Journal of Applied Analysis and Computation
Volume 11, Number 5, October 2021, 2279–2302

Website:http://www.jaac-online.com
DOI:10.11948/20200370

ANALYSIS OF A MULTI-GROUP ALCOHOLISM MODEL WITH PUBLIC HEALTH EDUCATION UNDER REGIME SWITCHING

Zhenfeng Shi[1], Daqing Jiang[2,3,†], Ningzhong Shi[1], Tasawar Hayat[2,4] and Ahmed Alsaedi[2]

Abstract In this paper, we study a multi-group stochastic alcoholism model with public health education, which is formulated as a piecewise deterministic Markov process. Through a rigorous analysis, we firstly show that the solution of the stochastic model is positive and global. Then we obtain sufficient conditions for the extinction of alcohol problems. In addition, sufficient conditions for the persistence in the mean of alcoholism are derived. Specifically, in the case of persistence, we prove the existence of positive recurrence of the solution to the model by employing suitable stochastic Lyapunov functions.

Keywords Multi-group model, alcoholism, markovian switching, extinction, persistence in the mean, positive recurrence.

MSC(2010) 34F05, 37Hxx, 60Jxx, 92Bxx.

1. Introduction

Alcoholism has become one of the leading risk factors for population health worldwide. Excessive drinking not only leads to increased risk of health problems such as liver diseases and cancer, but also causes road injuries and violence. In 2016, the harmful use of alcohol resulted in some 3 million deaths worldwide and 132.6 million disability [43].

Mathematical modeling has proved especially useful in understanding complex social dynamics, notably those involving interactions between micro and macro level processes and the development of emergent behaviors [8]. Complex systems modelling tactics have the potential to combine our developing knowledge about multilevel determinants of population health, patterns of feedback and interaction between determinants at unique levels, and to inform our knowledge about how

[†]The corresponding author. Email: daqingjiang2010@hotmail.com(D. Jiang)
[1]School of Mathematics and Statistics, Key Laboratory of Applied Statistics of MOE, Northeast Normal University, Changchun 130024, Jilin Province, China
[2]Nonlinear Analysis and Applied Mathematics (NAAM)-Research Group, King Abdulaziz University, Jeddah 121589, Saudi Arabia
[3]College of Science, China University of Petroleum, Qingdao 266580, Shandong Province, China
[4]Department of Mathematics, Quaid-I-Azam University 45320, Islamabad 44000, Pakistan

unique policy interventions affect the pathways that shape the health of populations. Therefore the risky health behavior of drinking can be viewed as a treatable contagious disease [9]. Unlike the infectious disease model, the alcoholism model is difficult to capture the interactions of individuals whose behavior is in many instances highly subjective and not entirely rational, and therefore very difficult to quantify and calibrate [8]. However, mathematicians have formulated and studied the dynamics of several alcoholism models [1–3, 28–30]. A model for the spread of alcoholic drinking is formulated in [1] which regard alcoholism as a contagious disease. Bani et al. [2] studied the role of environmental factors on the long term dynamics of an alcohol drinking population and found that heavy drinking can be reduced if the drinking reproduction number $R_d < 1$. Mulone and Straughan [29] developed a two-stage model for youths with serious drinking problems and their treatment and they established a threshold such that two steady states will be stable.

In usual epidemics models, researchers assumed that each individual has the same probability to be infected. However, considering the heterogeneity (e.g. sex, age, space, etc.) of host population, more and more attention has been paid to multi-group models [7, 16, 18, 21, 35, 36, 44]. For instance, Li et al. [18] investigated a class of multi-group SEIR models with distributed delays and indicated that the endemic equilibrium is globally asymptotically stable if the basic reproduction number $\mathcal{R}_0 > 1$. Kuniya [16] studied that the global behavior of a multi-group SIR epidemic model with age structure is completely determined by the basic reproduction number.

For alcoholism, women drink less alcohol and have fewer alcohol-related problems than men [33] and underage drinkers consume more drinks per drinking occasion than adult drinkers [4]. Hence multi-group alcoholism epidemic model is worthy of being studied, Ma et al. [31] have formulated the following multi-group SEA alcoholism model with public health education

$$\begin{cases} \dfrac{\mathrm{d}S_i}{\mathrm{d}t} = (1-q_i)\Lambda_i - \displaystyle\sum_{j=1}^{n} \beta_{ij} S_i A_j - (\mu_i^S + p_i) S_i, \\ \dfrac{\mathrm{d}E_i}{\mathrm{d}t} = p_i S_i - \displaystyle\sum_{j=1}^{n} \sigma_i \beta_{ij} E_i A_j - (\mu_i^E + \varepsilon_i) E_i, \\ \dfrac{\mathrm{d}A_i}{\mathrm{d}t} = (S_i + \sigma_i E_i) \displaystyle\sum_{j=1}^{n} \beta_{ij} A_j - (\mu_i^A + a_i + \gamma_i) A_i, \quad i = 1, 2, \ldots, n. \end{cases} \tag{1.1}$$

In this model, the heterogeneous host population is divided into n homogeneous groups. For ith $(1 \le i \le n)$ group, the susceptible drinkers $S_i(t)$ who do not drink or drink only moderately and do not accept the public health education, but may one day develop problems with alcohol; the educated susceptible drinkers $E_i(t)$ who do not drink or consume alcohol in moderation and have accepted the public health education, but may one day also develop problems with alcohol; the alcoholics $A_i(t)$ who have drinking problems or addictions. All parameters of model (1.1) are defined in Table 1. Since alcohol abuse is harmful to human health, in this paper, we assume that $\mu_i^S < \mu_i^A + a_i$. Denote $\mathbb{R}_+^d = \{x = (x_1, x_2, \ldots x_d) \in \mathbb{R}^d : x_i > 0, 1 \le i \le d\}$. Define $\tilde{\mu}_i = \min\{\mu_i^S, \mu_i^E + \varepsilon_i\} < \mu_i^A + a_i + \gamma_i$ and $\bar{\mu}_i = \max\{\mu_i^E + \varepsilon_i, \mu_i^A + a_i + \gamma_i\} > \mu_i^S$.

Table 1. Description of the variables and associated parameters

Symbol	Description
Λ_i	Influx of individuals
$1-q_i \in (0,1)$	Fraction at which influx of individuals entering into the susceptible compartment S_i
β_{ij}	Rate of becoming alcoholics through contacts between susceptible individuals (S_i or E_i) and heavy drinkers (A_j)
p_i	the rate of susceptible S_i who accepted the public health education entering into the educated E_i class
$\sigma_i \in (0,1)$	Multiplier to the force of infection to susceptible individuals by public health education
μ_i^S	Natural death rate of susceptible individuals
μ_i^E	Natural death rate of educated individuals
μ_i^A	Natural death rate of drinking individuals
ε_i	Fraction of educated individuals E_i who do not drink or who quit drink due to
	the effect of the public health educational campaigns
a_i	Additional death rate of alcoholics due to excessive drinking
γ_i	Recovery rate of heavy drinkers

In [31], authors presented the positively invariant set of model (1.1) as follows

$$\begin{aligned}\Gamma =&\Big\{(S_1,E_1,A_1,\dots,S_n,E_n,A_n)\in\mathbb{R}_+^{3n} : 0\le S_i\le \frac{(1-q_i)\Lambda_i}{\mu_i^S+p_i},\\ &0\le S_i+E_i+A_i\le \frac{(1-q_i)\Lambda_i}{\tilde{\mu}_i}, 1\le i\le n\Big\}.\end{aligned}$$

System (1.1) has two equilibria: the alcohol-free equilibrium $P^0=(S_1^0,E_1^0,0,\dots,S_n^0,E_n^0,0)$, where

$$S_i^0=\frac{(1-q_i)\Lambda_i}{\mu_i^S+p_i},\quad E_i^0=\frac{p_iS_i^0}{\mu_i^E+\varepsilon_i}=\frac{p_i(1-q_i)\Lambda_i}{(\mu_i^S+p_i)(\mu_i^E+\varepsilon_i)},\quad i=1,2,\dots,n,$$

and the alcohol-present equilibrium $P^*=(S_1^*,E_1^*,A_1^*,\dots,S_n^*,E_n^*,A_n^*)$. Let basic reproduction number $R_0=\rho(M_0)$, where

$$M_0=M(S_1^0,E_1^0,\dots,S_n^0,E_n^0)=\Big(\frac{\beta_{ij}(S_i^0+\sigma_iE_i^0)}{\mu_i^A+a_i+\gamma_i}\Big)_{n\times n},$$

and $\rho(M_0)$ denotes the spectral radius of the matrix.

If $R_0<1$, the alcohol-free equilibrium P^0 is unique equilibrium of system (1.1), and it is globally asymptotically stable (GAS) in Γ. If $R_0>1$, the alcohol-free equilibrium P^0 is unstable, and there exists a alcohol-present equilibrium P^* which is GAS in Γ.

In system (1.1), authors assumed that host individuals live in a deterministic environment. However ecosystem dynamics are inevitably affected by random fluctuations in the real world, many studies confirm that, it is more realistic to include the effect of stochasticity rather than to study a entirely deterministic model [17, 19, 20, 32, 34, 37, 38, 40, 45]. For instance, Lv et al. [20] investigated a stochastic predator-prey model with a functional response, and obtained the existence of stationary distribution to the model. An impulsive stochastic chemostat model with saturated growth rate in a polluted environment is proposed in [32], and authors obtained conditions for the extinction and the permanence of their model. Wang et al. [41] studied the asymptotic behaviors of a stochastic social epidemic model with multi-perturbation, and they investigated the long-term stochastic dynamics behaviors of random disturbance on the stability of the alcohol abuse model [42]. Recent study indicated that colder weather and fewer sunshine hours are possible causal agents for higher alcohol consumption worldwide [39]. Therefore, climate exerts a strong influence on the alcoholism transmission coefficient β_{ij} $(i, j = 1, 2, \ldots, n)$ of system (1.1). For example, when the weather is cold, people are more willing to drink high-quality alcohol, which leads to alcoholism, but not when the weather is cold. In other words, due to changes in climate and temperature, the coefficient β_{ij} may switch from one environmental state to another. The usual modeling cannot model environmental changes as solutions to differential equations. They are random discrete events that occur at random epochs [6]. It is an emerging approach to use a continuous-time Markov chain taking values in a finite state space to model the random switching of environmental regimes. The continuous-time Markov chain produces the changes of the main parameters of epidemic models with state switchings of the Markov chain. Hence it is interesting and meaningful to study the effect of the random switching of environmental regimes on the spread of alcoholism.

Although environmental noise plays an important role in epidemic models, researchers have paid little attention in this area. Gary et al. [10] firstly studied a piecewise deterministic SIS epidemic model with Markovian switching. D. Li, S. Liu and J. Cui [22,23] studied the dynamics and ergodicity of Markovian switching and semi-Markov switching on the deterministic SIRS epidemic models, respectively. For multi-group epidemic models, a multigroup SIS epidemic model with standard incidence rates and Markovian switching was investigated by Liu et al. [24], they establish sufficient conditions for extinction and persistence in the mean of the diseases. In addition, they obtained sufficient conditions for the existence of positive recurrence of the solutions to the model. However, the above multigroup SIS model is $2n$-dimensional, it is more realistic and meaningful to study higher dimensional models.

