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EXTINCTION AND STATIONARY DISTRIBUTION OF A STOCHASTIC PREDATOR-PREY MODEL WITH HOLLING II FUNCTIONAL RESPONSE AND STAGE STRUCTURE OF PREY*

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Abstract The interaction between predator and prey is an important part of ecological diversity. This paper presents a stage-structured predator-prey model to study how stochastic environments affect population dynamics. Holling II functional response is also incorporated in the proposed theoretical framework. Specifically, by using the theory of stochastic stability, we provide conditions for the stochastic system to suffer extinction or to have a unique ergodic stationary distribution. Besides, numerical simulations are also employed to verify the validity of the theoretical results.

Keywords Stage-structured predator-prey model, Holling II functional response, ergodicity, stationary distribution, extinction.

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1. Introduction

In the natural world, the interactions among populations mainly include competition, predation, parasitism and mutualism [4, 6, 14, 28] and so on. The law of the jungle is the basic survival rule for predators. The Lotka-Volterra mathematical model depicting the predator-prey interaction was initially put forward in the 1920s [7]. This classic model has a profound effect on the development of modern ecological theory. Since then, a large number of mathematical models have been widely used in the study of various species ecosystems [1, 9, 15, 21, 23, 29].

As we all know, the growth of many animals needs to go through the juvenile and adult stages. However, when they are young, they are less fertile and are more likely to be captured by natural enemies. Therefore, mathematical models with the stage structure have aroused much concern during the past few decades [2, 13, 17, 19]. For example, the stage-structured predator-prey model was developed by Zhang et al. [34], in which the prey had two-stage structures and the stage structure of predator population was no longer subdivided. As the strong side, the predator population not only has to prey on the bait to maintain its own material needs, because of the

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limited external resources, they will inevitably compete within species for food. As the relatively weak side, the juvenile prey is in a more passive position and often has to face the risk of predation alone. The model given in the literature [34] is as follows:

$$\begin{cases} \dot{x_1} = ex_2 - d_1x_1 - \alpha x_1 - \beta_1 x_1^2 - \eta x_1 y, \\ \dot{x_2} = \alpha x_1 - d_2 x_2, \\ \dot{y} = y(-d + k\eta x_1 - \beta y), \end{cases} \tag{1.1}$$

where x_1, x_2, y denote the population densities of juvenile, mature preys and predators, respectively. The parameter e represents the juvenile prey's birth rate; the death rates of juvenile, mature preys and predators are indicated by d_1, d_2 and d; α implies the rate at which juvenile preys develop into mature preys; β_1 and β are the intraspecific competition coefficients of juvenile prey population and predator population, respectively; predators feed on the juvenile preys and so ηx_1 shows predation rate of the predators; k is the nutrient conversion rate for the predator. All coefficients involved above are positive.

The model (1.1) adopts a linear function as the functional response, which assumes that the amount of predation per predator is proportional to the number of prey. However, in the real world, even if prey is plentiful, the predation may reach saturation and thereby the predators reduce the feeding rate. Consequently, in the research and analysis of biological systems, the Holling type II scheme has been broadly applied [33,35,36]. The cubs of many animals do not have the ability to hunt alone, but mainly rely on their parents for feeding. Especially when the food is sufficient, the intraspecific competition of the cubs is weak. Some amphibians, such as frog larvae, tadpoles do not forage when they just hatch, but mainly depend on the nutrition brought from the yolk to maintain their lives. At this time, the intraspecific competition formed by tadpoles competing for food can be ignored. Literature [22] studied the global stability of a predator-prey system with stage structure for prey. The author did not consider the intraspecific constraints of juvenile prey (see model (1.3) in Literature [22]). Therefore, this paper mainly focuses on the predation effect of predators on juvenile prey without considering the intraspecific competition of juvenile prey. Based on the above discussion, a predator-prey model with Holling type II scheme and phase structure of prey is built as follows:

$$\begin{cases} \dot{x_1} = ex_2 - d_1x_1 - \alpha x_1 - \dfrac{ax_1}{b + x_1} y, \\ \dot{x_2} = \alpha x_1 - d_2 x_2, \\ \dot{y} = y\left(-d + k\dfrac{ax_1}{b + x_1} - \beta y\right), \end{cases} \tag{1.2}$$

where a is the effect of predation on the juvenile prey and b means the so-called half-saturation constant, other parameters are the same as the model (1.1).

As species live in nature, they are unavoidably affected by all sorts of environmental noises. For example, environmental factors such as water temperature, water quality and sunlight can affect tadpole growth and development. Thus, it is vital to consider the effects of random disturbances on population dynamics [5,8,10,20,24,25,30–32,37]. Considering the influence of white noise and colored noise, literature [20] established two stochastic non-autonomous SEIS infectious disease models with latent and active patients, and discussed the existence conditions of periodic solution and ergodic stationary distribution of the models. Assuming that the birth rates of species are disturbed by white noise, literature [37] proposed a competitive n-species stochastic model with delayed diffusions, and discussed dynamic behaviors such as persistence in mean and the asymptotic stability in distribution, as well as optimal harvest strategy. By transforming the Itô's integral into an equivalent Stratonovich integral, Chang et al. [5] presented a new method to study the dynamic behavior of a stochastic SIS model with multiplicative noise.

There are many ways to construct a stochastic differential equation model, such as adding random perturbations to the parameters of deterministic system [25,37], or introducing proportional perturbations to state variables [20,31]. References [12,27] provide detailed modeling methods for Itô's-type stochastic differential equations disturbed by white noise. Next, on the basis of the deterministic model (1.2), we construct the stochastic model. First, we introduce a discrete time Markov chain: For a fixed time increment $\Delta t > 0$, we define a process $Z^{(\Delta t)}(t) = (x_1^{(\Delta t)}(t), x_2^{(\Delta t)}(t), y^{(\Delta t)}(t))$ for $t = 0, \Delta t, 2\Delta t, \cdots$ Let $Z^{(\Delta t)}(0) = (x_1(0), x_2(0), y(0)) \in \mathbb{R}_+^3$ is a deterministic initial value. Then, assume that on each interval $[k\Delta t, (k+1)\Delta t)$, the effect of random influences on the species can be captured by $\xi_i^{(\Delta t)}(k), i = 1, 2, 3$. Here the random variables $\left\{\xi_i^{(\Delta t)}(k)\right\}_{k=0}^{\infty}$ are identically distributed and for each k satisfy

$$E\left[\xi_i^{(\Delta t)}(k)\right] = 0, \quad E\left[\xi_i^{(\Delta t)}(k)\right]^2 = \sigma_i^2 \Delta t, \quad i = 1, 2, 3; k = 0, 1, \cdots .$$

