Journal of Applied Analysis and Computation
Volume 12, Number 2, April 2022, 790–806
Website:http://www.jaac-online.com
DOI:10.11948/20210357

ON NONLINEAR EVOLUTION MODEL FOR DRINKING BEHAVIOR UNDER CAPUTO-FABRIZIO DERIVATIVE

Fang Jin[1], Zi-Shan Qian[2], Yu-Ming Chu[3,4] and Mati ur Rahman[5,†]

Abstract The investigation of this research article is the development of studying the dynamical behavior of the drinking population through the fractional drinking model in the sense of Caputo-Fabrizio (CF) arbitrary order operator along with the special non-singular kernel. The proposed system is analyzed for existence result and uniqueness of solution by applying fixed point theory and Picard's technique. Also on utilizing Adams-Bashforth method (ABM) of numerical analysis to interpret the approximate results through plots to observe dynamical behavior corresponding to different fractional order. For the mentioned simulation some real initial and parameter data are used.

Keywords Mathematical model of drinking, Picard's analysis, Caputo-Fabrizio operator, numerical simulation, Adams-Bashforth Method.

MSC(2010) 26A33, 34A08, 65L07.

1. Introduction

Various mathematical expressions and formulae have been tested for the dealing of infectious diseases like the transmission of the epidemic, endemic and pandemic [12,21]. These models have also been used for the last few decades to describe various habits of the society, like drinking of alcohol and swine, habits of smoking, cocaine, obesity, corruption, cooperation, ideological conflicts, dynamics of tax evasion and radicalization phenomena, see [13,16,22,23,39,41]. The most important dealings in this regard of Mathematical modeling and expression are of transmission of alcohol taking and its tendency towards the population of different societies [24,35].

By Medical and biological aspects, drinking of alcohol and its taking [45] are divided into the groups of three different individuals, non-consumer (not taking any alcohol), seldom or the moderate user (taking alcohol for some time and not

†The corresponding author. Email: mati-maths_374@sjtu.edu.cn(M. ur Rahman)
1College of Science, Hunan City University, Yiyang 413000, China
2School of Physical Sciences, University of California, Irvine CA 92617, USA
3Department of Mathematics, Huzhou University, Huzhou 313000, China
4Institute for Advanced Study Honoring Chen Jian Gong, Hangzhou Normal University, Hangzhou 311121, China
5School of Mathematical Sciences, Shanghai Jiao Tong University, Shanghai, China

every time) and the last group is of high or risk users (taking a high quantity of alcohol and every time). So, expressing communication and ensuing deviation mathematically by making assumptions about the parameters and the population of the society. The most beneficial and important term in formulating the drinking of alcohol consumption by assuming that the people of society slightly increase the rate of taking alcohol. This is not because of various social-gathering but due to under forceful or depressing situations. The study also shows that consumption of alcohol plays a fundamental role in depression, see [40]. This implies that a moderate drinker becomes a high quantity drinker which may be related to the drinker's population density and the two individuals are involved to change it. If anyone takes the recent globe situation because of the COVID-19, then the aforementioned self-induced growing of alcohol consumption is not only up to date but also very well see [17, 18].

Mathematical models are powerful tools in this approach that help us to investigate such infectious type disease models more accurately. The drinking behavior evolution model was studied by Nuno Crokidakis and Lucas Sigaud [19] and presented as

$$\begin{cases} \dfrac{d\mathcal{S}(t)}{dt} = -\zeta\mathcal{S}(t)\mathcal{M}(t) - \zeta\mathcal{S}(t)\mathcal{R}(t) + \rho\mathcal{S}(t)\mathcal{R}(t), \\ \dfrac{d\mathcal{M}(t)}{dt} = \zeta\mathcal{S}(t)\mathcal{M}(t) + \zeta\mathcal{S}(t)\mathcal{R}(t) - \eta\mathcal{M}(t)\mathcal{R}(t) - \mu\mathcal{M}(t), \\ \dfrac{d\mathcal{R}(t)}{dt} = \mu\mathcal{M}(t) + \eta\mathcal{M}(t)\mathcal{R}(t) - \rho\mathcal{S}(t)\mathcal{R}(t), \end{cases} \tag{1.1}$$

where the details of the used parameters are given below:

- $\mathcal{S}(t)$ (non-consumer population): People who have never taken alcohol or have used it in the past time and then become alter of this habits. In such a situation, we named these populations as Susceptible class or population, i.e., susceptible to drinking again or only at this time;
- $\mathcal{M}(t)$ (non-risk population): Population with continuous but with less quantity usage. We named it Moderated takers;
- $\mathcal{R}(t)$ (risk Population): Population with continuous more quantity usage. We named it Risk-takers;
- ζ: shows an "infection" chance, i.e., the chance that a user ($\mathcal{M}(t)$ or $\mathcal{R}(t)$) classes turn a non-consumer population into the consumers of drinking;
- η: represents the chance of the moderate takers $\mathcal{M}$ into risk drinkers $\mathcal{R}$;
- μ: The rate of infection from $\mathcal{M} \to \mathcal{R}$.
- ρ: Shows the chance of transferring the risk drinkers $\mathcal{R}$ into non-consumer population $\mathcal{S}$.

The area of fractional calculus has attained extreme attention in the last three decades. Enormous scientists have provided their contribution in this aspect by introducing different fractional operators in different articles [25, 30]. Modern calculus provides more realistic result as compared to classical calculus. It describes the dynamics of different real-world phenomena lying between two integers. Further, the fractional operators have more degree of freedom and it generalizes the integer differential operators. Up to now, various researchers have published more research articles, books and different monographs which touch the said area. The dynamics of real-world phenomena for classical models of differentiation and integration have been investigated through the modern approach of arbitrary-order by different systematic approaches [37].