To the best of our knowledge, there are few investigations on the dynamics of the multi-group alcoholism model, which is formulated as a piecewise deterministic Markov process. Motivated by the referred works, in this paper, we will study the dynamics of the multi-group SEA alcoholism with public health education and Markovian switching. In this model, the alcoholism transmission coefficient β_{ij} is obtained by a homogeneous continuous-time Markov chain $\{r(t), t > 0\}$ and the other parameters are the same as in system (1.1). Then, the stochastic model can be described as the following system

$$\begin{cases} \dfrac{\mathrm{d}S_i}{\mathrm{d}t} = (1-q_i)\Lambda_i - \displaystyle\sum_{j=1}^{n} \beta_{ij}(r(t)) S_i A_j - (\mu_i^S + p_i) S_i, \\ \dfrac{\mathrm{d}E_i}{\mathrm{d}t} = p_i S_i - \displaystyle\sum_{j=1}^{n} \sigma_i \beta_{ij}(r(t)) E_i A_j - (\mu_i^E + \varepsilon_i) E_i, \\ \dfrac{\mathrm{d}A_i}{\mathrm{d}t} = (S_i + \sigma_i E_i) \displaystyle\sum_{j=1}^{n} \beta_{ij}(r(t)) A_j - (\mu_i^A + a_i + \gamma_i) A_i, \quad i = 1, 2, \ldots, n. \end{cases} \tag{1.2}$$

Let $r(t)$ be a homogeneous continuous-time Markov chain, taking values in a finite state space $\mathbb{S} = \{1, 2, \ldots, N\}$, with the generator $\Gamma^* = (\gamma_{ij})_{N\times N}$ given by [27]

$$P\{r(t+\Delta t) = j | r(t) = i\} = \begin{cases} \gamma_{ij}\Delta t + o(\Delta t), & \text{if } i \neq j, \\ 1 + \gamma_{ii}\Delta t + o(\Delta t), & \text{if } i = j, \end{cases}$$

where $\Delta t > 0$ denotes a small time increment, $\gamma_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$, while $\sum_{j=1}^{N} \gamma_{ij} = 0$. For each $l \in \mathbb{S}$, $\beta_{ij}(l)$ is positive constant.

The organization of the rest of this paper is as follows. In Section 2, we prove that there exists a unique global positive solution of system (1.2) with any positive initial value. Sufficient conditions for extinction of the diseases are established in Section 3. In Section 4, we obtain sufficient conditions for the diseases being persistent in the mean and prove the existence of positive recurrence of the solutions to system (1.2) by constructing appropriate stochastic Lyapunov functions. Section 5 shows numerical simulations to illustrate how the Markovian switching affect the behavior of the stochastic systems. Finally, we conclude our results to end this paper.

2. Existence and uniqueness of the global positive solution and preliminaries

Throughout this paper, unless otherwise specified, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). For any vector $g = (g(1), g(2), \ldots, g(N))$, define $\hat{g} = \min_{k\in\mathbb{S}} g(k)$, $\check{g} = \max_{k\in\mathbb{S}} g(k)$. If M is a matrix, its transpose is denoted by M^T and $\rho(M)$ denotes the spectral radius of M.

Assume further that the Markov chain $r(t)$ is irreducible [25] such that the system can switch from any regime to any other one, which implies the ergodicity property according to Markov theory for finite states. Since Γ^* always has a trivial eigenvalue. The algebraic interpretation of irreducibility denotes that the rank of Γ^* is $N-1$. On the basis of these conditions, the Markov chain $r(t)$ has a unique stationary distribution $\pi = (\pi_1, \pi_2, \ldots, \pi_N)$ which can be determined by equation

$$\pi\Gamma^* = 0, \tag{2.1}$$

subject to $\sum_{h=1}^{N} \pi_h = 1$, and $\pi_h > 0$, for any $h \in \mathbb{S}$.

Firstly, we give a theorem to show the existence and uniqueness of the global positive solution of the stochastic model (1.2).

Theorem 2.1. *For any initial value $(S_1(0), E_1(0), A_1(0), \ldots, S_n(0), E_n(0), A_n(0), r(0)) \in \mathbb{R}^{3n}_+ \times \mathbb{S}$, there is a unique solution $(S_1(t), E_1(t), A_1(t), \ldots, S_n(t), E_n(t), A_n(t), r(t))$ of system (1.2) on $t \geq 0$ and the solution will remain in $\mathbb{R}^{3n}_+ \times \mathbb{S}$ with probability one, which means $(S_1(t), E_1(t), A_1(t), \ldots, S_n(t), E_n(t), A_n(t)) \in \mathbb{R}^{3n}_+ \times \mathbb{S}$ for all $t \geq 0$ almost surely (a.s.).*

Proof. We omit the beginning of proof, which is similar to [15]. We only show the essential stochastic Lyapunov function here.

In view of model (1.2), for $t \leq \tau_w$, we have

$$\begin{aligned}(S_i + E_i + A_i)' =& (1-q_i)\Lambda_i - \mu_i^S S_i - (\mu_i^E + \varepsilon_i)E_i - (\mu_i^A + a_i + \gamma_i)A_i \\ \leq& (1-q_i)\Lambda_i - \tilde{\mu}_i(S_i + E_i + A_i),\end{aligned}$$

thus $S_i(t) + E_i(t) + A_i(t) \le P_i$, where

$$P_i = \begin{cases} \dfrac{(1-q_i)\Lambda_i}{\tilde{\mu}_i}, & \text{if } S_i(0) + E_i(0) + A_i(0) \le \dfrac{(1-q_i)\Lambda_i}{\tilde{\mu}_i}, \\ S_i(0) + E_i(0) + A_i(0), & \text{if } S_i(0) + E_i(0) + A_i(0) > \dfrac{(1-q_i)\Lambda_i}{\tilde{\mu}_i}. \end{cases}$$

On the other hand, for $t \le \tau_w$, one gets

$$\begin{aligned}(S_i + E_i + A_i)' =& (1-q_i)\Lambda_i - \mu_i^S S_i - (\mu_i^E + \varepsilon_i)E_i - (\mu_i^A + a_i + \gamma_i)A_i \\ \ge& (1-q_i)\Lambda_i - \bar{\mu}_i(S_i + E_i + A_i),\end{aligned}$$

therefore, $S_i(t) + E_i(t) + A_i(t) \ge Q_i$, where

$$Q_i = \begin{cases} S_i(0) + E_i(0) + A_i(0), & \text{if } S_i(0) + E_i(0) + A_i(0) \le \dfrac{(1-q_i)\Lambda_i}{\bar{\mu}_i}, \\ \dfrac{(1-q_i)\Lambda_i}{\bar{\mu}_i}, & \text{if } S_i(0) + E_i(0) + A_i(0) > \dfrac{(1-q_i)\Lambda_i}{\bar{\mu}_i}. \end{cases}$$

Define a C^2-function V_1 on $\mathbb{R}_+^{3n}$ to $\mathbb{R}_+ \bigcup\{0\}$ as follows

$$\begin{aligned}& V_1(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \\ =& \sum_{i=1}^{n}[(S_i - 1 - \log S_i) + (E_i - 1 - \log E_i) + (A_i - 1 - \log A_i)].\end{aligned}$$

The nonnegativity of V_1 can be seen from $x - 1 - \log x >$ for any $x > 0$.

Applying Itô's formula [26] to V_1, we have

$$\mathrm{d}V_1(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) = \mathcal{L}V_1(S_1, E_1, A_1, \ldots, S_n, E_n, A_n)\mathrm{d}t,$$

where $\mathcal{L}V_1 : \mathbb{R}_+^{3n} \to \mathbb{R}$ is calculated by

$$\begin{aligned}\mathcal{L}V_1 =& \sum_{i=1}^{n}\Big[\Big(1 - \frac{1}{S_i}\Big)\Big((1-q_i)\Lambda_i - \sum_{j=1}^{n}\beta_{ij}(k)S_iA_j - (\mu_i^S + p_i)S_i\Big)\Big] \\ &+ \sum_{i=1}^{n}\Big[\Big(1 - \frac{1}{E_i}\Big)\Big(p_iS_i - \sum_{j=1}^{n}\sigma_i\beta_{ij}(k)E_iA_j - (\mu_i^E + \varepsilon_i)E_i\Big)\Big] \\ &+ \sum_{i=1}^{n}\Big[\Big(1 - \frac{1}{A_i}\Big)\Big((S_i + \sigma_iE_i)\sum_{j=1}^{n}\beta_{ij}(k)A_j - (\mu_i^A + a_i + \gamma_i)A_i\Big)\Big] \\ \le& \sum_{i=1}^{n}\Big[(1-q_i)\Lambda_i - \mu_i^S S_i - (\mu_i^E + \varepsilon_i)E_i - (\mu_i^A + a_i + \gamma_i)A_i\Big] \\ &+ \sum_{i=1}^{n}\Big[\sum_{j=1}^{n}\check{\beta}_{ij}A_j + \mu_i^S + p_i + \sum_{j=1}^{n}\sigma_i\check{\beta}_{ij}A_j + \mu_i^E + \varepsilon_i + \mu_i^A + a_i + \gamma_i\Big] \\ \le& \sum_{i=1}^{n}\Big[(1-q_i)\Lambda_i + \mu_i^S + p_i + \mu_i^E + \varepsilon_i + \mu_i^A + a_i + \gamma_i + (1+\sigma_i)\sum_{j=1}^{n}\check{\beta}_{ij}P_j\Big] \\ :=& M_1,\end{aligned}$$

where M_1 is a positive constant. The remainder of the proof is similar to Ji et al. [15], we omit it here. This completes the proof. □

Remark 2.1. Theorem 2.1 demonstrates that for any initial value $(S_1(0), E_1(0), A_1(0), \dots, S_n(0), E_n(0), A_n(0)) \in \mathbb{R}_+^{3n} \times \mathbb{S}$, there exists a unique global solution $(S_1(t), E_1(t), A_1(t), \dots, S_n(t), E_n(t), A_n(t)) \in \mathbb{R}_+^{3n} \times \mathbb{S}$ *a.s.* of system (1.2).

Since

$$S_i' = (1-q_i)\Lambda_i - \sum_{j=1}^{n} \beta_{ij}(r(t))S_iA_j - (\mu_i^S + p_i)S_i \le (1-q_i)\Lambda_i - (\mu_i^S + p_i)S_i,$$

and

$$S_i(t) \le S_i^0 + e^{-(\mu_i^S + p_i)t}(S_i(0) - S_i^0),$$

If $0 < S_i(0) < S_i^0$, then $0 < S_i(t) < S_i^0$ a.s..

In the case of $0 < S_i(0) < S_i^0$, we have

$$E_i' = p_iS_i - \sum_{j=1}^{n} \sigma_i\beta_{ij}(r(t))E_iA_j - (\mu_i^E + \varepsilon_i)E_i \le p_iS_i^0 - (\mu_i^E + \varepsilon_i)E_i,$$

thus if $0 < E_i(0) < E_i^0$, then $0 < E_i(t) < E_i^0$ a.s..

Moreover,

$$(1-q_i)\Lambda_i - \bar{\mu}_i(S_i + E_i + A_i) \le (S_i + E_i + A_i)' \le (1-q_i)\Lambda_i - \tilde{\mu}_i(S_i + E_i + A_i).$$

Hence if $\frac{(1-q_i)\Lambda_i}{\bar{\mu}_i} < S_i(0) + E_i(0) + A_i(0) < \frac{(1-q_i)\Lambda_i}{\tilde{\mu}_i}$, then $\frac{(1-q_i)\Lambda_i}{\bar{\mu}_i} < S_i(t) + E_i(t) + A_i(t) < \frac{(1-q_i)\Lambda_i}{\tilde{\mu}_i}$ a.s..

