_1-8\delta_1\kappa^2a_0a_1+12\delta_3a_0^2a_1+4\alpha^2\delta_1a_0a_1m+16\delta_4a_0^3a_1=0,\\ -\alpha^2\delta_1a_1^2m^2-4\delta_1\sigma\omega a_0^2+4\delta_4a_0^4-4\delta_1\kappa^2a_0^2+4\delta_2+4\delta_3a_0^3=0. \end{cases} \tag{5.10}$$

On solving this system of equations with help of Maple, we attain the values of involved parameters as follows. We construct solution corresponding to these values.

$$\kappa=\kappa,\ \alpha=\alpha,\ m=m,$$
$$\omega=\frac{-32\delta_1\delta_4\kappa^2-9\delta_3^2+16\alpha^2\delta_1m\delta_4}{32\delta_4},$$
$$a_0=-\frac{3\delta_3}{8\delta_4},\ a_1=\sqrt{-\frac{3\delta_1}{4\delta_4}}\ \alpha,$$
$$\delta_2=-\frac{576\delta_1^2\sigma\delta_3^2\kappa^2\delta_4+162\delta_3^4\delta_1\sigma-288\delta_3^2\alpha^2\delta_1^2m\sigma\delta_4-135\delta_3^4-576\delta_1\kappa^2\delta_3^2\delta_4+768\delta_1^2\alpha^4m^2\delta_4^2}{4096\delta_4^3}.$$

Putting these values in Eq. (5.9), we have

$$u(\xi)=-\frac{3\delta_3}{8\delta_4}+\sqrt{-\frac{3\delta_1}{4\delta_4}}\ \alpha\eta, \tag{5.11}$$

where, $\xi=\alpha(x-ct)\Rightarrow\xi=\alpha(x+2\delta_1\kappa t)$, using Eq. (4.4).

Now put this value of u in Eq. (4.6), ultimately in Eq. (4.1) and also considering solutions described in Step 5 of Section 2.11, we have following optical soliton solutions to Eq. (2.1) as

Case 1 For $m < 0$, solution to Eq. (2.1) are

$$q(x,t) = \left[-\frac{3\delta_3}{8\delta_4} - \sqrt{-\frac{3\delta_1}{4\delta_4}}\ \alpha\sqrt{-m}\tanh\{\sqrt{-m}\ \alpha(x+2\delta_1\kappa t)\} \right]^{\frac{1}{2}} \times e^{\iota(-\kappa x+\omega t)}, \tag{5.12}$$

and

$$q(x,t) = \left[-\frac{3\delta_3}{8\delta_4} - \sqrt{-\frac{3\delta_1}{4\delta_4}}\ \alpha\sqrt{-m}\coth\{\sqrt{-m}\ \alpha(x+2\delta_1\kappa t)\} \right]^{\frac{1}{2}} \times e^{\iota(-\kappa x+\omega t)}, \tag{5.13}$$

which are dark and singular soliton solutions respectively.

Case 2 For $m = 0$, solution to Eq. (2.1) is

$$q(x,t) = \left[-\frac{3\delta_3}{8\delta_4} - \sqrt{-\frac{3\delta_1}{4\delta_4}}\ \left\{ \frac{1}{x+2\delta_1\kappa t} \right\} \right]^{\frac{1}{2}} \times e^{\iota(-\kappa x+\omega t)}. \tag{5.14}$$

Case 3 For $m > 0$, solution to Eq. (2.1) are

$$q(x,t) = \left[-\frac{3\delta_3}{8\delta_4} + \sqrt{-\frac{3\delta_1}{4\delta_4}}\ \alpha\sqrt{m}\tan\{\sqrt{m}\ \alpha(x+2\delta_1\kappa t)\} \right]^{\frac{1}{2}} \times e^{\iota(-\kappa x+\omega t)}, \tag{5.15}$$

and

$$q(x,t) = \left[-\frac{3\delta_3}{8\delta_4} - \sqrt{-\frac{3\delta_1}{4\delta_4}}\ \alpha\sqrt{m}\cot\{\sqrt{m}\ \alpha(x+2\delta_1\kappa t)\} \right]^{\frac{1}{2}} \times e^{\iota(-\kappa x+\omega t)}, \tag{5.16}$$

which represent singular periodic soliton solutions.