We make further assumptions that within the same time period $[k\Delta t, (k+1)\Delta t)$, $Z^{(\Delta t)}(t) = (x_1^{(\Delta t)}(t), x_2^{(\Delta t)}(t), y^{(\Delta t)}(t))$ grows according to the deterministic model (1.2) and, in addition, by the random amount $(\xi_1^{(\Delta t)}(k)x_1^{(\Delta t)}(k\Delta t), \xi_2^{(\Delta t)}(k)x_2^{(\Delta t)}(k\Delta t),$ $\xi_3^{(\Delta t)}(k)y^{(\Delta t)}(k\Delta t))$, that is

$$\begin{aligned} x_1^{(\Delta t)}((k+1)\Delta t) =& x_1^{(\Delta t)}(k\Delta t) + \Delta t \left\{ e x_2^{(\Delta t)}(k\Delta t) - d_1 x_1^{(\Delta t)}(k\Delta t) - \alpha x_1^{(\Delta t)}(k\Delta t) \right. \\ & \left. - \frac{a x_1^{(\Delta t)}(k\Delta t)}{b + x_1^{(\Delta t)}(k\Delta t)} y^{(\Delta t)}(k\Delta t) \right\} + \xi_1^{(\Delta t)}(k) x_1^{(\Delta t)}(k\Delta t), \\ x_2^{(\Delta t)}((k+1)\Delta t) =& x_2^{(\Delta t)}(k\Delta t) + \Delta t \left\{ \alpha x_1^{(\Delta t)}(k\Delta t) - d_2 x_2^{(\Delta t)}(k\Delta t) \right\} \\ & + \xi_2^{(\Delta t)}(k) x_2^{(\Delta t)}(k\Delta t) \end{aligned}$$

and

$$\begin{aligned} y^{(\Delta t)}((k+1)\Delta t) =& y^{(\Delta t)}(k\Delta t) + \Delta t \left\{ y^{(\Delta t)}(k\Delta t)(-d + \frac{k a x_1^{(\Delta t)}(k\Delta t)}{b + x_1^{(\Delta t)}(k\Delta t)} \right. \\ & \left. - \beta y^{(\Delta t)}(k\Delta t)) \right\} + \xi_3^{(\Delta t)}(k) y^{(\Delta t)}(k\Delta t). \end{aligned}$$

The following derivation process is similar to literatures [12,27], and we omit it. Finally, the random model corresponding to (1.2) is obtained as follows:

$$\begin{cases} dx_1 = \left(e x_2 - d_1 x_1 - \alpha x_1 - \dfrac{a x_1}{b + x_1} y \right) dt + \sigma_1 x_1 dB_1(t), \\ dx_2 = (\alpha x_1 - d_2 x_2)\, dt + \sigma_2 x_2 dB_2(t), \\ dy = \left[y \left(-d + k \dfrac{a x_1}{b + x_1} - \beta y \right) \right] dt + \sigma_3 y dB_3(t). \end{cases} \tag{1.3}$$

Here, $B_i(t), i = 1, 2, 3$ are mutually independent standard Brownian motions.

Remark 1.1. Assuming that the mortality parameters of species are disturbed by environmental noise, model (1.3) can also be obtained by cooperating white noise directly into model (1.2):

$$-d_1 \to -d_1 + \sigma_1 \dot{B}_1(t), \quad -d_2 \to -d_2 + \sigma_2 \dot{B}_2(t), \quad -d \to -d + \sigma_3 \dot{B}_3(t).$$

The remaining parts are indicated as below: Next, the existence and uniqueness of the global positive solution of model (1.3) will be proved. We derive the criteria for the existence of ergodic stationary distribution of model (1.3) in section 3. The fourth part discusses the sufficient conditions for systematic extinction. Ultimately, the article is concluded by several examples and numerical simulations.

2. Preliminaries

If (x_1^*, x_2^*, y^*) is an equilibrium point of system (1.2), then the Jacobian matrix at (x_1^*, x_2^*, y^*) is

$$J|_{(x_1^*, x_2^*, y^*)} = \left.\begin{pmatrix} -(\alpha + d_1) - \frac{aby}{(b+x_1)^2} & e & -\frac{ax_1}{b+x_1} \\ \alpha & -d_2 & 0 \\ \frac{kaby}{(b+x_1)^2} & 0 & -d - 2\beta y + \frac{kax_1}{b+x_1} \end{pmatrix}\right|_{(x_1^*, x_2^*, y^*)}.$$

Thus, the Jacobian matrix of system (1.2) at its trivial equilibrium point $(0, 0, 0)$ is

$$J|_{(0,0,0)} = \begin{pmatrix} -(\alpha + d_1) & e & 0 \\ \alpha & -d_2 & 0 \\ 0 & 0 & -d \end{pmatrix}.$$

Introduce parameter

$$R_1 = \frac{\alpha e}{d_2(\alpha + d_1)}.$$

Obviously, when $R_1 < 1$, the equilibrium point $P_0(0, 0, 0)$ is locally asymptotically stable, the system (1.2) becomes extinct.

We now have the following theorem concerning the existence of the solution of the stochastic system (1.3).

Theorem 2.1. *System (1.3) exists a unique solution $x_1(t), x_2(t), y(t)$ for any initial value on $\mathbb{R}_+^3$, and the solution remains in $\mathbb{R}_+^3$ for all $t \geq 0$ almost surely.*

Proof. Since the coefficients of model (1.3) are locally Lipschitz continuous, there exists a unique local positive solution $(x_1(t), x_2(t), y(t))$ on $[0, \tau_e)$, in which τ_e means the explosion time [18].

We take a large enough constant $n_0 > 0$, such that $x_1(0), x_2(0), y(0)$ all belongs to $[\frac{1}{n_0}, n_0]$. Regarding any integer $n \geq n_0$, the stopping time is defined as [18]

$$\tau_n = \inf \left\{ t \in [0, \tau_e) : \ \min \{x_1(t), x_2(t), y(t)\} \leq \frac{1}{n} \ \text{or} \ \max \{x_1(t), x_2(t), y(t)\} \geq n \right\}.$$

Evidently, τ_n increases monotonically. Set $\tau_\infty = \lim_{n\to\infty}\tau_n$, then $\tau_\infty \le \tau_e$ a.s. If $\tau_\infty = \infty$ *a.s.*, we get $\tau_e = \infty$ and the theorem is proved. Otherwise, there exists $T > 0$ and $\epsilon \in (0,1)$, such that

$$\mathbb{P}\{\tau_\infty \le T\} > \epsilon.$$

Therefore, existing an integer $n_1 \ge n_0$ meets

$$\mathbb{P}\{\tau_n \le T\} \ge \epsilon, \ \forall n \ge n_1.$$

Define the Lyapunov function:

$$\overline{V}(x_1,x_2,y) = (x_1 - 1 - \ln x_1) + \frac{e}{d_2}(x_2 - 1 - \ln x_2) + \frac{1}{k}(y - 1 - \ln y).$$

Using the same method as in reference [26], theorem 2.1 can be proved, which is omitted here. □

3. The existence of stationary solution

Then, the long-term dynamic properties of model (1.3) are discussed, and the conditions for existence of ergodic stationary distribution are provided. Define the parameter

$$R_0^S = \frac{\alpha e}{\left(d_2 + \frac{\sigma_2^2}{2}\right)\left(\alpha + d_1 + \frac{\sigma_1^2}{2} + d + \frac{\sigma_3^2}{2}\right)}.$$

Theorem 3.1. *When given initial value on $\mathbb{R}_+^3$, if $R_0^S > 1$, then there exists a unique stationary distribution for model* (1.3) *and it possesses ergodicity.*

Proof. So as to verify the ergodicity of the system (1.3), it is enough to validate Lemma 2.1 of [16]. We firstly certify the condition H_1 of Lemma 2.1 in [16]. The diffusion matrix of model (1.3) is presented by

$$B = \begin{pmatrix} \sigma_1^2 x_1^2 & 0 & 0 \\ 0 & \sigma_2^2 x_2^2 & 0 \\ 0 & 0 & \sigma_3^2 y^2 \end{pmatrix}.$$

It is apparent that B is a positive definite matrix on $\mathbb{R}_+^3$, so the condition H_1 is true.