The region

$$\begin{aligned}\bar{\Gamma} = \Big\{ &(S_1, E_1, A_1, \dots, S_n, E_n, A_n) \in \mathbb{R}_+^{3n} : S_i < S_i^0, E_i < E_i^0, \\ &\frac{(1-q_i)\Lambda_i}{\bar{\mu}_i} < S_i + E_i + A_i < \frac{(1-q_i)\Lambda_i}{\tilde{\mu}_i}, i = 1, 2, \dots, n \Big\}\end{aligned}$$

is a positively invariant set of system (1.2).

From now on, we always assume that the initial value $(S_1(0), E_1(0), A_1(0), \dots, S_n(0), E_n(0), A_n(0), r(0)) \in \bar{\Gamma} \times \mathbb{S}$.

Then we present the graph-theoretical approach of Guo et al. [11–13].

Given a nonnegative matrix $A = (a_{ij})_{n\times n}$, the directed graph $\mathcal{G}(A)$ associated with A has vertices $1, 2, \dots, n$ with a directed arc (i, j) leading from initial vertex k to terminal vertex j if and only if $a_{ij} \neq 0$. A digraph $\mathcal{G}(A)$ is then said to be strongly connected if any two distinct vertices can be joined by an oriented path. A weighted digraph $\mathcal{G}(A)$ is strongly connected if and only if the matrix A is irreducible [5].

Lemma 2.1 ([5]). *If matrix A is nonnegative and irreducible, then the spectral radius $\rho(A)$ of A is a simple eigenvalue, and A has a positive eigenvector $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ corresponding to $\rho(A)$.*

Then we present the definitions of the persistence in the mean and the positive recurrence.

Definition 2.1. System (1.2) is said to be persistent in the mean if for any initial value $(S_1(0), E_1(0), A_1(0), \dots, S_n(0), E_n(0), A_n(0), r(0)) \in \bar{\Gamma} \times \mathbb{S}$, the solution $(S_1(t), E_1(t), A_1(t), \dots, S_n(t), E_n(t), A_n(t), r(t))$ of system (1.2) has the following property

$$\liminf_{t\to\infty} \frac{1}{t}\int_0^t A_i(s) > 0 \quad a.s., \quad i = 1, 2, \dots, n.$$

Definition 2.2 ([27]). The process X_t^x with $X_0 = x$ is recurrent with respect to D_ϵ, if for any $x \notin D_\epsilon$, $\mathbb{P}(\tau_{D_\epsilon} < \infty) = 1$, where τ_{D_ϵ} is the hitting time of D_ϵ for the process X_t^x, that is

$$\tau_{D_\epsilon} = \inf\{t > 0, X_t^x \in D_\epsilon\}.$$

The process X_t^x is said to be positive recurrent with respect to D_ϵ if $\mathbb{E}(\tau_{D_\epsilon}) < \infty$ for any $x \notin D_\epsilon$.

3. Extinction

For the dynamical behavior of the stochastic alcoholism model, we are firstly interested in the state that the alcohol problems die out in a long term. In this section, we shall establish sufficient conditions for extinction of the alcohol problems for model (1.2).

Theorem 3.1. *Assume that* $B_1 = (\check{\beta}_{ij})_{n\times n}$ *is irreducible. If* $R_0^e = \rho(M_0^e) < 1$, *then there exists a positive solution* $(S_1(t), E_1(t), A_1(t), \ldots, S_n(t), E_n(t), A_n(t), r(t))$ *of system (1.2) which has the following property*

$$\limsup_{t\to\infty} \frac{1}{t} \log\Big(\sum_{i=1}^{n} \frac{\bar{\omega}_i A_i}{\mu_i^A + a_i + \gamma_i}\Big) < 0, \quad a.s.,$$

which means the alcohol problems die out exponentially with probability one, i.e.,

$$\lim_{t\to\infty} A_i(t) = 0 \quad a.s., \quad i = 1, 2, \ldots, n,$$

where $\rho(M_0^e)$ *denotes the spectral radius of* $M_0^e = \left(\frac{\check{\beta}_{ij}(S_i^0+\sigma_i E_i^0)}{\mu_i^A+a_i+\gamma_i}\right)_{n\times n}$ *and* $\bar{\omega} = (\bar{\omega}_1, \bar{\omega}_2, \ldots, \bar{\omega}_n)$ *is a left positive eigenvector of* M_0^e *corresponding to* $\rho(M_0^e)$.

4. Persistence and positive recurrence

In this section, we will study the persistence of model (1.2), i.e. the alcohol problem will persist in all groups.

Theorem 4.1. *Assume that* $B_2 = (\hat{\beta}_{ij})_{n\times n}$ $(i, j = 1, 2, \ldots, n)$ *is irreducible. If* $R_0^p = \rho(M_0^p) > 1$, *then there exists a positive solution* $(S_1(t), E_1(t), A_1(t), \ldots, S_n(t), E_n(t), A_n(t), r(t))$ *of system (1.2) which has the following property*

$$\liminf_{t\to\infty} \int_0^t A_p(s)\mathrm{d}s \geq \frac{1}{D_p} \min_{1\leq i\leq n}\{\mu_i^A + a_i + \gamma_i\}(R_0^p - 1) > 0, \quad a.s., \quad p = 1, \ldots, n,$$

where $\rho(M_0^p)$ *denotes the spectral radius of* $M_0^p = \left(\frac{\hat{\beta}_{ij}(S_i^0+\sigma_i E_i^0)}{\mu_i^A+a_i+\gamma_i}\right)_{n\times n}$, $\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)$ *is a left positive eigenvector of* M_0^p *corresponding to* $\rho(M_0^p)$ *and for* $p = 1, 2, \ldots, n$,

$$D_p = \frac{\max_{1\leq i,j\leq n}\{\check{\beta}_{ij}\}^2 n(S_i^0 + \sigma_i E_i^0)}{\min\{\mu_i^S, \mu_i^E + \varepsilon_i\}}\Big[1 + \frac{\sigma_p \sum_{j=2}^n \hat{\beta}_{pj} \frac{(1-q_j)\Lambda_j}{\bar{\mu}_j} + \mu_p^A + a_p + \gamma_p}{\frac{\sigma_p \Lambda_p (1-q_p) \min_{2\leq j\leq n}\{\hat{\beta}_{pj}\}}{\bar{\mu}_p}}\Big].$$

In the case of persistence, we will find a domain $D_\epsilon \subset \bar{\Gamma}$ which is positive recurrence for the process $(S_1(t), E_1(t), A_1(t), \ldots, S_n(t), E_n(t), A_n(t), r(t))$.

Theorem 4.2. *Assume that* $B_2 = (\hat{\beta}_{ij})_{n\times n}$ $(i,j = 1,2,\dots,n)$ *is irreducible. If* $R_0^p = \rho(M_0^p) > 1$, *then there exists a positive solution* $(S_1(t), E_1(t), A_1(t), \dots, S_n(t), E_n(t), A_n(t), r(t))$ *of system (1.2) which is positive recurrence with respect to the domain* D_ϵ, *where* R_0^p *is given in Theorem 4.1,*

$$
\begin{aligned}
D_\epsilon = \Big\{ & (S_1, E_1, A_1, \dots, S_n, E_n, A_n) \in \bar{\Gamma} : \epsilon \le S_1 \le S_1^0 - \epsilon^2, \epsilon \le S_j \le S_j^0 - \epsilon^2, \\
& \epsilon^2 \le E_1 \le E_1^0 - \epsilon^2, \epsilon^2 \le E_j \le E_j^0 - \epsilon^2, \epsilon \le A_1, \epsilon^3 \le A_j, \\
& \frac{(1-q_1)\Lambda_1}{\bar{\mu}_1} + \epsilon^2 \le S_1 + E_1 + A_1 \le \frac{(1-q_1)\Lambda_1}{\tilde{\mu}_1} - \epsilon^2, \\
& \frac{(1-q_j)\Lambda_j}{\bar{\mu}_j} + \epsilon^2 \le S_j + E_j + A_j \le \frac{(1-q_j)\Lambda_j}{\tilde{\mu}_j} - \epsilon^4, \quad j = 2,3,\dots,n \Big\},
\end{aligned}
$$

and ϵ *is a sufficiently small positive number.*

5. Numerical simulations

In this section, applying the the EM method [14], we shall verify our theoretical results. Assume that the total population is divided into two distinct groups, i.e., $n = 2$. We choose the initial value as $(S_1(0), E_1(0), A_1(0), S_2(0), E_2(0), A_2(0)) = (5, 0.5, 0.5, 5, 0.5, 0.5)$. From [31], we take the following set of values of parameters for model (1.1):

$$
\begin{aligned}
& [\Lambda_1, \Lambda_2] = [2, 1.5], \quad [\mu_1^S, \mu_2^S] = [0.25, 0.125], \quad [\mu_1^E, \mu_2^E] = [0.2, 0.125], \\
& [\mu_1^R, \mu_2^R] = [0.2, 0.125], \quad [q_1, q_2] = [0.5, 1/3], \quad [\varepsilon_1, \varepsilon_2] = [0.2, 0.125], \\
& [p_1, p_2] = [0.5, 0.25], \quad [\mu_1^A, \mu_2^A] = [0.5, 0.25], \quad [\gamma_1, \gamma_2] = [0.5, 1/3], \\
& [a_1, a_2] = [0.5, 0.4]
\end{aligned}
\tag{5.1}
$$

and $\beta_{kj}(r(t))$ takes different values in different examples, where $r(t)$ takes values $\mathbb{S} = \{1, 2, 3\}$. The generator of the Markov chain is

$$
\Gamma^* = \begin{pmatrix} -1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{5} & -\frac{2}{5} & \frac{1}{5} \\ \frac{1}{7} & \frac{1}{7} & -\frac{2}{7} \end{pmatrix},
$$

thus by solving the linear equation (2.1), we obtain the unique stationary distribution $\pi = (\pi_1, \pi_2, \pi_3) = \left(\frac{1}{7}, \frac{5}{14}, \frac{1}{2}\right)$.

Example 5.1. For the state $l = 1, 2, 3$, Let the transmission coefficient $\beta_{kj}(l)$ as

$$
\begin{pmatrix} \beta_{11}(1) & \beta_{12}(1) \\ \beta_{21}(1) & \beta_{22}(1) \end{pmatrix} = \begin{pmatrix} 0.08 & 0.03 \\ 0.03 & 0.08 \end{pmatrix}, \quad \begin{pmatrix} \beta_{11}(2) & \beta_{12}(2) \\ \beta_{21}(2) & \beta_{22}(2) \end{pmatrix} = \begin{pmatrix} 0.1 & 0.05 \\ 0.05 & 0.1 \end{pmatrix},
$$
$$
\begin{pmatrix} \beta_{11}(3) & \beta_{12}(3) \\ \beta_{21}(3) & \beta_{22}(3) \end{pmatrix} = \begin{pmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{pmatrix}.
$$

Therefore, we obtain $R_0^e = 0.9658 < 1$. Thus the alcohol problems die out exponentially with probability one (Theorem 3.1). For deterministic model (1.1), choose $\begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} = \begin{pmatrix} 0.1 & 0.05 \\ 0.05 & 0.1 \end{pmatrix}$, then we compute $R_0 = 0.2788 < 1$. Fig. 1 shows the Markovian chain of the extinction system. Fig. 2 confirms that the disease will die out in both the deterministic model (1.1) and the stochastic model (1.2).