In all solutions, ω is as given above. These solitons exist provided that

$$\delta_1\delta_4 < 0. \tag{5.17}$$

6. Results and Graphical Representation

In this study, we successfully obtained new exact traveling wave soliton solutions of the NLSE with anti-cubic nonlinearity. These solutions are in the exponential functions form and the hyperbolic functions form. For the physical interpretation, three-dimensional (3D) and two-dimensional (2D) graphs of the solutions of the NLSE having AC nonlinearity are represented in Fig. 1 to Fig. 7. The solution acquired by Exp_a-function method as given in Eq. (5.4) is bright soliton, as plotted in Fig. 1 for arbitrary values of the involved parameters as

$$a_1 = 1,\ b_0 = 12,\ b_1 = -3,\ \delta_1 = 4,\ \delta_3 = 2,\ \delta_4 = -1,\ k = \frac{1}{2},\ a = 3.2.$$

The solution deduced by modified Kudryashov method, as given in Eq. (5.8) is kink soliton, drawn in Fig. 2 for arbitrary values of the involved parameters as

$$a_1 = 0.5,\ \alpha = 2,\ \lambda = 1,\ \delta_3 = 1,\ \delta_4 = -1,\ k = 1,\ a = 3.5.$$

The graphs of all solutions acquired by generalized tanh-method, are plotted, by giving specific values to the involved parameters as follows;

$$\alpha = -2,\ \kappa = 2,\ \delta_1 = 1,\ \delta_3 = 4,\ \delta_4 = -3.$$

Fig. 3 depicts the graphs of the solution given by Eq. (5.12), which is kinked dark soliton, drawn for m=-1. Fig. 4 shows the graphs of the solution given by Eq. (5.13), which is singular soliton, for m=-1. Fig. 5 demonstrates the graph of solution given in Eq. (5.14), which is singular kink-shaped soliton, for m=0. Fig. 6 and Fig. 7 represent the graphs of solutions given in Eq. (5.15) and Eq. (5.16), which are singular periodic solitons, for m=1.

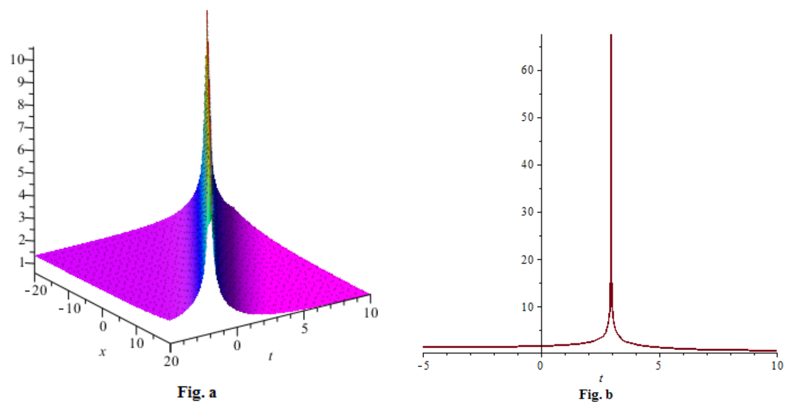

Figure 1. (a) 3-D plot of solution (5.4) for $-20 \leq x \leq 20$ and $-5 \leq t \leq 10$ (b) 2-D plot of solution (5.4) for $x = 0$ and $-5 \leq t \leq 10$

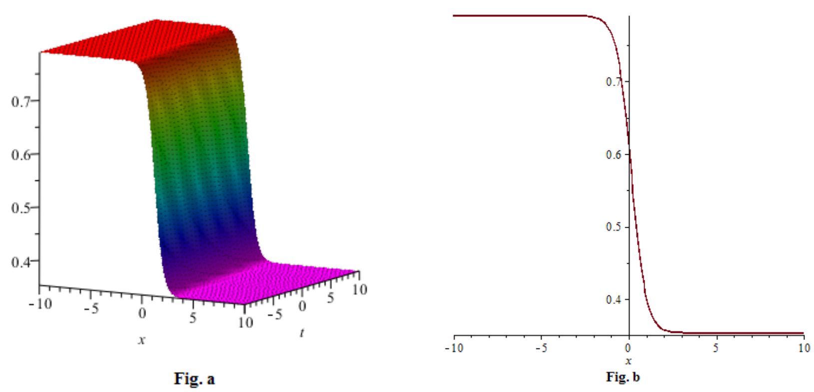

Figure 2. (a) 3-D plot of solution (5.8) for $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$(b) 2-D plot of solution (5.8) for $-10 \leq x \leq 10$ and $t = 0$