Various techniques have been applied to discuss such modern derivatives, e.g., the techniques applied by Akbari *et al.* [26] and Talaee *et al.* [42]. Several other significant uses of the modern differential equation may be found in the famous research work published in [14, 33]. In this manuscript, we have used the new fractional CFD [15] to investigate the mathematical modeling for simulating the transmission of drinking model (1.1). Currently, several researcher have studied and published related work to the fractional CFD, see, [1, 2, 8, 10, 11, 20, 27, 30, 36, 43]. The notion of CFD [15] has been extensively used to investigate treated cancer problems [3], the dynamical behavior of hepatitis B and E [28, 29], effecting cancer cell by immune system cells [32], the dynamical behavior of TB [44] and the epidemic dengue [38]. This derivative with fractional order is applied for the investigation of different iterative solutions of various physical phenomena, as given in [6,7,9,31,34]. The stability, existence and uniqueness of solutions for these modern mathematical models dealing with the dynamical behavior of HBV infectious epidemic have been cited in [28, 38]. Up to now, the novel CF fractional operator is not used to deal with the analysis of drinking dynamical behavior.

Motivated from the above-mentioned literature, in the current work, we use the novel fractional-order CFD to examine the drinking system in [19] for numerical simulations and qualitative analysis. We reconsider the model described in (1.1) under CFD of fractional order $\gamma \in (0, 1)$ as

$$\begin{cases} {}^{CF}D_t^{\gamma}\mathcal{S}(t) = -\zeta\mathcal{S}(t)\mathcal{M}(t) - \zeta\mathcal{S}(t)\mathcal{R}(t) + \rho\mathcal{S}(t)\mathcal{R}(t), \\ {}^{CF}D_t^{\gamma}\mathcal{M}(t) = \zeta\mathcal{S}(t)\mathcal{M}(t) + \zeta\mathcal{S}(t)\mathcal{R}(t) - \eta\mathcal{M}(t)\mathcal{R}(t) - \mu\mathcal{M}(t), \\ {}^{CF}D_t^{\gamma}\mathcal{R}(t) = \mu\mathcal{M}(t) + \eta\mathcal{M}(t)\mathcal{R}(t) - \rho\mathcal{S}(t)\mathcal{R}(t), \end{cases} \tag{1.2}$$

along with the initial conditions, $\mathcal{S}(0) = \mathcal{S}_0$, $\mathcal{M}(0) = \mathcal{M}_0$, $\mathcal{R}(0) = \mathcal{R}_0$. In this paper, we explore an existence theory for the system (1.2) using a fixed point theory to ensure that the considered model has at least one solution. Also, we utilize the ABM to derive the general procedure of solution to the model (1.2) under the CFD and traditional Caputo derivative. For the novelty of the paper we add that fractional order mathematical model has been investigated for different fractional order derivatives having more information as compared to integer order model. It provides the total density for each compartment lying between 0 and 1. The qualitative analysis of the proposed problem is derived with the help of fixed point theory.

The structure of this manuscript is as follows. In Section 2, we present some basic definitions and notation from the fractional calculus. In Section 3, we explain our main work and discuss the existence, uniqueness of the solution and stability result for the proposed model. By using a well know Adams-Bashforth method for the approximate solution for the considered system. In Section 4, we perform the numerical simulation by using the initial condition and the date available in the table. Finally, we conclude our work in Section 5.

2. Preliminaries

Let $H^1[0,t] = \{f : f \in L^2[0,T] \text{ and } f' \in L^2[0,T]\}$,where $L^2[0,T]$ is the space of square integrable functions on the $[0,T]$. For the sake of simplicity let us represent the exponential kernel as $K(t,\varrho) = \exp\left[-\gamma\frac{t-\varrho}{1-\gamma}\right]$.

Definition 2.1 ([4]). If $\zeta(t) \in H^1[0,T], T > 0, \gamma \in (0,1)$,then the CFD of $\zeta(t)$ is defined as:

$$^{CF}D_t^{\gamma}\left[\zeta(t)\right] = \frac{M(\gamma)}{1-\gamma}\int_0^t \zeta^{'}(\varrho)K(t,\varrho)d\varrho,$$

$M(\gamma)$ represent normalization function such that $M(1) = M(0) = 1$. However if $\zeta(t) \notin H^1[0,T]$, then one has

$$^{CF}D_t^{\gamma}\left[\zeta(t)\right] = \frac{M(\gamma)}{1-\gamma}\int_0^t \left[\zeta(t) - \zeta(\varrho)\right]K(t,\varrho)d\varrho.$$

Definition 2.2 ([15]). The Caputo-Fabrizio fractional integral of $\zeta(t)$ is presented as:

$$^{CF}I_t^{\gamma}\left[\zeta(t)\right] = \frac{1-\gamma}{M(\gamma)}\zeta(t) + \frac{\gamma}{M(\gamma)}\int_0^t \zeta(\varrho)d\varrho, \quad t \geq 0,\ \gamma \in (0,1). \tag{2.1}$$

Definition 2.3 ([15]). Taking $M(\gamma) = 1$, we define general formula for Laplace transform of CFD as:

$$\begin{aligned}\mathcal{L}\left\{^{CF}D_t^{\gamma+M}\left[\zeta(t)\right]\right\} &= \frac{1}{1-\gamma}\mathcal{L}\left[\zeta^{(h+\gamma)}(t)\right]\mathcal{L}\left[\exp\left(\frac{-\gamma t}{1-\gamma}\right)\right],\\ &= \frac{1}{\nu+\gamma(1-\nu)}\left[\nu^{h+1}\mathcal{L}\left[\zeta(t)\right] + \sum_{i=0}^{h}\nu^{h-i}\zeta^{(i)}(0)\right].\end{aligned} \tag{2.2}$$