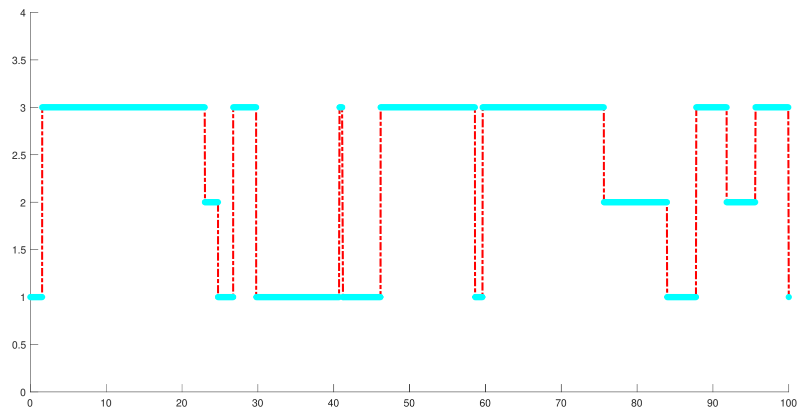

Figure 1. The Markovian chain of the extinction system with state space $\mathbb{S} = \{1, 2, 3\}$.

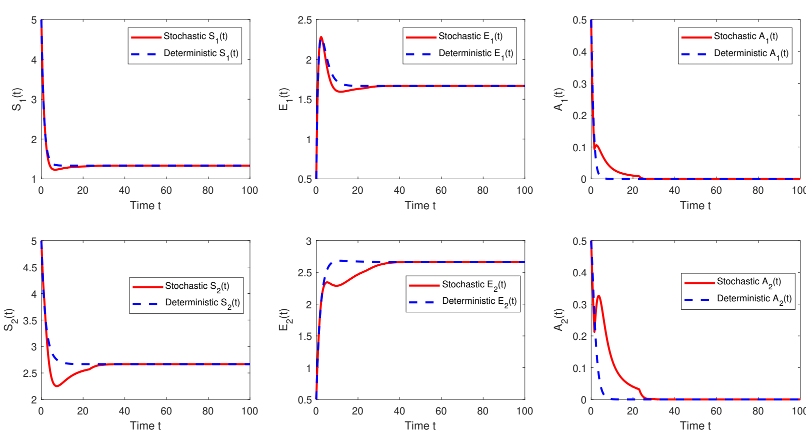

Figure 2. The diagrams track the populations size of the susceptible drinkers, educated susceptible drinkers and alcoholics of the deterministic model and the stochastic model over time, respectively.

Example 5.2. For the state $l = 1, 2, 3$, consider the transmission coefficient $\beta_{kj}(l)$

as

$$\begin{pmatrix} \beta_{11}(1) & \beta_{12}(1) \\ \beta_{21}(1) & \beta_{22}(1) \end{pmatrix} = \begin{pmatrix} 0.35 & 0.25 \\ 0.35 & 0.25 \end{pmatrix}, \quad \begin{pmatrix} \beta_{11}(2) & \beta_{12}(2) \\ \beta_{21}(2) & \beta_{22}(2) \end{pmatrix} = \begin{pmatrix} 0.4 & 0.3 \\ 0.3 & 0.5 \end{pmatrix},$$

$$\begin{pmatrix} \beta_{11}(3) & \beta_{12}(3) \\ \beta_{21}(3) & \beta_{22}(3) \end{pmatrix} = \begin{pmatrix} 0.5 & 0.4 \\ 0.4 & 0.5 \end{pmatrix}.$$

Therefore, we obtain $R_0^* = 1.1470 > 1$. Theorem 4.1 shows that the disease will persist in the mean for a long term. Moreover, for deterministic model (1.1), choose $\begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} = \begin{pmatrix} 0.4 & 0.3 \\ 0.3 & 0.5 \end{pmatrix}$, then we compute $R_0 = 1.5577 > 1$. Fig. 3 shows the Markovian chain of the persistence system. Fig. 4 displays that the alcohol problem will persist in both the deterministic model (1.1) and the stochastic model (1.2).

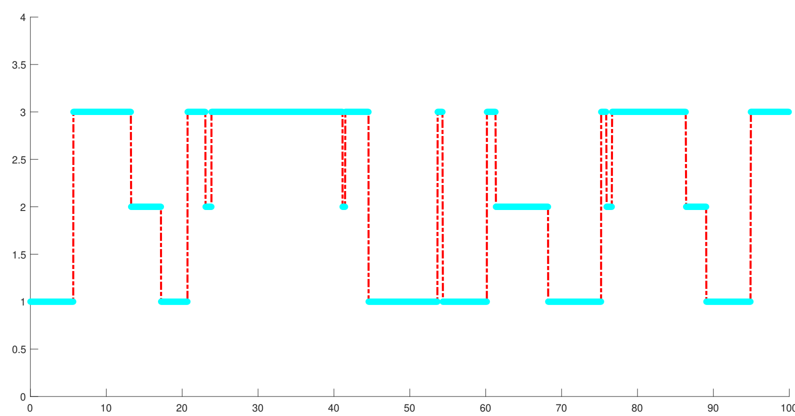

Figure 3. The Markovian chain of the persistence system with state space $\mathbb{S} = \{1, 2, 3\}$.

6. Conclusion

In this paper, we have studied a multi-group alcoholism model with public health education and Markovian switching. Firstly, the existence of the unique global positive solution is analyzed. In addition, we establish sufficient conditions R_0^e for extinction of the alcohol problem. By constructing suitable stochastic Lyapunov functions with regime switching, the critical condition R_0^p for persistence in the mean of alcoholism is identified, which is also shown to determine the existence of positive recurrence of the solutions to the model (1.2). More precisely, we have obtained the following results

★ Assume that $B_1 = (\check{\beta}_{ij})_{n\times n}$ is irreducible. If $R_0^e = \rho(M_0^e) < 1$, then there exists a positive solution $(S_1(t), E_1(t), A_1(t), \dots, S_n(t), E_n(t), A_n(t), r(t))$ of system

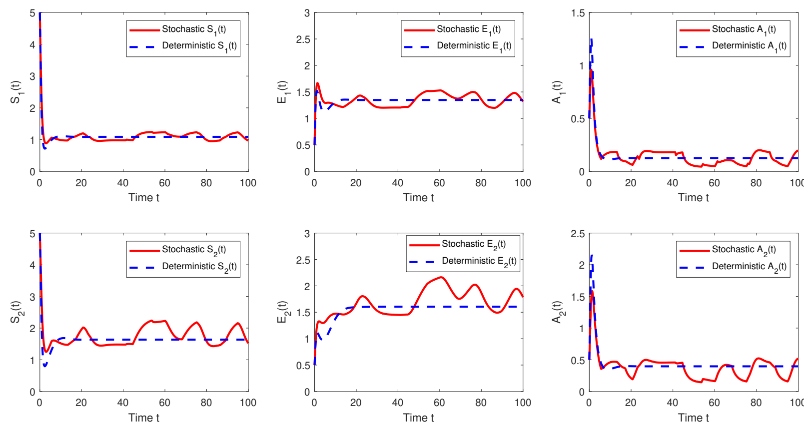

Figure 4. The diagrams track the populations size of the susceptible drinkers, educated susceptible drinkers and alcoholics of the deterministic model and the stochastic model over time, respectively.

(1.2) which has the following property

$$\limsup_{t\to\infty} \frac{1}{t} \log\Big(\sum_{i=1}^{n} \frac{\bar{\omega}_i A_i}{\mu_i^A + a_i + \gamma_i}\Big) < 0, \quad a.s..$$

In other words, the alcohol problems die out exponentially with probability one, i.e.,

$$\lim_{t\to\infty} A_i(t) = 0 \quad a.s., \quad i = 1, 2, \ldots, n.$$

★ Assume that $B_2 = (\hat{\beta}_{ij})_{n\times n}$ $(i, j = 1, 2, \ldots, n)$ is irreducible. If $R_0^p = \rho(M_0^p) > 1$, then there exists a positive solution $(S_1(t), E_1(t), A_1(t), \ldots, S_n(t), E_n(t), A_n(t), r(t))$ of system (1.2) which has the following property

$$\liminf_{t\to\infty} \int_0^t A_p(s)\mathrm{d}s \geq \frac{1}{D_p} \min_{1\leq i\leq n} \{\mu_i^A + a_i + \gamma_i\}(R_0^p - 1) > 0, \quad a.s., \quad p = 1, \ldots, n,$$

which means the alcohol problem will persist in all groups, where

$$D_p = \frac{\max_{1\leq i,j\leq n}\{\check{\beta}_{ij}\}^2 n(S_i^0 + \sigma_i E_i^0)}{\min\{\mu_i^S, \mu_i^E + \varepsilon_i\}} \Big[1 + \frac{\sigma_p \sum_{j=2}^{n} \hat{\beta}_{pj} \frac{(1-q_j)\Lambda_j}{\bar{\mu}_j} + \mu_p^A + a_p + \gamma_p}{\frac{\sigma_p \Lambda_p (1-q_p) \min_{2\leq j\leq n}\{\hat{\beta}_{pj}\}}{\bar{\mu}_p}}\Big],$$

for $p = 1, 2, \ldots, n$.

★ Furthermore, assume that $B_2 = (\hat{\beta}_{ij})_{n\times n}$ $(i, j = 1, 2, \ldots, n)$ is irreducible. If $R_0^p = \rho(M_0^p) > 1$, we obtain that there exists a positive solution $(S_1(t), E_1(t), A_1(t), \ldots, S_n(t), E_n(t), A_n(t), r(t))$ of system (1.2) which is positive recurrence with respect to the domain D_ϵ, where

$$\begin{aligned}D_\epsilon =&\Big\{(S_1,E_1,A_1,\ldots,S_n,E_n,A_n)\in\bar{\Gamma}:\epsilon\le S_1\le S_1^0-\epsilon^2,\epsilon\le S_j\le S_j^0-\epsilon^2,\\&\epsilon^2\le E_1\le E_1^0-\epsilon^2,\epsilon^2\le E_j\le E_j^0-\epsilon^2,\epsilon\le A_1,\epsilon^3\le A_j,\\&\frac{(1-q_1)\Lambda_1}{\bar{\mu}_1}+\epsilon^2\le S_1+E_1+A_1\le\frac{(1-q_1)\Lambda_1}{\tilde{\mu}_1}-\epsilon^2,\\&\frac{(1-q_j)\Lambda_j}{\bar{\mu}_j}+\epsilon^2\le S_j+E_j+A_j\le\frac{(1-q_j)\Lambda_j}{\tilde{\mu}_j}-\epsilon^4,\quad j=2,3,\ldots,n\Big\}.\end{aligned}$$

Appdenix A (Proof of Theorem 3.1)

Since $B_1=(\check{\beta}_{ij})_{n\times n}$ is irreducible, $\check{\beta}_{ij}>0$ and $\mu_i^A+a_i+\gamma_i>0$ and $S_i^0+\sigma_iE_i^0>0$, $(i,j=1,\ldots,n)$, then $M_0^e=\left(\frac{\check{\beta}_{ij}(S_i^0+\sigma_iE_i^0)}{\mu_i^A+a_i+\gamma_i}\right)_{n\times n}$ is irreducible and nonnegative. From Lemma 2.1, we obtain that there exists a left positive eigenvector $\bar{\omega}=(\bar{\omega}_1,\bar{\omega}_2,\ldots,\bar{\omega}_n)$ of M_0^e corresponding to $\rho(M_0^e)$ such that

$$(\bar{\omega}_1,\bar{\omega}_2,\ldots,\bar{\omega}_n)\rho(M_0^e)=(\bar{\omega}_1,\bar{\omega}_2,\ldots,\bar{\omega}_n)M_0^e. \tag{A.1}$$