Afterwards condition H_2 of Lemma 2.1 in [16] will be tested. Define

$$V_1(x_1,x_2,y) = -\ln x_1 - Q\ln x_2 - \ln y + \frac{a+b\beta}{bd}y,$$

here Q is a positive value which will be determined later. From Itô's formula, we have

$$\begin{aligned} LV_1 \le\ & \alpha + d_1 + \frac{\sigma_1^2}{2} - \frac{ex_2}{x_1} + \frac{a}{b+x_1}y - \frac{Q\alpha x_1}{x_2} + Q\left(d_2 + \frac{\sigma_2^2}{2}\right) + \beta y - \frac{kax_1}{b+x_1} \\ & + d + \frac{\sigma_3^2}{2} - \frac{a+b\beta}{b}y + \frac{ka(a+b\beta)}{b^2 d}x_1 y \end{aligned}$$

$$\leq \alpha + d_1 + \frac{\sigma_1^2}{2} - 2\sqrt{Q\alpha e} + Q\left(d_2 + \frac{\sigma_2^2}{2}\right) + d + \frac{\sigma_3^2}{2} + \frac{ka(a+b\beta)}{b^2 d}x_1 y. \quad (3.1)$$

Choose

$$Q = \frac{\alpha e}{\left(d_2 + \frac{\sigma_2^2}{2}\right)^2},$$

next by (3.1), one can get

$$\begin{aligned} LV_1 &\leq -\frac{\alpha e}{d_2 + \frac{\sigma_2^2}{2}} + \alpha + d_1 + \frac{\sigma_1^2}{2} + d + \frac{\sigma_3^2}{2} + \frac{ka(a+b\beta)}{b^2 d}x_1 y \\ &= -\left(\alpha + d_1 + \frac{\sigma_1^2}{2} + d + \frac{\sigma_3^2}{2}\right)(R_0^S - 1) + \frac{ka(a+b\beta)}{b^2 d}x_1 y \\ &= -\lambda + \frac{ka(a+b\beta)}{b^2 d}x_1 y, \end{aligned} \quad (3.2)$$

where

$$\lambda := \left(\alpha + d_1 + \frac{\sigma_1^2}{2} + d + \frac{\sigma_3^2}{2}\right)(R_0^S - 1) > 0.$$

Define

$$V_2(x_2) = -\ln x_2, \quad V_3(x_1, x_2, y) = \frac{1}{\theta + 2}\left(x_1 + \frac{me}{d_2}x_2 + \frac{y}{k}\right)^{\theta+2},$$

in which $\theta > 0$ is a small enough constant and $m > 0$ is a sufficiently large value. Thus we can obtain

$$\begin{aligned} LV_2 &= -\frac{1}{x_2}(\alpha x_1 - d_2 x_2) + \frac{\sigma_2^2}{2} \\ &= -\frac{\alpha x_1}{x_2} + d_2 + \frac{\sigma_2^2}{2}, \end{aligned} \quad (3.3)$$

and

$$\begin{aligned} LV_3 = &\left(x_1 + \frac{me}{d_2}x_2 + \frac{y}{k}\right)^{\theta+1}\left(\frac{me\alpha}{d_2}x_1 - d_1 x_1 - \alpha x_1 - (m-1)e x_2 - \frac{\beta}{k}y^2 - \frac{d}{k}y\right) \\ &+ \frac{\theta+1}{2}\left(x_1 + \frac{me}{d_2}x_2 + \frac{y}{k}\right)^{\theta}\left(\sigma_1^2 x_1^2 + \frac{m^2 e^2}{d_2^2}\sigma_2^2 x_2^2 + \frac{\sigma_3^2}{k^2}y^2\right) \\ \leq &\frac{me\alpha}{d_2}x_1\left(x_1 + \frac{me}{d_2}x_2 + \frac{y}{k}\right)^{\theta+1} - (\alpha + d_1)x_1^{\theta+2} - (m-1)e\left(\frac{me}{d_2}\right)^{\theta+1} x_2^{\theta+2} \\ &- \frac{\beta}{k^{\theta+2}}y^{\theta+3} + \frac{\theta+1}{2} \times \left(x_1 + \frac{em}{d_2}x_2 + \frac{y}{k}\right)^{\theta}\left(\sigma_1^2 x_1^2 + \frac{m^2 e^2}{d_2^2} + \frac{\sigma_3^2}{k^2}y^2\right) \\ = &-\frac{\alpha + d_1}{2}x_1^{\theta+2} - (m-1)\theta e\left(\frac{me}{d_2}\right)^{\theta+1} x_2^{\theta+2} - \frac{\beta}{2k^{\theta+2}}y^{\theta+3} + \frac{me\alpha}{d_2}x_1 \\ &\times \left(x_1 + \frac{me}{d_2}x_2 + \frac{y}{k}\right)^{\theta+1} + \frac{\theta+1}{2}\left(x_1 + \frac{me}{d_2}x_2 + \frac{y}{k}\right)^{\theta} \\ &\times \left(\sigma_1^2 x_1^2 + \frac{m^2 e^2}{d_2^2}\sigma_2^2 x_2^2 + \frac{\sigma_3^2}{k^2}y^2\right) - \frac{\alpha + d_1}{2}x_1^{\theta+2} - (m-1)(1-\theta)e \end{aligned}$$

$$\times\left(\frac{me}{d_2}\right)^{\theta+1}x_2^{\theta+2}-\frac{\beta}{2k^{\theta+2}}y^{\theta+3}$$
$$\leq-\frac{\alpha+d_1}{2}x_1^{\theta+2}-(m-1)\theta e\left(\frac{me}{d_2}\right)^{\theta+1}x_2^{\theta+2}-\frac{\beta}{2k^{\theta+2}}y^{\theta+3}+A, \tag{3.4}$$

where

$$A=\sup_{(x_1,x_2,y)\in\mathbb{R}_+^3}\Bigg\{\frac{me\alpha}{d_2}x_1\left(x_1+\frac{me}{d_2}x_2+\frac{y}{k}\right)^{\theta+1}-\frac{\alpha+d_1}{2}x_1^{\theta+2}$$
$$-(m-1)(1-\theta)\left(\frac{me}{d_2}\right)^{\theta+1}\times ex_2^{\theta+2}-\frac{\beta}{2k^{\theta+2}}y^{\theta+3}$$
$$+\frac{\theta+1}{2}\left(x_1+\frac{me}{d_2}x_2+\frac{y}{k}\right)^{\theta}\left(\sigma_1^2x_1^2+\frac{m^2e^2}{d_2^2}\sigma_2^2x_2^2+\frac{\sigma_3^2}{k^2}y^2\right)\Bigg\}.$$