One can be obtain the following results for $h = 0, 1$ respectively

$$\begin{aligned}\mathcal{L}\left[^{CF}D_t^{\gamma}\left[\zeta(t)\right]\right] &= \frac{\nu\mathcal{L}\left[\zeta(t)\right]}{\nu+\gamma(1-\nu)},\\ \mathcal{L}\left[^{CF}D_t^{\gamma+1}\left[\zeta(t)\right]\right] &= \frac{\nu\mathcal{L}\left[\zeta(t)\right] + \nu\zeta(0) - \zeta^{'}(0)}{\nu+\gamma(1-\nu)},\end{aligned}$$

Definition 2.4 ([5]). The Laplace transform of $^{C}D_t^{\gamma}\left[\zeta(t)\right]$ is define as:

$$\mathcal{L}\left[^{C}D_t^{\gamma}\left[\zeta(t)\right]\right] = \nu^{\gamma}\mathcal{L}\left[\zeta(t)\right] - \sum_{i=0}^{k-1}\nu^{k-i-1}\zeta^{(i)}(0).$$

3. Main work

Here, Picard-Lindelof and the fixed-point approach is used for the existence and uniqueness of the solution to the proposed model. Also, the stability of the suggested model has been proven by using the aforesaid tools. The numerical results are constructed by using three steps ABM. The numerical solutions are provided for the different compartments in different fractional order.

3.1. Existence and uniqueness results

Through fixed point approach we derive require results as:

$$\begin{aligned}
&\Xi_1\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right)=-\zeta\mathcal{S}(t)\mathcal{M}(t)-\zeta\mathcal{S}(t)\mathcal{R}(t)+\rho\mathcal{S}(t)\mathcal{R}(t),\\
&\Xi_2\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right)=\zeta\mathcal{S}(t)\mathcal{M}(t)+\zeta\mathcal{S}(t)\mathcal{R}(t)-\eta\mathcal{M}(t)\mathcal{R}(t)-\mu\mathcal{M}(t),\\
&\Xi_3\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right)=\mu\mathcal{M}(t)+\eta\mathcal{M}(t)\mathcal{R}(t)-\rho\mathcal{S}(t)\mathcal{R}(t),
\end{aligned}$$

so the system (1.2) becomes

$$\begin{cases}
{}^{CF}D_t^{\gamma}\mathcal{S}(t)=\Xi_1\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right),\\
{}^{CF}D_t^{\gamma}\mathcal{M}(t)=\Xi_2\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right),\\
{}^{CF}D_t^{\gamma}\mathcal{R}(t)=\Xi_3\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right),
\end{cases}\tag{3.1}$$

let

$$\eta_n=\sup_{C[d,b_n]}\left\|\Xi_n\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right)\right\|,\qquad \text{for } n=1,2,3,$$

with

$$C\left[d,b_n\right]=[t-d,t+d]\times[u-c_n,u+c_k]=D\times D_n,\quad \text{for } n=1,2,3.$$

Assume a uniform norm on $C\left[d,b_n\right]$, for $k=1,2,3$ as follows:

$$\|\mathscr{W}\|_{\infty}=\sup_{t\in[t-d,t+d]}|\mathscr{W}(t)|.\tag{3.2}$$

Applying ${}^{CF}I_t^{\gamma}$ on both sides of (3.1), we have

$$\begin{cases}
\mathcal{S}(t)-\mathcal{S}(0)={}^{CF}I_t^{\gamma}\Xi_1\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right),\\
\mathcal{M}(t)-\mathcal{M}(0)={}^{CF}I_t^{\gamma}\Xi_2\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right),\\
\mathcal{R}(t)-\mathcal{R}(0)={}^{CF}I_t^{\gamma}\Xi_3\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right),
\end{cases}\tag{3.3}$$

$$\begin{cases}
\mathcal{S}(t)=\mathcal{S}(0)+\dfrac{1-\gamma}{M(\gamma)}\left[\Xi_1\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right)-\Xi_1\left(0,\mathcal{S}(0),\mathcal{M}(0),\mathcal{R}(0)\right)\right]\\
\qquad+\dfrac{\gamma}{M(\gamma)}\displaystyle\int_0^t\Xi_1\left(\varrho,\mathcal{S},\mathcal{M},\mathcal{R}\right)d\varrho,\\
\mathcal{M}(t)=\mathcal{M}(0)+\dfrac{1-\gamma}{M(\gamma)}\left[\Xi_2\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right)-\Xi_2\left(0,\mathcal{S}(0),\mathcal{M}(0),\mathcal{R}(0)\right)\right]\\
\qquad+\dfrac{\gamma}{M(\gamma)}\displaystyle\int_0^t\Xi_2\left(\varrho,\mathcal{S},\mathcal{M},\mathcal{R}\right)d\varrho,\\
\mathcal{R}(t)=\mathcal{R}(0)+\dfrac{1-\gamma}{M(\gamma)}\left[\Xi_3\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right)-\Xi_3\left(0,\mathcal{S}(0),\mathcal{M}(0),\mathcal{R}(0)\right)\right]\\
\qquad+\dfrac{\gamma}{M(\gamma)}\displaystyle\int_0^t\Xi_3\left(\varrho,\mathcal{S},\mathcal{M},\mathcal{R}\right)d\varrho.
\end{cases}\tag{3.4}$$