Applying Itô's formula to $\frac{A_i}{\mu_i^A+a_i+\gamma_i}$ for model (1.2), one gets

$$\mathcal{L}\Big(\frac{A_i}{\mu_i^A+a_i+\gamma_i}\Big)\le\frac{S_i+\sigma_iE_i}{\mu_i^A+a_i+\gamma_i}\sum_{j=1}^n\check{\beta}_{ij}A_j-A_i.$$

Define a C^2-function V_2 on $\bar{\Gamma}$ to $\mathbb{R}$

$$V_2=\sum_{i=1}^n\frac{\bar{\omega}_iA_i}{\mu_i^A+a_i+\gamma_i}.$$

The differential operator $\mathcal{L}$ acting on the function V_2 along the solutions and combining with (A.1), we have

$$\begin{aligned}\mathcal{L}V_2&\le\sum_{i=1}^n\sum_{j=1}^n\bar{\omega}_i\frac{\check{\beta}_{ij}(S_i^0+\sigma_iE_i^0)}{\mu_i^A+a_i+\gamma_i}A_j-\sum_{i=1}^n\bar{\omega}_iA_i\\&=(\bar{\omega}_1,\bar{\omega}_2,\ldots,\bar{\omega}_n)(M_0^eI_n-I_n)\begin{pmatrix}A_1\\A_2\\\vdots\\A_n\end{pmatrix}\\&=(\rho(M_0^e)-1)\sum_{i=1}^n\bar{\omega}_iA_i\\&=-\sum_{i=1}^n(1-R_0^e)\bar{\omega}_iA_i,\end{aligned}$$

where I_n denotes identity matrix. Then we obtain

$$\mathcal{L}(\log V_2) \leq -\frac{\sum_{i=1}^{n}(1-R_0^e)\bar{\omega}_i A_i}{\sum_{i=1}^{n}\frac{\bar{\omega}_i A_i}{\mu_i^A+a_i+\gamma_i}} \leq -\min_{1\leq i\leq n}\{\mu_i^A+a_i+\gamma_i\}(1-R_0^e). \tag{A.2}$$

Integrating (A.2) from 0 to t and then dividing by t on both sides leads to that

$$\frac{\log V_2(t)}{t} \leq \frac{\log V_2(0)}{t} - \min_{1\leq i\leq n}\{\mu_i^A+a_i+\gamma_i\}(1-R_0^e) < 0. \tag{A.3}$$

Taking the superior limit on both sides of (A.3), we obtain

$$\limsup_{t\to\infty}\frac{\log V_2(t)}{t} \leq -\min_{1\leq i\leq n}\{\mu_i^A+a_i+\gamma_i\}(1-R_0^e) < 0, \quad a.s.,$$

which means

$$\lim_{t\to\infty} A_i(t) = 0 \quad a.s., \quad i = 1,2,\ldots,n.$$

That is to say, the alcohol problems die out exponentially with probability one in the sense that alcoholism fractions go to zero from all the groups. This completes the proof.

Appdenix B (Proof of Theorem 4.1)

Since $B_2 = (\hat{\beta}_{ij})_{n\times n}$ is irreducible, $\hat{\beta}_{ij} > 0$, $\mu_i^A + a_i + \gamma_i > 0$ and $S_i^0 + \sigma_i E_i^0 > 0$, $(i,j = 1,\ldots,n)$, then $M_0^p = \left(\frac{\hat{\beta}_{ij}(S_i^0+\sigma_i E_i^0)}{\mu_i^A+a_i+\gamma_i}\right)_{n\times n}$ is irreducible and nonnegative. By Lemma 2.1, we obtain that there exists a left positive eigenvector $\tilde{\omega} = (\tilde{\omega}_1,\tilde{\omega}_2,\ldots,\tilde{\omega}_n)$ of M_0^p corresponding to $\rho(M_0^p)$, such that

$$(\tilde{\omega}_1,\tilde{\omega}_2,\ldots,\tilde{\omega}_n)\rho(M_0^p) = (\tilde{\omega}_1,\tilde{\omega}_2,\ldots,\tilde{\omega}_n)M_0^p. \tag{B.1}$$

In the view of model (1.2), one gets

$$\begin{aligned}
&\mathcal{L}\Big(\frac{A_i}{\mu_i^A+a_i+\gamma_i}\Big)\\
=&\frac{S_i+\sigma_i E_i}{\mu_i^A+a_i+\gamma_i}\sum_{j=1}^{n}\beta_{ij}(k)A_j - A_i\\
=&\frac{S_i^0+\sigma_i E_i^0}{\mu_i^A+a_i+\gamma_i}\sum_{j=1}^{n}\beta_{ij}(k)A_j - A_i - \frac{(S_i^0-S_i)+\sigma_i(E_i^0-E_i)}{\mu_i^A+a_i+\gamma_i}\sum_{j=1}^{n}\beta_{ij}(k)A_j.
\end{aligned}$$

Define a C^2-function V_3 on $\bar{\Gamma}$ to $\mathbb{R}$ as follows

$$V_3 = \sum_{i=1}^{n}\frac{\tilde{\omega}_i A_i}{\mu_i^A+a_i+\gamma_i}.$$

The differential operator $\mathcal{L}$ acting on the function V_3 along the solutions, we have

$$\begin{aligned}
\mathcal{L}V_3 \geq & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\hat{\beta}_{ij}\tilde{\omega}_i(S_i^0+\sigma_i E_i^0)}{\mu_i^A+a_i+\gamma_i}A_j-\sum_{i=1}^{n}\tilde{\omega}_i A_i-\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\check{\beta}_{ij}\tilde{\omega}_i[(S_i^0-S_i)+(E_i^0-E_i)]}{\mu_i^A+a_i+\gamma_i}A_j \\
&= (\tilde{\omega}_1,\tilde{\omega}_2,\ldots,\tilde{\omega}_n)(M_0^p I_n - I_n)\begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{pmatrix} - \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\check{\beta}_{ij}\tilde{\omega}_i[(S_i^0-S_i)+(E_i^0-E_i)]}{\mu_i^A+a_i+\gamma_i}A_j \\
&= (\rho(M_0^p-1))\sum_{i=1}^{n}\tilde{\omega}_i A_i - \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\check{\beta}_{ij}\tilde{\omega}_i[(S_i^0-S_i)+(E_i^0-E_i)]}{\mu_i^A+a_i+\gamma_i}A_j \\
&= (R_0^p-1)\sum_{i=1}^{n}\tilde{\omega}_i A_i - \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\check{\beta}_{ij}\tilde{\omega}_i[(S_i^0-S_i)+(E_i^0-E_i)]}{\mu_i^A+a_i+\gamma_i}A_j.
\end{aligned}$$

Applying Itô's formula to the functions $-\log V_3$ and $-(S_i+E_i)$, respectively, one gets

$$\begin{aligned}
-\mathcal{L}(\log V_3) \leq & -\frac{(R_0^p-1)\sum_{i=1}^{n}\tilde{\omega}_i A_i - \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\check{\beta}_{ij}\tilde{\omega}_i[S_i^0+\sigma_i E_i^0-(S_i+\sigma_i E_i)]}{\mu_i^A+a_i+\gamma_i}A_j}{\sum_{i=1}^{n}\frac{\tilde{\omega}_i A_i}{\mu_i^A+a_i+\gamma_i}} \\
&\leq -\min_{1\leq i\leq n}\{\mu_i^A+a_i+\gamma_i\}(R_0^p-1)+\max_{1\leq i,j\leq n}\{\check{\beta}_{ij}\}n[(S_i^0-S_i)+\sigma_i(E_i^0-E_i)]. \\
-\mathcal{L}(S_i+E_i) =& -(1-q_i)\Lambda_i+\mu_i S_i+(\mu_i^E+\varepsilon_i)E_i+(S_i+\sigma_i E_i)\sum_{j=1}^{n}\beta_{ij}(k)A_j \\
&\leq -\mu_i^S(S_i^0-S_i)-(\mu_i^E+\varepsilon_i)(E_i^0-E_i)+(S_i^0+\sigma_i E_i^0)\sum_{j=1}^{n}\check{\beta}_{ij}A_j \\
&\leq -\min\{\mu_i^S,\mu_i^E+\varepsilon_i\}[(S_i^0-S_i)+(E_i^0-E_i)]+(S_i^0+\sigma_i E_i^0)\sum_{j=1}^{n}\check{\beta}_{ij}A_j.
\end{aligned}$$

Then we define

$$V_4 = -\log V_3 - \frac{\max_{1\leq i,j\leq n}\{\check{\beta}_{ij}\}n}{\min\{\mu_i^S,\mu_i^E+\varepsilon_i\}}(S_i+E_i).$$

The differential operator $\mathcal{L}$ acting on the function V_4 along the solutions, we have

$$\mathcal{L}V_4 \leq -\min_{1\leq i\leq n}\{\mu_i^A+a_i+\gamma_i\}(R_0^p-1)+\frac{\max_{1\leq i,j\leq n}\{\check{\beta}_{ij}\}^2 n(S_i^0+\sigma_i E_i^0)}{\min\{\mu_i^S,\mu_i^E+\varepsilon_i\}}\sum_{j=1}^{n}A_j. \tag{B.2}$$

Applying Itô's formula to $-A_1$, one gets

$$\begin{aligned}-\mathcal{L}(A_1) =& -(S_1+\sigma_1 E_1)\sum_{j=1}^n \beta_{1j}(k)A_j + (\mu_1^A + a_1 + \gamma_1)A_1\\ \le& -\sigma_1(S_1+E_1+A_1)\sum_{j=2}^n \hat\beta_{1j}A_j + (\sigma_1\sum_{j=2}^n \hat\beta_{1j}A_j + \mu_1^A + a_1+\gamma_1)A_1\\ \le& -\frac{\sigma_1\Lambda_1(1-q_1)\min_{2\le j\le n}\{\hat\beta_{1j}\}}{\bar\mu_1}\sum_{j=2}^n A_j\\ &+\Big[\sigma_1\sum_{j=2}^n\hat\beta_{1j}\frac{(1-q_j)\Lambda_j}{\tilde\mu_j} + \mu_1^A + a_1+\gamma_1\Big]A_1.\end{aligned}$$

Define

$$V_5 = V_4 - \frac{\frac{\max_{1\le i,j\le n}\{\check\beta_{ij}\}^2 n(S_i^0+\sigma_i E_i^0)}{\min\{\mu_i^S,\mu_i^E+\varepsilon_i\}}}{\frac{\sigma_1\Lambda_1(1-q_1)\min_{2\le j\le n}\{\hat\beta_{1j}\}}{\bar\mu_1}}A_1.$$

The differential operator $\mathcal{L}$ acting on the function V_5 along the solutions, we have

$$\mathcal{L}V_5 \le -\min_{1\le i\le n}\{\mu_i^A + a_i+\gamma_i\}(R_0^p - 1) + D_1 A_1, \tag{B.3}$$

where

$$D_1 = \frac{\max_{1\le i,j\le n}\{\check\beta_{ij}\}^2 n(S_i^0+\sigma_i E_i^0)}{\min\{\mu_i^S,\mu_i^E+\varepsilon_i\}}\Big[1+\frac{\sigma_1\sum_{j=2}^n\hat\beta_{1j}\frac{(1-q_j)\Lambda_j}{\tilde\mu_j} + \mu_1^A + a_1+\gamma_1}{\frac{\sigma_1\Lambda_1(1-q_1)\min_{2\le j\le n}\{\hat\beta_{1j}\}}{\bar\mu_1}}\Big].$$