Next, define another function:

$$\tilde{V}(x_1,x_2,y)=MV_1(x_1,x_2,y)+V_2(x_2)+V_3(x_1,x_2,y),$$

here $M>0$ is a sufficiently large value, which satisfies

$$-M\lambda+d_2+\frac{\sigma_2^2}{2}+A\leq-2. \tag{3.5}$$

We shall observe that $\tilde{V}$ is both continuous and approaches to ∞ when the norm of (x_1,x_2,y) is close to infinity. Thus the function must exist low bound and achieve it at a point (x_1^0,x_2^0,y^0) in the interior of $\mathbb{R}_+^3$. Let

$$V(x_1,x_2,y)=MV_1(x_1,x_2,y)+V_2(x_2)+V_3(x_1,x_2,y)-\tilde{V}(x_1^0,x_2^0,y^0).$$

By (3.2), (3.3) and (3.4), we can get

$$LV\leq-M\lambda+\frac{Mka(a+b\beta)}{b^2d}x_1y-\frac{\alpha x_1}{x_2}-\frac{\alpha+d_1}{2}x_1^{\theta+2}-(m-1)\theta e\left(\frac{me}{d_2}\right)^{\theta+1}x_2^{\theta+2}$$
$$-\frac{\beta}{2k^{\theta+2}}y^{\theta+3}+d_2+\frac{\sigma_2^2}{2}+A$$
$$=-M\lambda+\frac{Mka(a+b\beta)}{b^2d}x_1y-\frac{\alpha x_1}{x_2}-\frac{\alpha+d_1}{4}x_1^{\theta+2}-(m-1)\theta e\left(\frac{me}{d_2}\right)^{\theta+1}x_2^{\theta+2}$$
$$-\frac{\beta}{4k^{\theta+2}}y^{\theta+3}-\frac{\alpha+d_1}{4}x_1^{\theta+2}-\frac{\beta}{4k^{\theta+2}}y^{\theta+3}+d_2+\frac{\sigma_2^2}{2}+A. \tag{3.6}$$

Choosing sufficiently small $0<\epsilon<1$ satisfies

$$\epsilon\leq\frac{b^2d(\theta+3)}{Mka(a+b\beta)(\theta+2)}, \tag{3.7}$$
$$\epsilon\leq\frac{\beta b^2d(\theta+3)}{4Mk^{\theta+3}a(a+b\beta)}, \tag{3.8}$$
$$\epsilon\leq\frac{b^2d(\theta+2)}{Mka(a+b\beta)(\theta+1)}, \tag{3.9}$$

$$\epsilon \leq \frac{(\alpha + d_1)b^2 d(\theta + 2)}{4Mka(a + b\beta)}, \tag{3.10}$$

$$-\frac{\alpha}{\epsilon} + G \leq -1, \tag{3.11}$$

$$-\frac{\alpha + d_1}{4\epsilon^{\theta+2}} + G \leq -1, \tag{3.12}$$

$$-\frac{\beta}{4k^{\theta+2}\epsilon^{\theta+3}} + G \leq -1, \tag{3.13}$$

$$-\frac{(m-1)m^{\theta+1}e^{\theta+2}\theta}{d_2^{\theta+1}\epsilon^{2\theta+4}} + G \leq -1, \tag{3.14}$$

where G can be presented later. To make the condition H_2 of Lemma 2.1 [16] hold, one consider the bounded open set

$$U_\epsilon = \left\{(x_1, x_2, y) \in \mathbb{R}_+^3 : \epsilon < x_1 < \frac{1}{\epsilon}, \epsilon^2 < x_2 < \frac{1}{\epsilon^2}, \epsilon < y < \frac{1}{\epsilon}\right\}.$$

Denote

$$U_1 = \left\{(x_1, x_2, y) \in \mathbb{R}_+^3 : x_1 \leq \epsilon\right\}, \quad U_2 = \left\{(x_1, x_2, y) \in \mathbb{R}_+^3 : y \leq \epsilon\right\},$$
$$U_3 = \left\{(x_1, x_2, y) \in \mathbb{R}_+^3 : x_1 > \epsilon, x_2 \leq \epsilon^2\right\}, \quad U_4 = \left\{(x_1, x_2, y) \in \mathbb{R}_+^3 : x_1 \geq \frac{1}{\epsilon}\right\},$$
$$U_5 = \left\{(x_1, x_2, y) \in \mathbb{R}_+^3 : y \geq \frac{1}{\epsilon}\right\}, \quad U_6 = \left\{(x_1, x_2, y) \in \mathbb{R}_+^3 : x_2 \geq \frac{1}{\epsilon^2}\right\}.$$

Clearly, $U_\epsilon^c = \mathbb{R}_+^3 \setminus U_\epsilon = \bigcup_{i=1}^{6} U_i$. Then validate $LV \leq -1$ for any $(x_1, x_2, y) \in U_\epsilon^c$.

Case 1: In domain U_1, according to $x_1 y \leq \epsilon y \leq \epsilon \frac{\theta+2+y^{\theta+3}}{\theta+3} = \frac{\theta+2}{\theta+3}\epsilon + \frac{\epsilon}{\theta+3} y^{\theta+3}$, we can get

$$\begin{aligned}
LV &\leq -M\lambda + \frac{Mka(a+b\beta)(\theta+2)\epsilon}{b^2 d(\theta+3)} - \left(\frac{\beta}{4k^{\theta+2}} - \frac{Mka(a+b\beta)\epsilon}{b^2 d(\theta+3)}\right) y^{\theta+3} \\
&\quad + d_2 + \frac{\sigma_2^2}{2} + A \\
&\leq -M\lambda + \frac{Mka(a+b\beta)(\theta+2)\epsilon}{b^2 d(\theta+3)} + d_2 + \frac{\sigma_2^2}{2} + A \\
&\leq -2 + 1 = -1,
\end{aligned} \tag{3.15}$$

which follows from (3.5), (3.7) and (3.8). Therefore

$$LV \leq -1 \quad \text{when} \quad (x_1, x_2, y) \in U_1.$$

Case 2: In domain U_2, since $x_1 y \leq \epsilon x_1 \leq \epsilon \frac{\theta+1+x_1^{\theta+2}}{\theta+2} = \frac{\theta+1}{\theta+2}\epsilon + \frac{\epsilon}{\theta+2} x_1^{\theta+2}$, we have

$$\begin{aligned}
LV &\leq -M\lambda + \frac{Mka(a+b\beta)(\theta+1)\epsilon}{b^2 d(\theta+2)} - \left(\frac{\alpha+d_1}{4} - \frac{Mka(a+b\beta)\epsilon}{b^2 d(\theta+2)}\right) x_1^{\theta+2} \\
&\quad + d_2 + \frac{\sigma_2^2}{2} + A \\
&\leq -M\lambda + \frac{Mka(a+b\beta)(\theta+1)\epsilon}{b^2 d(\theta+2)} + d_2 + \frac{\sigma_2^2}{2} + A
\end{aligned}$$