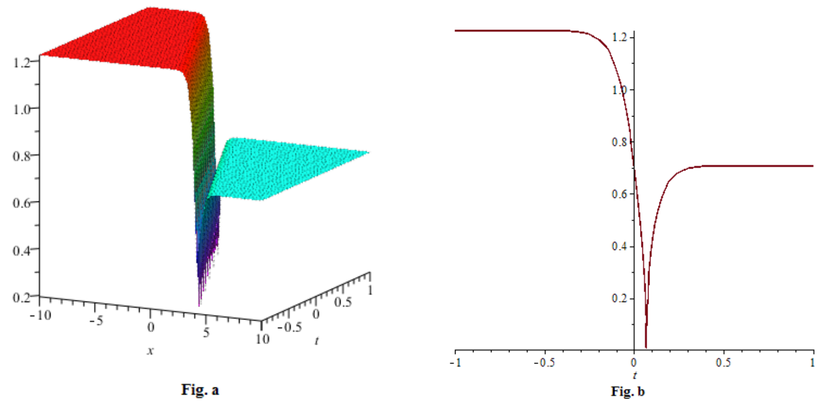

Figure 3. (a) 3-D plot of solution (5.12) for $-10 \leq x \leq 10$ and $-1 \leq t \leq 1$ (b) 2-D plot of solution (5.12) for $x = 0$ and $-1 \leq t \leq 1$

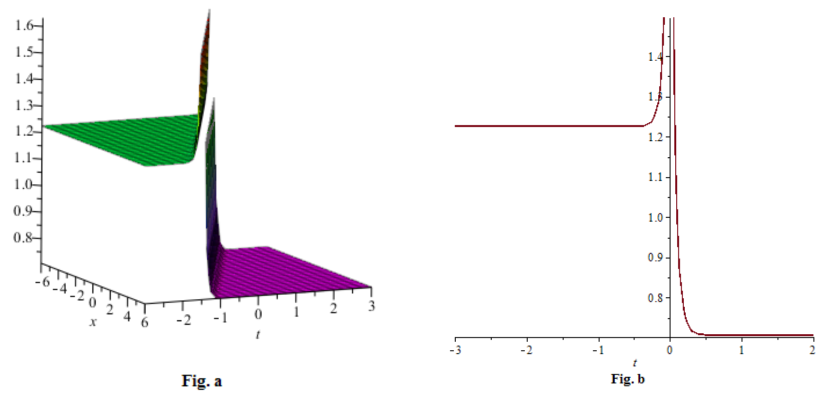

Figure 4. (a) 3-D plot of solution (5.13) for $-6 \leq x \leq 6$ and $-3 \leq t \leq 3$ (b) 2-D plot of solution (5.13) for $x = 0$ and $-3 \leq t \leq 3$

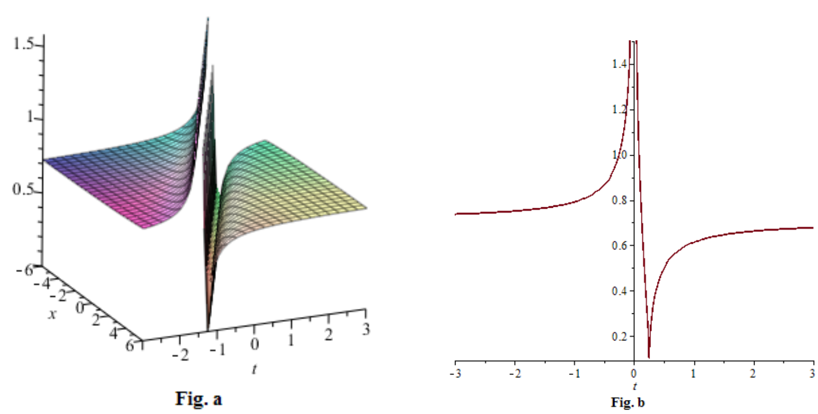

Figure 5. (a) 3D plot of solution (5.14) for $-6 \leq x \leq 6$ and $-3 \leq t \leq 3$ (b) 2D plot of solution (5.14) for $x = 0$ and $-3 \leq t \leq 3$