Integrating (B.3) from 0 to t and then dividing by t on both sides, we get

$$\frac{V_5(t)}{t} \le \frac{V_5(0)}{t} - \min_{1\le i\le n}\{\mu_i^A + a_i+\gamma_i\}(R_0^p-1) + \frac{D_1}{t}\int_0^t A_1(s)\mathrm{d}s. \tag{B.4}$$

Since $\frac{(1-q_i)\Lambda_i}{\bar\mu_i} < S_i + E_i + A_i < \frac{(1-q_i)\Lambda_i}{\tilde\mu_i}$, $(i=1,2,\dots,n)$, we have

$$\begin{aligned}V_5(S_1,E_1,A_1,\dots,S_n,E_n,A_n) &= V_4 + \frac{\sum_{i=1}^n\sum_{j=1}^n\frac{(\check\beta_{ij})^2(S_i^0+\sigma_i E_i^0)}{\min\{\mu_i^S,\mu_i^E+\varepsilon_i\}}}{\frac{\sigma_1\Lambda_1(1-q_1)\min_{2\le j\le n}\{\hat\beta_{1j}\}}{\bar\mu_1}}A_1\\ > -\log\Big(\sum_{i=1}^n\frac{\tilde\omega_i(1-q_i)\Lambda_i}{\tilde\mu_i(\mu_i^A+a_i+\gamma_i)}\Big) &- \sum_{i=1}^n\sum_{j=1}^n\frac{\check\beta_{ij}(1-q_i)\Lambda_i}{\tilde\mu_i\min\{\mu_i^S,\mu_i^E+\varepsilon_i\}}\\ -\frac{\sum_{i=1}^n\sum_{j=1}^n\frac{(\check\beta_{ij})^2(S_i^0+\sigma_i E_i^0)}{\min\{\mu_i^S,\mu_i^E+\varepsilon_i\}}}{\frac{\sigma_1\Lambda_1(1-q_1)\min_{2\le j\le n}\{\hat\beta_{1j}\}}{\bar\mu_1}}&\times\frac{(1-q_1)\Lambda_1}{\tilde\mu_1}\\ :=M_2 \text{ is a constant.}\end{aligned}$$

Therefore

$$\liminf_{t\to\infty}\frac{V_5(t)}{t}\ge 0. \tag{B.5}$$

Taking the inferior limit on both sides of (B.4) and combining with (B.5), we have

$$\liminf_{t\to\infty}\int_0^t A_1(s)\mathrm{d}s \geq \frac{1}{D_1}\min_{1\leq i\leq n}\{\mu_i^A + a_i + \gamma_i\}(R_0^p - 1) > 0, \quad a.s..$$

Using the same method as above, we can also get that

$$\liminf_{t\to\infty}\int_0^t A_p(s)\mathrm{d}s \geq \frac{1}{D_p}\min_{1\leq i\leq n}\{\mu_i^A + a_i + \gamma_i\}(R_0^p - 1) > 0, \quad a.s., \quad p = 2, \ldots, n,$$

where

$$D_p = \frac{\max_{1\leq i,j\leq n}\{\check{\beta}_{ij}\}^2 n(S_i^0 + \sigma_i E_i^0)}{\min\{\mu_i^S, \mu_i^E + \varepsilon_i\}}\Big[1 + \frac{\sigma_p \sum_{j=2}^n \hat{\beta}_{pj}\frac{(1-q_j)\Lambda_j}{\tilde{\mu}_j} + \mu_p^A + a_p + \gamma_p}{\frac{\sigma_p\Lambda_p(1-q_p)\min_{2\leq j\leq n}\{\hat{\beta}_{pj}\}}{\bar{\mu}_p}}\Big].$$

This completes the proof.

Appdenix C (Proof of Theorem 4.2)

From Theorem 2.1, it follows that for any initial value $(S_1(0), E_1(0), A_1(0), \ldots, S_n(0), E_n(0), A_n(0), r(0)) \in \bar{\Gamma}\times\mathbb{S}$, the solution of system (1.2) is regular.

Define a C^2-function $\bar{V}$ on $\bar{\Gamma}$ to $\mathbb{R}$ as follows

$$\begin{aligned}\bar{V} = MV_5 &- \sum_{i=1}^n \log S_i - \sum_{i=1}^n \log(S_i^0 - S_i) - \sum_{i=1}^n \log E_i - \sum_{i=1}^n \log(E_i^0 - E_i) - \sum_{i=2}^n \log A_i\\ &- \sum_{i=1}^n \log[\frac{(1-q_i)\Lambda_i}{\tilde{\mu}_i} - S_i - E_i - A_i] - \sum_{i=1}^n \log[S_i + E_i + A_i - \frac{(1-q_i)\Lambda_i}{\bar{\mu}_i}],\end{aligned}$$

where $M > 0$ is a sufficiently large number satisfying the following condition

$$-M[\min_{1\leq i\leq n}\{\mu_i^A + a_i + \gamma_i\}(R_0^p - 1)] + Q \leq -2, \tag{C.1}$$

and Q will be determined later.

Note that $\bar{V}(S_1, E_1, A_1, \ldots, S_n, E_n, A_n)$ is not only continuous, but also tends to ∞ as $(S_1, E_1, A_1, \ldots, S_n, E_n, A_n)$ approaches the boundary of $\bar{\Gamma}$. So it must be lower bounded and achieve this lower bound at a point $(S_1(\bar{k}), E_1(\bar{k}), A_1(\bar{k}), \ldots, S_n(\bar{k}), E_n(\bar{k}), A_n(\bar{k}))$ in the interior of $\bar{\Gamma}$.

Then we define a C^2-function V on $\bar{\Gamma}$ to $\mathbb{R}_+\bigcup\{0\}$ as follows

$$\begin{aligned}&V(S_1, E_1, A_1, \ldots, S_n, E_n, A_n)\\ =&\bar{V}(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) - \bar{V}(S_1(\bar{k}), E_1(\bar{k}), A_1(\bar{k}), \ldots, S_n(\bar{k}), E_n(\bar{k}), A_n(\bar{k}))\\ =&MV_5 - \sum_{i=1}^n \log S_i - \sum_{i=1}^n \log(S_i^0 - S_i) - \sum_{i=1}^n \log E_i - \sum_{i=1}^n \log(E_i^0 - E_i) - \sum_{i=2}^n \log A_i\\ &- \sum_{i=1}^n \log(\frac{(1-q_i)\Lambda_i}{\tilde{\mu}_i} - S_i - E_i - A_i) - \sum_{i=1}^n \log(S_i + E_i + A_i - \frac{(1-q_i)\Lambda_i}{\bar{\mu}_i})\\ &- \bar{V}(S_1(\bar{k}), E_1(\bar{k}), A_1(\bar{k}), \ldots, S_n(\bar{k}), E_n(\bar{k}), A_n(\bar{k}))\\ :=&MV_5(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) + V_6(S_1, \ldots, S_n) + V_7(S_1, \ldots, S_n)\end{aligned}$$

$$+ V_8(E_1,\dots,E_n) + V_9(E_1,\dots,E_n) + V_{10}(A_2,\dots,A_n)$$
$$+ V_{11}(S_1,E_1,A_1,\dots,S_n,E_n,A_n) + V_{12}(S_1,E_1,A_1,\dots,S_n,E_n,A_n),$$

where

$$\begin{aligned}
&V_6(S_1,\dots,S_n) = -\sum_{i=1}^n \log S_i, \quad V_7(S_1,\dots,S_n) = -\sum_{i=1}^n \log(S_i^0 - S_i),\\
&V_8(E_1,\dots,E_n) = -\sum_{i=1}^n \log E_i, \quad V_9(E_1,\dots,E_n) = -\sum_{i=1}^n \log(E_i^0 - E_i),\\
&V_{10}(A_2,\dots,A_n) = -\sum_{i=2}^n \log A_i,\\
&V_{11}(S_1,E_1,A_1,\dots,S_n,E_n,A_n) = -\sum_{i=1}^n \log(\frac{(1-q_i)\Lambda_i}{\tilde{\mu}_i} - S_i - E_i - A_i),
\end{aligned}$$

and $V_{12}(S_1,E_1,A_1,\dots,S_n,E_n,A_n) = -\sum_{i=1}^n \log(S_i+E_i+A_i-\frac{(1-q_i)\Lambda_i}{\bar{\mu}_i}) - \bar{V}(S_1(\bar{l}), E_1(\bar{l}), A_1(\bar{l}),\dots,S_n(\bar{l}),E_n(\bar{l}),A_n(\bar{l}))$.

Applying Itô's formula to V_6, V_7, V_8, V_9, V_{10}, V_{11} and V_{12}, respectively, we have

$$\begin{aligned}
\mathcal{L}V_6 &= -\sum_{i=1}^n \frac{(1-q_i)\Lambda_i}{S_i} + \sum_{i=1}^n\sum_{j=1}^n \beta_{ij}(k)A_j + \sum_{i=1}^n(\mu_i^S + p_i)\\
&\le -\sum_{i=1}^n \frac{(1-q_i)\Lambda_i}{S_i} + \sum_{i=1}^n\sum_{j=1}^n \frac{\check{\beta}_{ij}(1-q_j)\Lambda_j}{\tilde{\mu}_j} + \sum_{i=1}^n(\mu_i^S + p_i),
\end{aligned} \tag{C.2}$$

$$\begin{aligned}
\mathcal{L}V_7 &= \sum_{i=1}^n \frac{1}{S_i^0 - S_i}\Big[(1-q_i)\Lambda_i - \sum_{j=1}^n \beta_{ij}(k)S_iA_j - (\mu_i^S + p_i)S_i\Big]\\
&\le \sum_{i=1}^n \frac{1}{S_i^0 - S_i}\Big[(\mu_i^S + p_i)(S_i^0 - S_i) + \sum_{j=1}^n \beta_{ij}(k)(S_i^0 - S_i)A_j - \sum_{j=1}^n \beta_{ij}(k)S_i^0A_j\Big]\\
&\le \sum_{i=1}^n \frac{1}{S_i^0 - S_i}\Big[(\mu_i^S + p_i)(S_i^0 - S_i) + \sum_{j=1}^n \beta_{ij}(k)(S_i^0 - S_i)A_j - \hat{\beta}_{i1}S_i^0A_1\Big]\\
&\le -\sum_{i=1}^n \frac{\hat{\beta}_{i1}S_i^0A_1}{S_i^0 - S_i} + \sum_{i=1}^n(\mu_i^S + p_i) + \sum_{i=1}^n\sum_{j=1}^n \frac{\check{\beta}_{ij}\Lambda_j(1-q_j)}{\tilde{\mu}_j},
\end{aligned} \tag{C.3}$$

$$\begin{aligned}
\mathcal{L}V_8 &= -\sum_{i=1}^n \frac{p_iS_i}{E_i} + \sum_{i=1}^n\sum_{j=1}^n \sigma_i\beta_{ij}(k)A_j + \sum_{i=1}^n(\mu_i^E + \varepsilon_i)\\
&\le -\sum_{i=1}^n \frac{p_iS_i}{E_i} + \sum_{i=1}^n\sum_{j=1}^n \frac{\check{\beta}_{ij}\sigma_i\Lambda_j(1-q_j)}{\tilde{\mu}_j} + \sum_{i=1}^n(\mu_i^E + \varepsilon_i),
\end{aligned} \tag{C.4}$$