$$\leq -2+1=-1, \tag{3.16}$$

which follows from (3.5), (3.9) and (3.10). Consequently

$$LV \leq -1 \quad \text{when} \quad (x_1, x_2, y) \in U_2.$$

Back to (3.6), we can also get

$$\begin{aligned} LV \leq & -\frac{\alpha x_1}{x_2} - \frac{\alpha + d_1}{4} x_1^{\theta+2} - (m-1)\theta e \left(\frac{me}{d_2}\right)^{\theta+1} x_2^{\theta+2} - \frac{\beta}{4k^{\theta+2}} y^{\theta+3} - \frac{\alpha + d_1}{4} \\ & \times x_1^{\theta+2} - \frac{\beta}{4k^{\theta+2}} y^{\theta+3} + \frac{Mka(a+b\beta)}{b^2 d} x_1 y + d_2 + \frac{\sigma_2^2}{2} + A \\ \leq & -\frac{\alpha x_1}{x_2} - \frac{\alpha + d_1}{4} x_1^{\theta+2} - (m-1)\theta e \left(\frac{me}{d_2}\right)^{\theta+1} x_2^{\theta+2} - \frac{\beta}{4k^{\theta+2}} y^{\theta+3} + G, \end{aligned} \tag{3.17}$$

where

$$G = \sup_{(x_1, x_2, y) \in \mathbb{R}_+^3} \left\{ -\frac{\alpha + d_1}{4} x_1^{\theta+2} - \frac{\beta}{4k^{\theta+2}} y^{\theta+3} + \frac{Mka(a+b\beta)}{b^2 d} x_1 y + d_2 + \frac{\sigma_2^2}{2} + A \right\}.$$

Case 3: In domain U_3, considering about (3.17), one can reach

$$\begin{aligned} LV &\leq -\frac{\alpha x_1}{x_2} + G \leq -\frac{\alpha \epsilon}{\epsilon^2} + G \\ &= -\frac{\alpha}{\epsilon} + G \leq -1, \end{aligned} \tag{3.18}$$

which follows from (3.11). Hence

$$LV \leq -1 \quad \text{when} \quad (x_1, x_2, y) \in U_3.$$

Case 4: In domain U_4, due to (3.17), it follows that

$$\begin{aligned} LV &\leq -\frac{\alpha + d_1}{4} x_1^{\theta+2} + G \\ &\leq -\frac{\alpha + d_1}{4\epsilon^{\theta+2}} + G \leq -1, \end{aligned} \tag{3.19}$$

which follows from (3.12). As a result

$$LV \leq -1 \quad \text{when} \quad (x_1, x_2, y) \in U_4.$$

Case 5: In domain U_5, by (3.17), it satisfies

$$\begin{aligned} LV &\leq -\frac{\beta}{4k^{\theta+2}} y^{\theta+3} + G \\ &\leq -\frac{\beta}{4k^{\theta+2}\epsilon^{\theta+3}} + G \leq -1, \end{aligned} \tag{3.20}$$

which follows from (3.13). Wherefore

$$LV \leq -1 \quad \text{when} \quad (x_1, x_2, y) \in U_5.$$

Case 6: In domain U_6, from (3.17), the following inequalities hold

$$\begin{aligned} LV &\le -(m-1)\theta e\left(\frac{me}{d_2}\right)^{\theta+1} x_2^{\theta+2} + G \\ &\le -\frac{(m-1)m^{\theta+1}e^{\theta+2}\theta}{d_2^{\theta+1}\epsilon^{2\theta+4}} + G \le -1, \end{aligned} \tag{3.21}$$

which follows from (3.14). Accordingly

$$LV \le -1 \quad \text{when} \quad (x_1, x_2, y) \in U_6.$$

Thus, from (3.15) to (3.21), one can obviously observe that for any sufficiently small ϵ,

$$LV \le -1 \quad \text{when} \quad (x_1, x_2, y) \in \mathbb{R}_+^3 \setminus U_\epsilon.$$

That is, we finish the proof of assumption H_2 of Lemma 2.1 in [16]. As a consequence, model (1.3) admits a stationary distribution and it possesses ergodicity. □

4. Extinction of the preys and predators

In the part, we shall provide the sufficient condition for extinction of predators and preys. The concerned theorems are displayed later.

Theorem 4.1. *Let* $(x_1(t), x_2(t), y(t))$ *be the solution of model* (1.3) *with any initial value on* $\mathbb{R}_+^3$. *If* $ka < d + \frac{\sigma_3^2}{2}$, *then the predator population* y *will become extinguish.*

Proof. It can be obtained from the Itô's formula,

$$d(\ln y(t)) = \left(-d + \frac{kax_1}{b+x_1} - \beta y - \frac{\sigma_3^2}{2}\right)dt + \sigma_3 dB_3(t). \tag{4.1}$$

Integrating (4.1) from 0 to t, there is

$$\begin{aligned} \ln y(t) - \ln y(0) &= \left(-d - \frac{\sigma_3^2}{2}\right)t + ka\int_0^t \frac{x_1(s)}{b+x_1(s)}ds - \beta\int_0^t y(s)ds + \sigma_3 B_3(t) \\ &\le \left(ka - d - \frac{\sigma_3^2}{2}\right)t + \sigma_3 B_3(t). \end{aligned} \tag{4.2}$$

Dividing by t on (4.2), we shall obtain

$$\frac{\ln y(t) - \ln y(0)}{t} \le ka - d - \frac{\sigma_3^2}{2} + \frac{\sigma_3 B_3(t)}{t}. \tag{4.3}$$

Note that

$$\lim_{t\to\infty} \frac{B_3(t)}{t} = 0 \quad a.s.$$

Thus, taking the superior limit of (4.3) yields

$$\limsup_{t\to\infty} \frac{\ln y(t)}{t} \le ka - d - \frac{\sigma_3^2}{2} < 0 \quad a.s.,$$

which represents that

$$\lim_{t\to\infty} y(t) = 0 \quad a.s.$$

To put it differently, the predator population will be die out. □

Theorem 4.2. *Let* $(x_1(t), x_2(t), y(t))$ *be the solution to model* (1.3) *with any initial value on* $\mathbb{R}^3_+$*, there is*

$$\limsup_{t\to\infty} \frac{1}{t} \ln\left(\frac{\sqrt{R_1}}{\alpha+d_1}x_1(t) + \frac{R_1}{\alpha}x_2(t)\right) \le \varrho \quad a.s.,$$

in which $\varrho = \min\{\alpha+d_1, d_2\}\left(\sqrt{R_1}-1\right)\mathrm{I}_{\left\{\sqrt{R_1}\le 1\right\}} + \max\{\alpha+d_1, d_2\}\left(\sqrt{R_1}-1\right)$ $\times \mathrm{I}_{\left\{\sqrt{R_1}>1\right\}} - \left(2\left(\sigma_1^{-2}+\sigma_2^{-2}\right)\right)^{-1}$*. Particularly, while* $\varrho < 0$*, three species will die out exponentially, namely*

$$\lim_{t\to\infty} x_1(t) = 0, \quad \lim_{t\to\infty} x_2(t) = 0, \quad \lim_{t\to\infty} y(t) = 0 \quad a.s.$$

Proof. Considering Theorem 1.4 of [3], there is

$$\sqrt{R_1}(\omega_1, \omega_2) = (\omega_1, \omega_2)E_0,$$

in which $E_0 = \begin{pmatrix} 0 & \dfrac{e}{\alpha+d_1} \\ \dfrac{\alpha}{d_2} & 0 \end{pmatrix}$ and $(\omega_1, \omega_2) = \left(\sqrt{R_1}, \dfrac{e}{\alpha+d_1}\right)$.