Define the Picard operator

$$\mathit{\Pi}:C\left(\mathcal{P},\mathcal{P}_1,\mathcal{P}_2,\mathcal{P}_3\right)\to C\left(\mathcal{P},\mathcal{P}_1,\mathcal{P}_2,\mathcal{P}_3\right),\tag{3.5}$$

given as follows:

$$\varPi\left(\mathscr{W}(t)\right)=\mathscr{W}_0(t)+\left[\varPsi\left(t,\mathscr{W}(t)\right)-\varPsi_0(t)\right]\frac{1-\gamma}{M(\gamma)}+\frac{\gamma}{M(\gamma)}\int_0^t\varPsi\left(\varrho,\mathscr{W}(\varrho)\right)d\varrho, \quad (3.6)$$

where

$$\mathscr{W}(t)=\begin{cases}\mathcal{S}(t),\\ \mathcal{M}(t),\\ \mathcal{R}(t),\end{cases}\qquad \mathscr{W}_0(t)=\begin{cases}\mathcal{S}(0),\\ \mathcal{M}(0),\\ \mathcal{R}(0),\end{cases}$$

$$\varPsi\left(t,\mathscr{W}(t)\right)=\begin{cases}\Xi_1\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right),\\ \Xi_2\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right),\\ \Xi_3\left(t,\mathcal{S},\mathcal{M},\mathcal{R}\right),\end{cases}\qquad \varPsi_0(t)=\begin{cases}\Xi_1\left(0,\mathcal{S}(0),\mathcal{M}(0),\mathcal{R}(0)\right),\\ \Xi_2\left(0,\mathcal{S}(0),\mathcal{M}(0),\mathcal{R}(0)\right),\\ \Xi_3\left(0,\mathcal{S}(0),\mathcal{M}(0),\mathcal{R}(0)\right).\end{cases}$$

Assume that the proposed model obeys:

$$\|\mathscr{W}(t)\|_\infty\le\max\{d_1,d_2,d_3\}. \quad (3.7)$$

Let $\eta=\max\{\eta_1,\eta_2,\eta_3\}$ and there exists $t_0=\max\{t\in D\}$, one gets

$$\begin{aligned}\|\varPi\mathscr{W}-\mathscr{W}_0\|&=\max_{t\in[0,T]}\left|\varPsi\left(t,\mathscr{W}(t)\right)\frac{(1-\gamma)}{M(\gamma)}+\frac{\gamma}{M(\gamma)}\int_0^t\varPsi\left(\varrho,\mathscr{W}(\varrho)\right)d\varrho\right|,\\ &\le\frac{(1-\gamma)}{M(\gamma)}\max_{t\in[0,T]}\left|\varPsi\left(t,\mathscr{W}(t)\right)\right|+\frac{\gamma}{M(\gamma)}\max_{t\in[0,T]}\int_0^t\left|\varPsi\left(\varrho,\mathscr{W}(\varrho)\right)\right|d\varrho,\\ &\le\frac{(1-\gamma)}{M(\gamma)}\eta+\frac{\gamma}{M(\gamma)}t_0\eta,\\ &\le d\eta\le\max\{d_1,d_2,d_3\}=d',\end{aligned}$$

where $d=\frac{1+\gamma t_0}{M(\gamma)}$, and satisfies $d<\frac{d'}{\eta}$. Also for given relation

$$\|\varPi\mathscr{W}_1-\varPi\mathscr{W}_2\|=\sup_{t\in D}\left|\mathscr{W}_1(t)-\mathscr{W}_2(t)\right|, \quad (3.8)$$

using definition of Picard operator yields

$$\begin{aligned}\|\varPi\mathscr{W}_1-\varPi\mathscr{W}_2\|&=\max_{t\in[0,T]}\left|\frac{(1-\gamma)}{M(\gamma)}\left[\varPsi\left(t,\mathscr{W}_1(t)\right)-\varPsi\left(t,\mathscr{W}_2(t)\right)\right]\right.\\ &\qquad\left.+\frac{\gamma}{M(\gamma)}\int_0^t\left[\varPsi\left(\varrho,\mathscr{W}_1(\varrho)\right)-\varPsi\left(\varrho,\mathscr{W}_2(\varrho)\right)\right]d\varrho\right|,\\ &\le\frac{(1-\gamma)}{M(\gamma)}\max_{t\in[0,T]}\vartheta\left|\mathscr{W}_1(t)-\mathscr{W}_2(t)\right|\\ &\qquad+\frac{\gamma\vartheta}{M(\gamma)}\max_{t\in[0,T]}\int_0^t\left|\mathscr{W}_1(\varrho)-\mathscr{W}_2(\varrho)\right|d\varrho,\\ &\le\left\{\frac{(1-\gamma)}{M(\gamma)}\vartheta+\frac{\gamma\vartheta t_0}{M(\gamma)}\right\}\|\mathscr{W}_1-\mathscr{W}_2\|,\\ &\le d\vartheta\|\mathscr{W}_1-\mathscr{W}_2\|,\end{aligned}$$

with $\vartheta<1$. For $\varPi$ to fulfill contraction condition we must have $d\vartheta<1$. Thus the Picard operator $\varPi$ obeys the contraction condition. Therefore, the consider model has a unique solution.

3.2. Stability analysis

Here, we will explore the stability of the Picard iteration by using fixed point theory. On applying ${}^{CF}I_t^{\gamma}$ to both sides of (1.2), we obtain

$$\begin{cases} \mathcal{S}(t)-\mathcal{S}(0)=\dfrac{1-\gamma}{M(\gamma)}\Big[-\zeta\mathcal{S}\mathcal{M}-\zeta\mathcal{S}\mathcal{R}+\rho\mathcal{S}\mathcal{R}\Big] \\ \qquad +\dfrac{\gamma}{M(\gamma)}\displaystyle\int_0^t\big[-\zeta\mathcal{S}(\varrho)\mathcal{M}(\varrho)-\zeta\mathcal{S}(\varrho)\mathcal{R}(\varrho)+\rho\mathcal{S}(\varrho)\mathcal{R}(\varrho)\big]d\varrho, \\ \mathcal{M}(t)-\mathcal{M}(0)=\dfrac{1-\gamma}{M(\gamma)}\Big[\zeta\mathcal{S}\mathcal{M}+\zeta\mathcal{S}\mathcal{R}-\eta\mathcal{M}\mathcal{R}-\mu\mathcal{M}\Big] \\ \qquad +\dfrac{\gamma}{M(\gamma)}\displaystyle\int_0^t\big[\zeta\mathcal{S}(\varrho)\mathcal{M}(\varrho)+\zeta\mathcal{S}(\varrho)\mathcal{R}(\varrho)-\eta\mathcal{M}(\varrho)\mathcal{R}(\varrho)-\mu\mathcal{M}(\varrho)\big]d\varrho, \\ \mathcal{R}(t)-\mathcal{R}(0)=\dfrac{1-\gamma}{M(\gamma)}\Big[\mu\mathcal{M}+\eta\mathcal{M}\mathcal{R}-\rho\mathcal{S}\mathcal{R}\Big] \\ \qquad +\dfrac{\gamma}{M(\gamma)}\displaystyle\int_0^t\big[\mu\mathcal{M}(\varrho)+\eta\mathcal{M}(\varrho)\mathcal{R}(\varrho)-\rho\mathcal{S}(\varrho)\mathcal{R}(\varrho)\big]d\varrho. \end{cases} \tag{3.9}$$