$$
\begin{aligned}
\mathcal{L}V_9 =& \sum_{i=1}^{n} \frac{1}{E_i^0 - E_i}\Big[p_i S_i - \sum_{j=1}^{n} \sigma_i \beta_{ij}(k) E_i A_j - (\mu_i^E + \varepsilon_i) E_i\Big] \\
\leq& \sum_{i=1}^{n} \frac{1}{E_i^0 - E_i}\Big[(\mu_i^E + \varepsilon_i) E_i^0 - (\mu_i^E + \varepsilon_i) E_i - \sigma_i \beta_{i1}(k) E_i^0 A_1 \\
&+ \sum_{j=1}^{n} \sigma_i \beta_{ij}(k)(E_i^0 - E_i) A_j\Big] \\
\leq& - \sum_{i=1}^{n} \frac{\hat{\beta}_{i1} \sigma_i E_i^0 A_1}{E_i^0 - E_i} + \sum_{i=1}^{n} (\mu_i^E + \varepsilon_i) + \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\check{\beta}_{ij} \sigma_i \Lambda_j (1 - q_j)}{\tilde{\mu}_j},
\end{aligned} \tag{C.5}
$$

$$
\begin{aligned}
\mathcal{L}V_{10} =& - \sum_{i=2}^{n}\sum_{j=1}^{n} \frac{\beta_{ij}(k)(S_i + \sigma_i E_i) A_j}{A_i} + \sum_{i=2}^{n} (\mu_i^A + a_i + \gamma_i) \\
\leq& - \sum_{i=2}^{n} \frac{\hat{\beta}_{i1} S_i A_1}{A_i} + \sum_{i=2}^{n} (\mu_i^A + a_i + \gamma_i),
\end{aligned} \tag{C.6}
$$

$$
\begin{aligned}
\mathcal{L}V_{11} =& \sum_{i=1}^{n} \frac{(1 - q_i)\Lambda_i - \mu_i^S S_i - (\mu_i^E + \varepsilon_i) E_i - (\mu_i^A + a_i + \gamma_i) A_i}{\frac{(1-q_i)\Lambda_i}{\tilde{\mu}_i} - S_i - E_i - A_i} \\
\leq& \sum_{i=1}^{n} \frac{(1 - q_i)\Lambda_i - \tilde{\mu}_i (S_i + E_i + A_i) - (\mu_i^A + a_i + \gamma_i - \tilde{\mu}_i) A_i}{\frac{(1-q_i)\Lambda_i}{\tilde{\mu}_i} - S_i - E_i - A_i} \\
=& - \sum_{i=1}^{n} \frac{(\mu_i^A + a_i + \gamma_i - \tilde{\mu}_i) A_i}{\frac{(1-q_i)\Lambda_i}{\tilde{\mu}_i} - S_i - E_i - A_i} + \sum_{i=1}^{n} \tilde{\mu}_i,
\end{aligned} \tag{C.7}
$$

$$
\begin{aligned}
\mathcal{L}V_{12} =& - \sum_{i=1}^{n} \frac{(1 - q_i)\Lambda_i - \mu_i^S S_i - (\mu_i^E + \varepsilon_i) E_i - (\mu_i^A + a_i + \gamma_i) A_i}{S_i + E_i + A_i - \frac{(1-q_i)\Lambda_i}{\bar{\mu}_i}} \\
\leq& \sum_{i=1}^{n} \frac{\bar{\mu}_i (S_i + E_i + A_i) - (1 - q_i)\Lambda_i - (\bar{\mu}_i - \mu_i^S) S_i}{S_i + E_i + A_i - \frac{(1-q_i)\Lambda_i}{\bar{\mu}_i}} \\
=& - \sum_{i=1}^{n} \frac{(\bar{\mu}_i - \mu_i^S) S_i}{S_i + E_i + A_i - \frac{(1-q_i)\Lambda_i}{\bar{\mu}_i}} + \sum_{i=1}^{n} \bar{\mu}_i.
\end{aligned} \tag{C.8}
$$

Hence, from (B.3), (C.2), (C.3), (C.4), (C.5), (C.6), (C.7) and (C.8), it follows that

$$
\begin{aligned}
\mathcal{L}V =& \mathcal{L}(MV_5 + V_6 + V_7 + V_8 + V_9 + V_{10} + V_{11} + V_{12}) \\
\leq& - M[\min_{1 \leq i \leq n} \{\mu_i^A + a_i + \gamma_i\}(R_0^p - 1)] - \sum_{i=1}^{n} \frac{(1 - q_i)\Lambda_i}{S_i} - \sum_{i=1}^{n} \frac{p_i S_i}{E_i} + M D_1 A_1 \\
&- \sum_{i=2}^{n} \frac{\hat{\beta}_{i1}(S_i + \sigma_i E_i) A_1}{A_i} - \sum_{i=1}^{n} \frac{\hat{\beta}_{i1} S_i^0 A_1}{S_i^0 - S_i} - \sum_{i=1}^{n} \frac{\hat{\beta}_{i1} E_i^0 A_1}{E_i^0 - E_i} \\
&- \sum_{i=1}^{n} \frac{(\mu_i^A + a_i + \gamma_i - \tilde{\mu}_i) A_i}{\frac{(1-q_i)\Lambda_i}{\tilde{\mu}_i} - S_i - E_i - A_i} - \sum_{i=1}^{n} \frac{(\bar{\mu}_i - \mu_i^S) S_i}{S_i + E_i + A_i - \frac{(1-q_i)\Lambda_i}{\bar{\mu}_i}} + Q,
\end{aligned} \tag{C.9}
$$

where

$$
Q = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{2\check{\beta}_{ij} \Lambda_j (1 + \sigma_i)(1 - q_j)}{\tilde{\mu}_j} + \sum_{i=1}^{n} \Big[2(\mu_i^S + p_i + \mu_i^E + \varepsilon_i) + \tilde{\mu}_i + \bar{\mu}_i\Big]
$$

$$+\sum_{i=2}^{n}(\mu_i^A + a_i + \gamma_i).$$

Then we define a bounded closed set as follows

$$\begin{aligned}D_\epsilon =&\Big\{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : \epsilon \le S_1 \le S_1^0 - \epsilon^2, \epsilon \le S_j \le S_j^0 - \epsilon^2,\\ &\epsilon^2 \le E_1 \le E_1^0 - \epsilon^2, \epsilon^2 \le E_j \le E_j^0 - \epsilon^2, \epsilon \le A_1, \epsilon^3 \le A_j,\\ &\frac{(1-q_1)\Lambda_1}{\bar{\mu}_1} + \epsilon^2 \le S_1 + E_1 + A_1 \le \frac{(1-q_1)\Lambda_1}{\tilde{\mu}_1} - \epsilon^2,\\ &\frac{(1-q_j)\Lambda_j}{\bar{\mu}_j} + \epsilon^2 \le S_j + E_j + A_j \le \frac{(1-q_j)\Lambda_j}{\tilde{\mu}_j} - \epsilon^4, \quad j = 2, 3, \ldots, n\Big\},\end{aligned}$$

where ϵ is a sufficiently small positive constant satisfying the following inequalities

$$-\frac{(1-q_1)\Lambda_1}{\epsilon} + \frac{MD_1(1-q_1)\Lambda_1}{\tilde{\mu}_1} + Q \le -1, \tag{C.10}$$

$$-\frac{\sum_{j=2}^{n}(1-q_j)\Lambda_j}{\epsilon} + \frac{MD_1(1-q_1)\Lambda_1}{\tilde{\mu}_1} + Q \le -1, \tag{C.11}$$

$$-\frac{p_1}{\epsilon} + \frac{MD_1(1-q_1)\Lambda_1}{\tilde{\mu}_1} + Q \le -1, \tag{C.12}$$

$$-\frac{\sum_{j=2}^{n} p_j}{\epsilon} + \frac{MD_1(1-q_1)\Lambda_1}{\tilde{\mu}_1} + Q \le -1, \tag{C.13}$$

$$\epsilon \le \frac{1}{MD_1}, \tag{C.14}$$

$$-\frac{\sum_{j=2}^{n} \hat{\beta}_{j1}}{\epsilon} + \frac{MD_1(1-q_1)\Lambda_1}{\tilde{\mu}_1} + Q \le -1, \tag{C.15}$$

$$-\frac{\hat{\beta}_{11}S_1^0}{\epsilon} + \frac{MD_1(1-q_1)\Lambda_1}{\tilde{\mu}_1} + Q \le -1, \tag{C.16}$$

$$-\frac{\sum_{j=2}^{n} \hat{\beta}_{j1}S_j^0}{\epsilon} + \frac{MD_1(1-q_1)\Lambda_1}{\tilde{\mu}_1} + Q \le -1, \tag{C.17}$$

$$-\frac{\hat{\beta}_{11}E_i^0}{\epsilon} + \frac{MD_1(1-q_1)\Lambda_1}{\tilde{\mu}_1} + Q \le -1, \tag{C.18}$$

$$-\frac{\sum_{j=2}^{n} \hat{\beta}_{j1}E_j^0}{\epsilon} + \frac{MD_1(1-q_1)\Lambda_1}{\tilde{\mu}_1} + Q \le -1, \tag{C.19}$$

$$-\frac{(\mu_1^A + a_1 + \gamma_1 - \tilde{\mu}_1)}{\epsilon} + \frac{MD_1(1-q_1)\Lambda_1}{\tilde{\mu}_1} + Q \le -1, \tag{C.20}$$

$$-\frac{\sum_{j=2}^{n}(\mu_j^A + a_j + \gamma_j - \tilde{\mu}_j)}{\epsilon} + \frac{MD_1(1-q_1)\Lambda_1}{\tilde{\mu}_1} + Q \le -1, \tag{C.21}$$

$$-\frac{\bar{\mu}_1 - \mu_1^S}{\epsilon} + \frac{MD_1(1-q_1)\Lambda_1}{\tilde{\mu}_1} + Q \le -1, \tag{C.22}$$

$$-\frac{\sum_{j=2}^{n}(\bar{\mu}_j - \mu_j^S)}{\epsilon} + \frac{MD_1(1-q_1)\Lambda_1}{\tilde{\mu}_1} + Q \le -1. \tag{C.23}$$

For convenience, we can divide $\bar{\Gamma} \setminus D_\epsilon$ into the following fourteen domains,

$$D_\varepsilon^1 = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : S_1 < \epsilon\},$$

$$
\begin{aligned}
&D_\varepsilon^2 = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : S_j < \epsilon\},\\
&D_\varepsilon^3 = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : E_1 < \epsilon^2, S_1 \geq \epsilon\},\\
&D_\varepsilon^4 = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : E_j < \epsilon^2, S_j \geq \epsilon^2\},\\
&D_\varepsilon^5 = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : A_1 < \epsilon\},\\
&D_\varepsilon^6 = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : A_j < \epsilon^4, S_j \geq \epsilon, E_j \geq \epsilon^2, A_1 \geq \epsilon\},\\
&D_\varepsilon^7 = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : S_1^0 - \epsilon^2 < S_1, A_1 \geq \epsilon\},\\
&D_\varepsilon^8 = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : S_j^0 - \epsilon^2 < S_j, A_1 \geq \epsilon\}\\
&D_\varepsilon^9 = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : E_1^0 - \epsilon^2 < E_1, A_1 \geq \epsilon\},\\
&D_\varepsilon^{10} = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : E_j^0 - \epsilon^2 < E_j, A_1 \geq \epsilon\}\\
&D_\varepsilon^{11} = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : \frac{(1-q_1)\Lambda_1}{\tilde{\mu}_1} - \epsilon^3 < S_1 + E_1 + A_1,\\
&\qquad E_1 \geq \epsilon^2, A_1 \geq \epsilon\},\\
&D_\varepsilon^{12} = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : \frac{(1-q_j)\Lambda_j}{\tilde{\mu}_j} - \epsilon^5 < S_j + E_j + A_j,\\
&\qquad E_j \geq \epsilon^2, A_j \geq \epsilon^4\},\\
&D_\varepsilon^{13} = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : S_1 + E_1 + A_1 < \frac{(1-q_1)\Lambda_1}{\bar{\mu}_1} + \epsilon^3,\\
&\qquad E_1 \geq \epsilon^2, A_1 \geq \epsilon\},\\
&D_\varepsilon^{14} = \{(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} : S_j + E_j + A_j < \frac{(1-q_j)\Lambda_j}{\tilde{\mu}_j} + \epsilon^5,\\
&\qquad E_j \geq \epsilon^2, A_j \geq \epsilon^4\},
\end{aligned}
$$

for $j = 2, 3, \ldots, n$.