A function can be defined as

$$V(x_1, x_2) = \gamma_1 x_1 + \gamma_2 x_2,$$

in which $\gamma_1 = \dfrac{\omega_1}{\alpha+d_1}, \gamma_2 = \dfrac{\omega_2}{d_2}$. Employing Itô's formula to $\ln V$, thus there is

$$d(\ln V) = L(\ln V)dt + \frac{1}{V}\left(\gamma_1\sigma_1 x_1 dB_1(t) + \gamma_2\sigma_2 x_2 dB_2(t)\right). \tag{4.4}$$

Here

$$L(\ln V) = \frac{\gamma_1}{V}\left(ex_2 - d_1x_1 - \alpha x_1 - \frac{ax_1}{b+x_1}y\right) + \frac{\gamma_2}{V}\left(\alpha x_1 - d_2x_2\right) - \frac{\gamma_1^2\sigma_1^2x_1^2}{2V^2} - \frac{\gamma_2^2\sigma_2^2x_2^2}{2V^2}.$$

Besides we can reach that

$$V^2 = \left(\gamma_1\sigma_1x_1\frac{1}{\sigma_1} + \gamma_2\sigma_2x_2\frac{1}{\sigma_2}\right)^2 \le (\gamma_1^2\sigma_1^2x_1^2 + \gamma_2^2\sigma_2^2x_2^2)\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right), \tag{4.5}$$

and

$$\begin{aligned}
&\frac{1}{V}\left\{\gamma_1\left[ex_2 - d_1x_1 - \alpha x_1 - \frac{ax_1}{b+x_1}y\right] + \gamma_2\left[\alpha x_1 - d_2x_2\right]\right\} \\
&\le \frac{1}{V}\left\{\frac{\omega_1}{\alpha+d_1}\left[ex_2 - (\alpha+d_1)x_1\right] + \frac{\omega_2}{d_2}\left[\alpha x_1 - d_2x_2\right]\right\} \\
&= \frac{1}{V}(\omega_1, \omega_2)\left[E_0(x_1, x_2)^T - (x_1, x_2)^T\right] \\
&= \frac{1}{V}\left(\sqrt{R_1}-1\right)(\omega_1x_1 + \omega_2x_2) \\
&= \frac{1}{V}\left(\sqrt{R_1}-1\right)\left[(\alpha+d_1)\gamma_1x_1 + d_2\gamma_2x_2\right]
\end{aligned}$$

$$\leq \min\{\alpha+d_1,d_2\}\left(\sqrt{R_1}-1\right)I_{\left\{\sqrt{R_1}\leq 1\right\}}+\max\{\alpha+d_1,d_2\}$$
$$\times\left(\sqrt{R_1}-1\right)I_{\left\{\sqrt{R_1}>1\right\}}. \tag{4.6}$$

Combining (4.5) with (4.6), one is able to obtain

$$L(\ln V)\leq \min\{\alpha+d_1,d_2\}\left(\sqrt{R_1}-1\right)I_{\left\{\sqrt{R_1}\leq 1\right\}}+\max\{\alpha+d_1,d_2\}\left(\sqrt{R_1}-1\right)$$
$$\times I_{\left\{\sqrt{R_1}>1\right\}}-\left(2\left(\sigma_1^{-2}+\sigma_2^{-2}\right)\right)^{-1}.$$

Thence,

$$d(\ln V)\leq [\min\{\alpha+d_1,d_2\}\left(\sqrt{R_1}-1\right)I_{\left\{\sqrt{R_1}\leq 1\right\}}+\max\{\alpha+d_1,d_2\}\left(\sqrt{R_1}-1\right)$$
$$\times I_{\left\{\sqrt{R_1}>1\right\}}-\left(2\left(\sigma_1^{-2}+\sigma_2^{-2}\right)\right)^{-1}]dt+\frac{\gamma_1\sigma_1x_1}{V}dB_1(t)$$
$$+\frac{\gamma_2\sigma_2x_2}{V}dB_2(t). \tag{4.7}$$

Integrating and then dividing by t, we shall gain

$$\frac{\ln V(t)}{t}\leq\frac{\ln V(0)}{t}+\min\{\alpha+d_1,d_2\}\left(\sqrt{R_1}-1\right)I_{\left\{\sqrt{R_1}\leq 1\right\}}+\max\{\alpha+d_1,d_2\}$$
$$\times\left(\sqrt{R_1}-1\right)I_{\left\{\sqrt{R_1}>1\right\}}-\left(2\left(\sigma_1^{-2}+\sigma_2^{-2}\right)\right)^{-1}+\frac{M_1(t)}{t}+\frac{M_2(t)}{t}. \tag{4.8}$$

Here $M_i(t)=\int_0^t\frac{\gamma_i\sigma_ix_i(s)}{V(s)}dB_i(s)(i=1,2)$ is local martingale and $\langle M_i,M_i\rangle_t=\sigma_i^2\int_0^t(\frac{\gamma_ix_i(s)}{V(s)})^2ds\leq\sigma_i^2t$. Taking advantage of the strong law of large numbers for local martingales [18] yields

$$\lim_{t\to\infty}\frac{M_i(t)}{t}=0\ \ a.s. \tag{4.9}$$

Taking the superior limit on both sides of (4.8) and combining with (4.9) lead to

$$\limsup_{t\to\infty}\frac{\ln V(t)}{t}\leq\min\{\alpha+d_1,d_2\}\left(\sqrt{R_1}-1\right)I_{\left\{\sqrt{R_1}\leq 1\right\}}+\max\{\alpha+d_1,d_2\}$$
$$\times\left(\sqrt{R_1}-1\right)I_{\left\{\sqrt{R_1}>1\right\}}-\left(2\left(\sigma_1^{-2}+\sigma_2^{-2}\right)\right)^{-1}$$
$$:=\varrho.$$

Consequently,

$$\limsup_{t\to\infty}\frac{1}{t}\ln\left(\frac{\sqrt{R_1}}{\alpha+d_1}x_1(t)+\frac{R_1}{\alpha}x_2(t)\right)\leq\varrho\ \ a.s.$$

Moreover, while $\varrho<0$, we shall readily get

$$\limsup_{t\to\infty}\frac{\ln x_1(t)}{t}<0\ \text{ and }\ \limsup_{t\to\infty}\frac{\ln x_2(t)}{t}<0\ \ a.s.,$$

which signifies that

$$\lim_{t\to\infty}x_1(t)=0,\quad \lim_{t\to\infty}x_2(t)=0\ \ a.s.$$

Therefore, existing t_0 such that when $t \geq t_0$, $\frac{x_1}{b+x_1} \leq \frac{\epsilon}{b+\epsilon}$ *a.s.*

Meanwhile, applying Itô's formula to $\ln y$, we have

$$\begin{aligned} d\ln y &= \left(-d - \beta y + \frac{kax_1}{b+x_1} - \frac{\sigma_3^2}{2}\right)dt + \sigma_3 dB_3(t) \\ &\leq \left(-d - \frac{\sigma_3^2}{2} + \frac{kax_1}{b+x_1}\right)dt + \sigma_3 dB_3(t). \end{aligned} \tag{4.10}$$