Let $\mathcal{S}_0(t)=\mathcal{S}(0)$, $\mathcal{M}_0(t)=\mathcal{M}(0)$ and $\mathcal{R}_0(t)=\mathcal{R}(0)$, then the Picard iteration is defined as:

$$\begin{cases} \mathcal{S}_{i+1}(t)=\dfrac{1-\gamma}{M(\gamma)}\Big[-\zeta\mathcal{S}_i\mathcal{M}_i-\zeta\mathcal{S}_i\mathcal{R}_i+\rho\mathcal{S}_i\mathcal{R}_i\Big] \\ \qquad +\dfrac{\gamma}{M(\gamma)}\displaystyle\int_0^t\big[-\zeta\mathcal{S}_i(\varrho)\mathcal{M}_i(\varrho)-\zeta\mathcal{S}_i(\varrho)\mathcal{R}_i(\varrho)+\rho\mathcal{S}_i(\varrho)\mathcal{R}_i(\varrho)\big]d\varrho, \\ \mathcal{M}_{i+1}(t)=\dfrac{1-\gamma}{M(\gamma)}\Big[\zeta\mathcal{S}_i\mathcal{M}_i+\zeta\mathcal{S}_i\mathcal{R}_i-\eta\mathcal{M}_i\mathcal{R}_i-\mu\mathcal{M}_i\Big] \\ \qquad +\dfrac{\gamma}{M(\gamma)}\displaystyle\int_0^t\big[\zeta\mathcal{S}_i(\varrho)\mathcal{M}_i(\varrho)+\zeta\mathcal{S}_i(\varrho)\mathcal{R}_i(\varrho)-\eta\mathcal{M}_i(\varrho)\mathcal{R}_i(\varrho)-\mu\mathcal{M}_i(\varrho)\big]d\varrho, \\ \mathcal{R}_{i+1}(t)=\dfrac{1-\gamma}{M(\gamma)}\Big[\mu\mathcal{M}_i+\eta\mathcal{M}_i\mathcal{R}_i-\rho\mathcal{S}_i\mathcal{R}_i\Big] \\ \qquad +\dfrac{\gamma}{M(\gamma)}\displaystyle\int_0^t\big[\mu\mathcal{M}_i(\varrho)+\eta\mathcal{M}_i(\varrho)\mathcal{R}_i(\varrho)-\rho\mathcal{S}_i(\varrho)\mathcal{R}_i(\varrho)\big]d\varrho. \end{cases} \tag{3.10}$$

Theorem 3.1. Let $(\mathfrak{B},\|.\|)$ be a Banach space and Π self mapping of $\mathfrak{B}$ satisfying

$$\|\Pi_x-\Pi_y\|\le L\,\|x-\Pi_x\|+l\,\|x-y\|\,,$$

$\forall\ x,y\in\mathfrak{B}$,where $L\ge 0$ and $0\le l\le 1$. Then Π is Picard $\Pi-$stable.

Now, suppose the recursive formula for the proposed system (1.2) as follows:

$$\begin{aligned} \mathcal{S}_{i+1}(t)&=\mathcal{S}_i+\mathcal{L}^{-1}\bigg[\frac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}\Big[-\zeta\mathcal{S}\mathcal{M}-\zeta\mathcal{S}\mathcal{R}+\rho\mathcal{S}\mathcal{R}\Big]\bigg],\\ \mathcal{M}_{i+1}(t)&=\mathcal{M}_i+\mathcal{L}^{-1}\bigg[\frac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}\Big[\zeta\mathcal{S}\mathcal{M}+\zeta\mathcal{S}\mathcal{R}-\eta\mathcal{M}\mathcal{R}-\mu\mathcal{M}\Big]\bigg],\\ \mathcal{R}_{i+1}(t)&=\mathcal{R}_i+\mathcal{L}^{-1}\bigg[\frac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}\Big[\mu\mathcal{M}+\eta\mathcal{M}\mathcal{R}-\rho\mathcal{S}\mathcal{R}\Big]\bigg]. \end{aligned} \tag{3.11}$$

Theorem 3.2. If Π be a self mapping such that

$$\begin{cases} \Pi(\mathcal{S}_i(t)) = \mathcal{S}_{i+1}(t) = \mathcal{S}_i + \mathcal{L}^{-1}\left[\dfrac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}\left[-\zeta\mathcal{S}\mathcal{M} - \zeta\mathcal{S}\mathcal{R} + \rho\mathcal{S}\mathcal{R}\right]\right], \\ \Pi(\mathcal{M}_i(t)) = \mathcal{M}_{i+1}(t) = \mathcal{M}_i + \mathcal{L}^{-1}\left[\dfrac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}\left[\zeta\mathcal{S}\mathcal{M} + \zeta\mathcal{S}\mathcal{R} - \eta\mathcal{M}\mathcal{R} - \mu\mathcal{M}\right]\right], \\ \Pi(\mathcal{R}_i(t)) = \mathcal{R}_{i+1}(t) = \mathcal{R}_i + \mathcal{L}^{-1}\left[\dfrac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}\left[\mu\mathcal{M} + \eta\mathcal{M}\mathcal{R} - \rho\mathcal{S}\mathcal{R}\right]\right]. \end{cases} \tag{3.12}$$