Clearly, $\bar{\Gamma} \setminus D_\epsilon = \bigcup_{i=1}^{14} D_\epsilon^i$. In what follows we prove that $\mathcal{L}V(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \leq -1$ for any $(S_1, E_1, A_1, \ldots, S_n, E_n, A_n)$ on $\bar{\Gamma} \setminus D_\epsilon$, which is equivalent to proving it on the above fourteen domains, respectively.

From inequalities (C.10) to (C.23), we obtain that there is a closed set D_ϵ such that for any $(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \in \bar{\Gamma} \setminus D_\epsilon$

$$
\mathcal{L}V(S_1, E_1, A_1, \ldots, S_n, E_n, A_n) \leq -1. \tag{C.24}
$$

Let $(S_1(0), E_1(0), A_1(0), \ldots, S_n(0), E_n(0), A_n(0)) \in \bar{\Gamma} \setminus D_\epsilon$, from (C.24) we have

$$
\begin{aligned}
&\mathbb{E}[S_1(\tau_{D_\epsilon}), E_1(\tau_{D_\epsilon}), A_1(\tau_{D_\epsilon}), \ldots, S_n(\tau_{D_\epsilon}), E_n(\tau_{D_\epsilon}), A_n(\tau_{D_\epsilon})]\\
&- V(S_1(0), E_1(0), A_1(0), \ldots, S_n(0), E_n(0), A_n(0))\\
=&\ \mathbb{E}\int_0^{\tau_{D_\epsilon}} \mathcal{L}V(S_1(t), E_1(t), A_1(t), \ldots, S_n(t), E_n(t), A_n(t))\\
\leq&\ -\mathbb{E}(\tau_{D_\epsilon}).
\end{aligned}
$$

Thus, due to the positivity of V, it follows that

$$
\mathbb{E}(\tau_{D_\epsilon}) \leq V(S_1(0), E_1(0), A_1(0), \ldots, S_n(0), E_n(0), A_n(0)).
$$

This completes the proof.

Acknowledgments

The authors thank the reviewers and editors for their valuable suggestions that have improved the quality of this article.

References

[1] B. Benedict, *Modelling alcoholism as a contagious disease: how "infected" drinking buddies spread problem drinking*, SIAM News., 2007, 40(3), 1–3.

[2] R. Bani, R. Hameed, S. Szymanowski, P. Greenwood, C. Kribs-Zaleta and A. Mubayi, *Influence of environmental factors on college alcohol drinking patterns*, Math. Biosci. Eng., 2013, 10(5), 1281–1300.

[3] C. P. Bhunu and S. Mushayabasa, *A theoretical analysis of smoking and alcoholism*, J. Math. Model. Algor., 2012, 11(2), 387–408.

[4] R. J. Bonnie and M. E. O'Connell, *Reducing Underage Drinking: A Collective Responsibility*, The National Academies Press, Washington, DC, 2004.

[5] A. Berman and R. J. Plemmons, *Nonnegative Matrices in the Mathematical Sciences*, Academic Press, New York, 1979.

[6] N. Dang, N. Du and G. Yin, *Existence of stationary distributions for Kolmogorov systems of competitive type under telegraph noise*, J. Diff. Eqs., 2014, 257(6), 2078–2101.

[7] Z. Feng, W. Huang and C. Castillo-Chavez, *Global behavior of a multi-group SIS epidemic model with age structure*, J. Diff. Eqs., 2005, 218(2), 292–324.

[8] D. M. Gorman, J. Mezic, I. Mezic and P. J. Gruenewald, *Agent-based modeling of drinking behavior: a preliminary model and potential applications to theory and practice*, Am. J. Public Health., 2006, 96(11), 2055–2060.

[9] S. Galea, C. Hall and G. A. Kaplan, *Social epidemiology and complex system dynamic modelling as applied to health behaviour and drug use research*, Int. J. Drug Policy., 2009, 20(3), 209–216.

[10] A. Gray, D. Greenhalgh, X. Mao and J. Pan, *The SIS epidemic model with Markovian switching*, J. Math. Anal. Appl., 2012, 394(2), 496–516.

[11] H. Guo, M. Li and Z. Shuai, *Global stability of the endemic equilibrium of multigroup SIR epidemic models*, Can. Appl. Math. Q., 2006, 14(3), 259–284.

[12] H. Guo, M. Li and Z. Shuai, *A graph-theoretic approach to the method of global Lyapunov functions*, Proc. Amer. Math. Soc., 2008, 136(8), 2793–2802.

[13] H. Guo, M. Li and Z. Shuai, *Global dynamics of a general class of multistage models for infectious diseases*, SIAM J. Appl. Math., 2012, 72(1), 261–279.

[14] D. J. Higham, *An algorithmic introduction to numerical simulation of stochastic differential equations*, SIAM Rev., 2001, 43(3), 525–546.

[15] C. Ji, D. Jiang and N. Shi, *Multigroup SIR epidemic model with stochastic perturbation*, Physica A, 2011, 390(10), 1747–1762.

[16] T. Kuniya, *Global Behavior of a Multi-Group SIR Epidemic Model with Age Structure and an Application to the Chlamydia Epidemic in Japan*, SIAM J. Appl. Math., 2019, 79(1), 321–340.

[17] D. Kuang, Q. Yin and J. Li, *Stationary distribution and extinction of stochastic HTLV-I infection model with CTL immune response under regime switching*, J. Nonl. Mod. Anal., 2020, 2, 585–600.

[18] M. Li, Z. Shuai and C. Wang, *Global stability of multi-group epidemic models with distributed delays*, J. Math. Anal. Appl., 2010, 361(1), 38–47.

[19] Q. Liu, D. Jiang and N. Shi, *Threshold behavior in a stochastic SIQR epidemic model with standard incidence and regime switching*, Appl. Math. Comput., 2018, 316, 310–325.

[20] J. Lv, H. Liu and X. Zou, *Stationary distribution and persistence of a stochastic predator-prey model with a functional response*, J. Appl. Anal. Comput., 2019, 9(1), 1–11.

[21] M. Li, Z. Jin and G. Sun, *Modeling direct and indirect disease transmission using multi-group model*, J. Math. Anal. Appl., 2017, 446(2), 1292–1309.

[22] D. Li, S. Liu and J. Cui, *Threshold dynamics and ergodicity of an SIRS epidemic model with Markovian switching*, J. Diff. Eqs., 2017, 263(12), 8873–8915.

[23] D. Li, S. Liu and J. Cui, *Threshold dynamics and ergodicity of an SIRS epidemic model with semi-Markov switching*, J. Diff. Eqs., 2019, 266(7), 3973–4017.

[24] Q. Liu and D. Jiang, *Dynamics of a multigroup SIS epidemic model with standard incidence rates and Markovian switching*, Physica A, 2019, 527, 121270.

[25] Q. Luo and X. Mao, *Stochastic population dynamics under regime switching*, J. Math. Anal. Appl., 2007, 334(1), 69–84.

[26] X. Mao and C. Yuan, *Stochastic Differential Equations With Markovian Switching*, Imperial College Press, London, 2006.

[27] X. Mao, *Stability of stochastic differential equations with Markovian switching*, Stoch. Process. Their Appl., 1999, 79(1), 45–67.

[28] J. L. Manthey, A. Y. Aidoo and K. Y. Ward, *Campus drinking: an epidemiological model*, J. Biol. Dyn., 2008, 2(3), 346–356.

[29] G. Mulone and B. Straughan, *Modeling binge drinking*, Int. J. Biomath., 2012, 5(1), 1250005.

[30] A. Mubayi and P. E. Greenwood, *Contextual interventions for controlling alcohol drinking*, Math. Popul. Stud., 2013, 20(1), 27–53.

[31] S. Ma, H. Huo and H. Xiang, *Threshold dynamics of a multi-group SEAR alcoholism model with public health education*, Int. J. Biomath., 2019, 12(3), 1950025.

[32] X. Meng, L. Wang and T. Zhang, *Global dynamics analysis of a nonlinear impulsive stochastic chemostat system in a polluted environment*, J. Appl. Anal. Comput., 2016, 6(3), 865–875.

[33] S. Nolen-Hoeksema, *Gender differences in risk factors and consequences for alcohol use and problems*, Clin. Psychol. Rev., 2004, 24(8), 981–1010.

[34] H. Peng and X. Zhang, *The dynamics of stochastic predator-prey models with non-constant mortality rate and general nonlinear functional response*, J. Nonl. Mod. Anal., 2020, 2, 495–511.

[35] H. Shu, D. Fan and J. Wei, *Global stability of multi-group SEIR epidemic models with distributed delays and nonlinear transmission*, Nonlinear Anal.-Real World Appl., 2012, 13(4), 1581–1592.

[36] H. Song, J. Wang and W. Jiang, *Global dynamics in a multi-group epidemic model for disease with latency spreading and nonlinear transmission rate*, J. Appl. Anal. Comput., 2016, 6(1), 47–64.

[37] A. Settati and A. Lahrouz, *Stationary distribution of stochastic population systems under regime switching*, Appl. Math. Comput., 2014, 244, 235–243.

[38] Y. Takeuchi, N. Du, N. T. Hieu and K. Sato, *Evolution of predator-prey systems described by a Lotka-Volterra equation under random environment*, J. Math. Anal. Appl., 2006, 323, 938–957.

[39] M. Ventura-Cots et al., *Colder weather and fewer sunlight hours increase alcohol consumption and alcoholic cirrhosis worldwide*, Hepatology, 2018, 69(5), 1916–1930.

[40] L. Wang and D. Jiang, *Ergodic property of the chemostat: A stochastic model under regime switching and with general response function*, Nonlinear Anal.-Hybrid Syst., 2018, 27, 341–352.

[41] X. Wang, Q. Yang and H. Huo, *The asymptotic behaviors of a stochastic social epidemic model with multi-perturbation*, J. Appl. Anal. Comput., 2018, 8(1), 272–295. DOI: 10.11948/2018.272.

[42] X. Wang, P. Zhang and Q. Yang, *On the stochastic dynamics of a social epidemics model*, Dyn. Nat. Soc., 2017, 6297074. DOI: 10.1155/2017/6297074.

[43] World Health Organization, *Global status report on alcohol and health*, Geneva, 2018.

[44] Q. Yang and X. Mao, *Extinction and recurrence of multi-group SEIR epidemic models with stochastic perturbations*, Nonlinear Anal.-Real World Appl., 2013, 14(3), 1434–1456.

[45] X. Zhang, D. Jiang, A. Ahmed and T. Hayat, *Stationary distribution of stochastic SIS epidemic model with vaccination under regime switching*, Appl. Math. Lett., 2016, 59, 87–93.