Integrating and then dividing t on both sides of (4.10) yield

$$\begin{aligned} \frac{\ln y(t) - \ln y(0)}{t} &\leq -\left(d + \frac{\sigma_3^2}{2}\right) + \frac{ka}{t}\int_0^t \frac{x_1(s)}{b+x_1(s)}ds + \frac{\sigma_3 dB_3(t)}{t} \\ &\leq -\left(d + \frac{\sigma_3^2}{2}\right) + \frac{ka\epsilon}{b+\epsilon} + \frac{\sigma_3 dB_3(t)}{t}. \end{aligned} \tag{4.11}$$

Note that

$$\lim_{t\to\infty} \frac{B_3(t)}{t} = 0 \quad a.s.$$

Hence, the following conclusion is drawn

$$\limsup_{t\to\infty} \frac{\ln y(t)}{t} \leq -\left(d + \frac{\sigma_3^2}{2}\right) + \frac{ka\epsilon}{b+\epsilon}.$$

Letting $\epsilon \to 0$ results in

$$\limsup_{t\to\infty} \frac{\ln y(t)}{t} \leq -\left(d + \frac{\sigma_3^2}{2}\right) < 0 \quad a.s.$$

which means

$$\lim_{t\to\infty} y(t) = 0 \quad a.s.$$

To be specific, all the populations die out exponentially with probability one. □

5. Conclusions and numerical simulations

In this paper, Holling type II scheme has been used to model the stochastic predator-prey model with phase structure of prey. For the corresponding deterministic model, if $R_1 < 1$, the extinction equilibrium point $P_0(0,0,0)$ of the system is locally asymptotically stable. For the random system (1.3), while $R_0^S > 1$, the system will persist for a long time and there is a unique ergodic stationary distribution. Theorem 4.1 and Theorem 4.2 also provide new criteria for judging extinction of the model (1.3).

To make the theoretical analysis more convincing, mathematical software is utilized for numerical simulations. Using Milstein's Higher-Order method mentioned in [11] to get the discretization equations of model (1.3):

$$\begin{cases} x_1^{j+1} = x_1^j + \left(e x_2^j - d_1 x_1^j - \alpha x_1^j - \dfrac{a x_1^j}{b + x_1^j} y^j \right) \Delta t + \sigma_1 x_1^j \sqrt{\Delta t} \xi_{1,j} \\ \qquad\qquad + \dfrac{\sigma_1^2}{2} x_1^j (\xi_{1,j}^2 - 1) \Delta t, \\ x_2^{j+1} = x_2^j + \left(\alpha x_1^j - d_2 x_2^j \right) \Delta t + \sigma_2 x_2^j \sqrt{\Delta t} \xi_{2,j} + \dfrac{\sigma_2^2}{2} x_2^j (\xi_{2,j}^2 - 1) \Delta t, \\ y^{j+1} = y^j + y^j \left(-d + k \dfrac{a x_1^j}{b + x_1^j} - \beta y^j \right) \Delta t + \sigma_3 y^j \sqrt{\Delta t} \xi_{3,j} + \dfrac{\sigma_3^2}{2} y^j (\xi_{3,j}^2 - 1) \Delta t, \end{cases} \tag{5.1}$$

where $\xi_{i,j}, i = 1, 2, 3$ are the Gaussian random variables which follow the distribution $N(0, 1)$. Choose related parameters:

$$e = 0.5, d_1 = 0.1, \alpha = 0.2, a = 0.6, b = 0.07, d_2 = 0.05, d = 0.2, k = 1, \beta = 0.1.$$

Example 5.1. To get the existence of an ergodic stationary distribution numerically, choosing $\sigma_1^2 = 0.004, \sigma_2^2 = 0.01, \sigma_3^2 = 0.01$ and other parameters are as above. Compute and obtain

$$R_0^S = \frac{\alpha e}{\left(d_2 + \frac{\sigma_2^2}{2}\right)\left(\alpha + d_1 + \frac{\sigma_1^2}{2} + d + \frac{\sigma_3^2}{2}\right)} \approx 3.586 > 1.$$

Namely, the condition of Theorem 3.1 is true. It can be concluded that model (1.3) admits a unique ergodic stationary distribution. In other words, three species present a state of coexistence for a long time. Fig.1 confirms this and provides the corresponding phase diagram.

Example 5.2. To illustrate the extinction of the predator population, choosing $\sigma_1^2 = 0.45, \sigma_2^2 = 1.5, \sigma_3^2 = 8$ and the other parameters are the same as in Fig.1. Calculate and note

$$ka = 0.6 < 4.2 = d + \frac{\sigma_3^2}{2}.$$

Therefore, the condition of Theorem 4.1 holds. According to Theorem 4.1, one can obtain that the predator population will be die out. Fig.2 demonstrates this and gives the solution of undisturbed model (1.2) and the corresponding phase diagram.

Example 5.3. To verify the conclusion of theorem 4.2, choosing $\sigma_1^2 = 9.2, \sigma_2^2 = 2, \sigma_3^2 = 9$ and the other parameters are the same as in Fig.1. After calculations, we can get

$$R_1 = \frac{\alpha e}{d_2(\alpha + d_1)} \approx 6.67 > 1,$$

and

$$\begin{aligned} \varrho &= \max\{\alpha + d_1, d_2\}\left(\sqrt{R_1} - 1\right) - \left(2\left(\sigma_1^{-2} + \sigma_2^{-2}\right)\right)^{-1} \\ &\approx 0.475 - 1.910 = -1.435 < 0. \end{aligned}$$

In view of Theorem 4.2, we can obtain that the prey and predator populations become extinct. Fig. 3 displays this. It can be seen that large environmental interference is not conducive to the survival of population system.