Then the iteration (3.12) is $\Pi-$stable if the following conditions are received

$$\begin{cases} (1+\zeta\Upsilon_1 + (\zeta-\rho)C_2\Upsilon_2) < 1, \\ (1+2\zeta C_2\Upsilon_3 - \rho C_4\Upsilon_4 - \mu\Upsilon_5) < 1, \\ (1+\mu\Upsilon_6 + \rho C_5\Upsilon_7 - \eta C_2\Upsilon_8) < 1. \end{cases} \tag{3.13}$$

Proof. First, we need to show that Π has a fixed point. Thus, we compute $\Pi(\mathcal{S}_i) - \Pi(\mathcal{S}_j)$ for all $(i,j) \in N \times N$ as follows:

$$\begin{aligned} \Pi(\mathcal{S}_i) - \Pi(\mathcal{S}_j) &= \mathcal{S}_i - \mathcal{S}_j + \mathcal{L}^{-1}\left[\frac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}\left[-\zeta\mathcal{S}_i\mathcal{M}_i - \zeta\mathcal{S}_i\mathcal{R}_i + \rho\mathcal{S}_i\mathcal{R}_i\right]\right], \\ &\quad -\mathcal{L}^{-1}\left[\frac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}\left[-\zeta\mathcal{S}_j\mathcal{M}_j - \zeta\mathcal{S}_j\mathcal{R}_j + \rho\mathcal{S}_j\mathcal{R}_j\right]\right], \\ &= \mathcal{S}_i - \mathcal{S}_j + \mathcal{L}^{-1}\left[\frac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}\left[-\zeta\mathcal{S}_i\mathcal{M}_i - \zeta\mathcal{S}_i\mathcal{R}_i + \rho\mathcal{S}_i\mathcal{R}_i\right.\right. \\ &\quad \left.\left. +\zeta\mathcal{S}_j\mathcal{M}_j + \zeta\mathcal{S}_j\mathcal{R}_j - \rho\mathcal{S}_j\mathcal{R}_j\right]\right], \\ &= \mathcal{S}_i - \mathcal{S}_j + \mathcal{L}^{-1}\left[\frac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}\left[\zeta\left(\mathcal{S}_j\mathcal{M}_j - \mathcal{S}_i\mathcal{M}_i\right)\right.\right. \\ &\quad \left.\left. +\zeta\left(\mathcal{S}_j\mathcal{R}_j - \mathcal{S}_i\mathcal{R}_i\right) - \rho\left(\mathcal{S}_j\mathcal{R}_j - \mathcal{S}_i\mathcal{R}_i\right)\right]\right]. \end{aligned} \tag{3.14}$$

Taking maximum of both sides (3.14), we get

$$\begin{aligned} \|\Pi(\mathcal{S}_i) - \Pi(\mathcal{S}_j)\| &\le \|\mathcal{S}_i - \mathcal{S}_j\| + \left\|\mathcal{L}^{-1}\left[\frac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}\left[\zeta\big(\mathcal{S}_j\mathcal{M}_j - \mathcal{S}_i\mathcal{M}_i\big)\right.\right.\right. \\ &\quad \left.\left.\left. +\zeta\big(\mathcal{S}_j\mathcal{R}_j - \mathcal{S}_i\mathcal{R}_i\big) - \rho\big(\mathcal{S}_j(t)\mathcal{R}_j - \mathcal{S}_i\mathcal{R}_i\big)\right]\right]\right\|, \\ &\le \|\mathcal{S}_i - \mathcal{S}_j\| + \mathcal{L}^{-1}\left[\frac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}\left[\zeta\|\mathcal{S}_j\mathcal{M}_j - \mathcal{S}_i\mathcal{M}_i\|\right.\right. \\ &\quad \left.\left. +\zeta\|\mathcal{S}_j\mathcal{R}_j - \mathcal{S}_i\mathcal{R}_i\| - \rho\|\mathcal{S}_i\mathcal{R}_i - \mathcal{S}_j\mathcal{R}_j\|\right]\right], \\ &\le \|\mathcal{S}_i - \mathcal{S}_j\| + \mathcal{L}^{-1}\left[\frac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}\left[\zeta\|\mathcal{S}_j\mathcal{M}_j - \mathcal{S}_i\mathcal{M}_i\|\right.\right. \\ &\quad \left.\left. +(\zeta-\rho)\|\mathcal{S}_j\mathcal{R}_j - \mathcal{S}_i\mathcal{R}_i\|\right]\right]. \end{aligned} \tag{3.15}$$

Both of solutions have the same role so we assume that

$$\|\Pi(\mathcal{S}_i) - \Pi(\mathcal{S}_j)\| \cong \|\Pi(\mathcal{M}_i) - \Pi(\mathcal{M}_j)\| \cong \|\Pi(\mathcal{R}_i) - \Pi(\mathcal{R}_j)\|. \tag{3.16}$$

From equations (3.15) and (3.16), we get

$$\|\Pi(\mathcal{S}_i(t)) - \Pi(\mathcal{S}_j(t))\| \leq \|\mathcal{S}_i - \mathcal{S}_j\| + \mathcal{L}^{-1}\left[\frac{\nu + \gamma(1-\nu)}{\nu}\mathcal{L}\left[\zeta\|\mathcal{M}_j\|\|\mathcal{S}_j - \mathcal{S}_i\| + (\zeta - \rho)\|\mathcal{R}_j\|\|\mathcal{S}_j - \mathcal{S}_i\|\right]\right]. \tag{3.17}$$