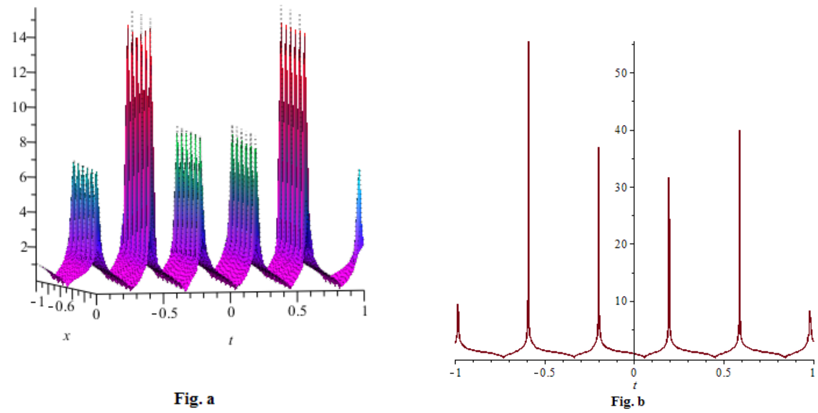

Figure 6. (a) 3D plot of solution (5.15) for $-1 \le x \le 0$ and $-1 \le t \le 1$ (b) 2D plot of solution (5.15) for $x = 0$ and $-1 \le t \le 1$

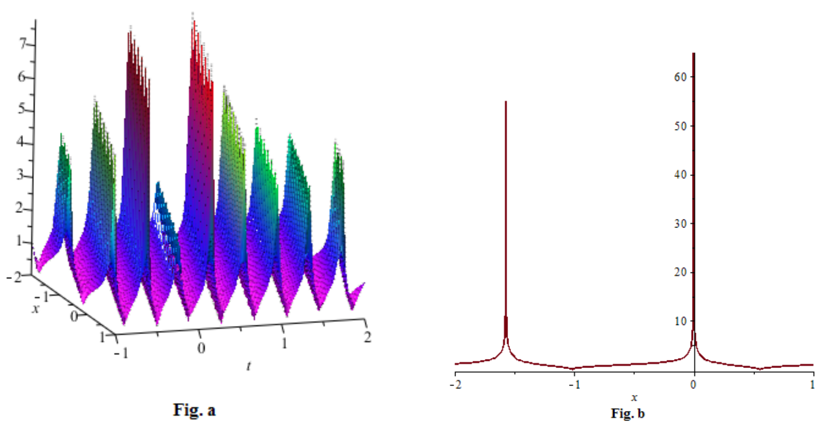

Figure 7. (a) 3D plot of solution (5.16) for $-2 \le x \le 1$ and $-1 \le t \le 2$ (b) 2D plot of solution (5.16) for $-2 \le x \le 1$ and $t = 0$

Remark 6.1. From this graphical representation, we suggest that these solutions for NLSE having anti-cubic nonlinearity would be very helpful in demonstration and analysis of all physical phenomena described by this equation.

Remark 6.2. We affirm that these reported solutions for NLSE having anti-cubic nonlinearity using $\mathrm{Exp_a}$-function method, the modified Kudryashov method and the generalized tanh-method are new and have not been stated earlier.

7. Conclusion

In this article, we established new optical soliton solutions for NLSE having anti-cubic nonlinearity with the help of $\mathrm{Exp_a}$-function method, the modified Kudryashov method and the generalized tanh-method. Maple is used for solving all the equations involved in this paper. All the solutions are also checked by putting them into the

PDE and proved to be correct. The results show that the reported approaches are very useful, efficient and accurate tools to get the soliton solutions of PDEs. The obtained solutions may be very helpful for demonstrating the certain nonlinear physical phenomena. All the graphs were plotted with help of Maple. Our graphical representation of acquired solutions shows that these solutions are well illustrative in describing the related physical phenomena. The solitons solutions obtained in this way would be very useful to investigate the signals through optical-fibers, theory of plasma physics and waves in electromagnetic field.

Conflict of interest

There is no conflict of interest among the authors.

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