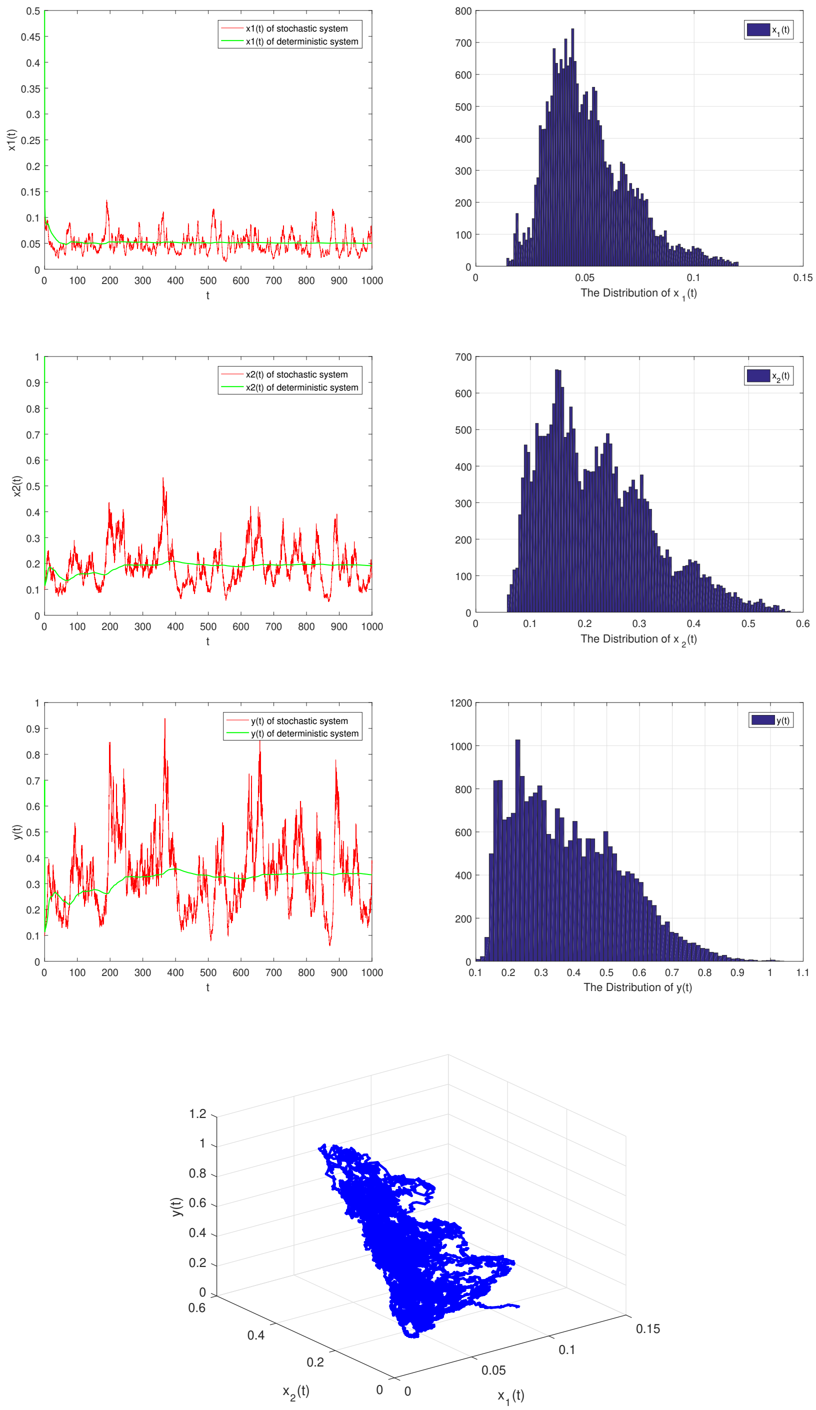

Figure 1. Three pictures in the left column are the paths of x_1, x_2, y of system (1.3) with the initial value $(0.1, 0.1, 0.1)$ under the noise intensities $\sigma_1^2 = 0.004, \sigma_2^2 = 0.01, \sigma_3^2 = 0.01$. The red lines mean the solution of (1.3) and the green lines mean the solution of the corresponding undisturbed system (1.2). Three in the right column show the histograms of the probability density functions of x_1, x_2, y. The seventh is the corresponding phase diagram, revealing the interplay of the three populations in phase space.

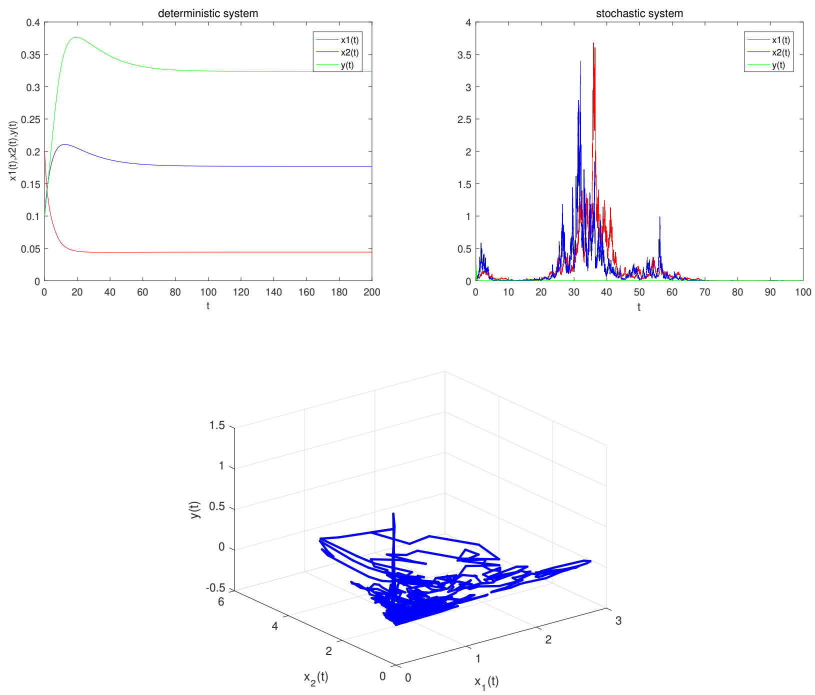

Figure 2. The left figure above shows the paths of x_1, x_2, y of the deterministic system (1.2) with initial value $(0.1, 0.1, 0.1)$. The right above is the paths of the corresponding stochastic system (1.3) under $\sigma_1^2 = 0.45, \sigma_2^2 = 1.5, \sigma_3^2 = 8$. The third is the corresponding phase diagram, which displays the interplay of the three populations in phase space.

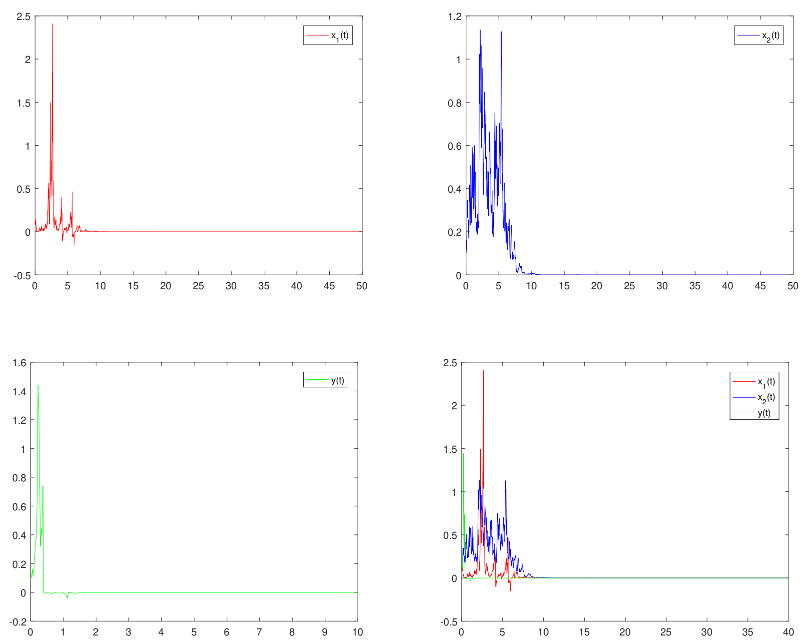

Figure 3. The paths of x_1, x_2, y of system (1.3) with the initial value $(0.1, 0.1, 0.1)$ under $\sigma_1^2 = 9.2, \sigma_2^2 = 2, \sigma_3^2 = 9$, which shows that preys and predators are extinguish.

This paper makes a concrete analysis about how environmental noise affects the survival of population. We give different criteria for species coexistence and extinction. The results display that when the noise intensity of the stochastic model is weak, the survival state of each species is not affected greatly, and they can coexist in the same ecological environment; as the noise intensity continues to increase and reaches a critical value, the population will be completely extinct. The above analysis has practical guiding significance for biological control.

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