Since $\mathcal{S}_i, \mathcal{S}_j, \mathcal{R}_i, \mathcal{R}_j$, $\mathcal{M}_i$ and $\mathcal{M}_j$ are convergent sequences, so they are bounded, there exists constants C_1, C_2, C_3, C_4, C_5 and C_6 for all t such that

$$\|\mathcal{S}_i\| \leq C_1, \quad \|\mathcal{S}_j\| \leq C_2, \quad \|\mathcal{R}_i\| \leq C_3, \quad \|\mathcal{R}_j\| \leq C_4, \;\; \|\mathcal{M}_i\| \leq C_5, \;\; \|\mathcal{M}_j\| \leq C_6.$$

Thus (3.17) becomes

$$\|\Pi(\mathcal{S}_i) - \Pi(\mathcal{S}_j)\| \leq \{1 + \zeta\Upsilon_1 + (\zeta - \rho)C_2\Upsilon_2\}\|\mathcal{S}_i - \mathcal{S}_j\|. \tag{3.18}$$

Similarly, we have

$$\|\Pi(\mathcal{M}_i) - \Pi(\mathcal{M}_j)\| \leq \{1 + 2\zeta C_2\Upsilon_3 - \rho C_4\Upsilon_4 - \mu\Upsilon_5\}\|\mathcal{M}_i - \mathcal{M}_j\|, \tag{3.19}$$

$$\|\Pi(\mathcal{R}_i) - \Pi(\mathcal{R}_j)\| \leq \|\mathcal{R}_i - \mathcal{R}_j\|. \tag{3.20}$$

Where Υ_m for $m = 1, 2, \cdots, 8$, are functions obtained from $\mathcal{L}^{-1}\left[\frac{\nu+\gamma(1-\nu)}{\nu}\mathcal{L}[*]\right]$. Now under the condition

$$\begin{cases} (1 + \zeta\Upsilon_1 + (\zeta - \rho)C_2\Upsilon_2) < 1, \\ (1 + 2\zeta C_2\Upsilon_3 - \rho C_4\Upsilon_4 - \mu\Upsilon_5) < 1, \\ (1 + \mu\Upsilon_6 + \rho C_5\Upsilon_7 - \eta C_2\Upsilon_8) < 1, \end{cases} \tag{3.21}$$

the self map Π is contraction, so Π posses a fixed point.

Next, we will show that Π fulfill the required conditions. To do so, we assume that

$$L = (0,0,0), \quad l = \begin{cases} (1 + \zeta\Upsilon_1 + (\zeta - \rho)C_2\Upsilon_2), \\ (1 + 2\zeta C_2\Upsilon_3 - \rho C_4\Upsilon_4 - \mu\Upsilon_5), \\ (1 + \mu\Upsilon_6 + \rho C_5\Upsilon_7 - \eta C_2\Upsilon_8). \end{cases}$$

Then all conditions are satisfied; hence, Π is Picardie $\Pi-$stable. □

3.3. Numerical results and simulations

In this part, we use three steps ABM to derive general numerical solution of the model (1.2). Consider the first equation of system (3.1) as

$$\begin{cases} {}^{CF}D_t^{\gamma}\mathcal{S}(t) = \Xi_1(t, \mathcal{S}, \mathcal{M}, \mathcal{R}), \\ \mathcal{S}(0) = \mathcal{S}_0. \end{cases} \tag{3.22}$$

Apply fractional integral in sense of Caputo-Fabrizio, we have

$$\mathcal{S}(t) - \mathcal{S}(0) = \frac{1-\gamma}{M(\gamma)}\Xi_1(t, \mathcal{S}, \mathcal{M}, \mathcal{R}) + \frac{\gamma}{M(\gamma)}\int_0^t \Xi_1(\tau, \mathcal{S}, \mathcal{M}, \mathcal{R})\,d\tau. \tag{3.23}$$

Now, we descriptive the interval $[0, t]$ by taking the step size l such that $t_0 = 0$, $t_{k+1} = t_k + l$, $k = 0, 1, 2, \cdots, n-1$. Replace $t = t_{k+1}$ and $t = t_k$ in equation (3.23) and take the difference of the resulting equations, we get

$$\begin{cases} \mathcal{S}(t_{k+1}) - \mathcal{S}(t_k) = & \dfrac{1-\gamma}{M(\gamma)} \left[\Xi_1\left(t_{k+1}, \mathcal{S}, \mathcal{M}, \mathcal{R}\right) - \Xi_1\left(t_k, \mathcal{S}, \mathcal{M}, \mathcal{R}\right)\right] \\ & + \dfrac{\gamma}{M(\gamma)} \displaystyle\int_{t_k}^{t_{k+1}} \Xi_1\left(\tau, \mathcal{S}, \mathcal{M}, \mathcal{R}\right) d\tau. \end{cases} \tag{3.24}$$

Now, we approximate the integral on the right side of equation (3.24) by considering the Lagrangian interpolation polynomial of degree two passing through the points

$$\left(t_{k-2}, \Xi_1\left(t_{k-2}, \mathcal{S}, \mathcal{M}, \mathcal{R}\right)\right), \left(t_{k-1}, \Xi_1\left(t_{k-1}, \mathcal{S}, \mathcal{M}, \mathcal{R}\right)\right),$$

and

$$\left(t_k, \Xi_1\left(t_k, \mathcal{S}, \mathcal{M}, \mathcal{R}\right)\right).$$

The Lagrangian interpolation polynomial $\mathcal{Q}_2(\tau)$ of degree two is given by

$$\mathcal{Q}_2(\tau) = \sum_{i=0}^{2} \Xi_1\left(t_{k-i}, \mathcal{S}, \mathcal{M}, \mathcal{R}\right) \mathcal{H}_i(\tau),$$

where the $\mathcal{H}_i(\tau)$ are the Lagrange polynomials on the points t_{k-2}, t_{k-1},and t_k. Let $\mathcal{S}_k = \mathcal{S}(t_k)$, now, to approximate the integral $\int_{t_k}^{t_{k+1}} \Xi_1\left(\tau, \mathcal{S}, \mathcal{M}, \mathcal{R}\right) d\tau$, we substitute $r = \frac{t_{k+1}-\tau}{l}$ in the Lagrange base polynomial and the integration, we get

$$\begin{aligned} \int_{t_k}^{t_{k+1}} \Xi_1\left(\tau, \mathcal{S}, \mathcal{M}, \mathcal{R}\right) d\tau = & l \int_0^1 \Xi_1\left(\tau_k, \mathcal{S}_k, \mathcal{M}_k, \mathcal{R}_k\right) \frac{(r-2)(r-3)}{(1-2)(1-3)}, \\ & + \Xi_1\left(\tau_{k-1}, \mathcal{S}_{k-1}, \mathcal{M}_{k-1}, \mathcal{R}_{k-1}\right) \frac{(r-1)(r-3)}{(2-1)(2-3)} \\ & + \Xi_1\left(\tau_{k-2}, \mathcal{S}_{k-2}, \mathcal{M}_{k-2}, \mathcal{R}_{k-2}\right) \frac{(r-2)(r-1)}{(3-2)(3-1)} d\tau, \\ = & \frac{23l}{12} \Xi_1\left(\tau_k, \mathcal{S}_k, \mathcal{M}_k, \mathcal{R}_k\right) - \frac{16l}{12} \Xi_1\left(\tau_{k-1}, \mathcal{S}_{k-1}, \mathcal{M}_{k-1}, \mathcal{R}_{k-1}\right) \\ & + \frac{5l}{12} \Xi_1\left(\tau_{k-2}, \mathcal{S}_{k-2}, \mathcal{M}_{k-2}, \mathcal{R}_{k-2}\right). \end{aligned}$$

Then equation (3.24) becomes

$$\begin{cases} \mathcal{S}(t_{k+1}) = & \mathcal{S}(t_k) + \left(\dfrac{1-\gamma}{M(\gamma)} + \dfrac{23\gamma l}{12M(\gamma)}\right) \Xi_1\left(\tau_k, \mathcal{S}_k, \mathcal{M}_k, \mathcal{R}_k\right) \\ & - \left(\dfrac{1-\gamma}{M(\gamma)} + \dfrac{16\gamma l}{12M(\gamma)}\right) \Xi_1\left(\tau_{k-1}, \mathcal{S}_{k-1}, \mathcal{M}_{k-1}, \mathcal{R}_{k-1}\right) \\ & + \dfrac{5\gamma l}{12M(\gamma)} \Xi_1\left(\tau_{k-2}, \mathcal{S}_{k-2}, \mathcal{M}_{k-2}, \mathcal{R}_{k-2}\right). \end{cases} \tag{3.25}$$

The iterative scheme for the remaining two compartments are given below

$$\begin{cases} \mathcal{M}(t_{k+1}) = \mathcal{M}(t_k) + \left(\dfrac{1-\gamma}{M(\gamma)} + \dfrac{23\gamma l}{12M(\gamma)}\right)\Xi_2\left(\tau_k, \mathcal{S}_k, \mathcal{M}_k, \mathcal{R}_k\right) \\ \qquad - \left(\dfrac{1-\gamma}{M(\gamma)} + \dfrac{16\gamma l}{12M(\gamma)}\right)\Xi_2\left(\tau_{k-1}, \mathcal{S}_{k-1}, \mathcal{M}_{k-1}, \mathcal{R}_{k-1}\right) \\ \qquad + \dfrac{5\gamma l}{12M(\gamma)}\Xi_2\left(\tau_{k-2}, \mathcal{S}_{k-2}, \mathcal{M}_{k-2}, \mathcal{R}_{k-2}\right), \end{cases} \tag{3.26}$$

$$\begin{cases} \mathcal{R}(t_{k+1}) = \mathcal{R}(t_k) + \left(\dfrac{1-\gamma}{M(\gamma)} + \dfrac{23\gamma l}{12M(\gamma)}\right)\Xi_3\left(\tau_k, \mathcal{S}_k, \mathcal{M}_k, \mathcal{R}_k\right) \\ \qquad - \left(\dfrac{1-\gamma}{M(\gamma)} + \dfrac{16\gamma l}{12M(\gamma)}\right)\Xi_3\left(\tau_{k-1}, \mathcal{S}_{k-1}, \mathcal{M}_{k-1}, \mathcal{R}_{k-1}\right) \\ \qquad + \dfrac{5\gamma l}{12M(\gamma)}\Xi_3\left(\tau_{k-2}, \mathcal{S}_{k-2}, \mathcal{M}_{k-2}, \mathcal{R}_{k-2}\right). \end{cases} \tag{3.27}$$

4. Numerical interpretation and discussion

For the numerical simulations, we use the numerical values given in the Table 1, and the initial conditions $\mathcal{S}(0) = 0.99$, $\mathcal{M}(0) = 0.01$, $\mathcal{R}(0) = 0$.

Table 1. "Parametric values for the numerical simulation"

Parameters	values
μ	0.03
η	0.07
ζ	0.07
ρ	0.10

Now in this part, we have to draw the graphical representation of all the three classes namely as $\mathcal{S}(t)$, $\mathcal{M}(t)$, and $\mathcal{R}(t)$ for the considered model via ABM under CFD of fractional order. We take the starting values as $\mathcal{S}(0) = 0.98$, $\mathcal{M}(0) = 0.015$ and $\mathcal{R}(0) = 0$. Four cases are investigated in which some parameters are fixed and some are changed from one case to another. In all the cases we have obtained the stable and convergent simulation.

Case-I In this case $d = 0.07$ and $a = 0.03$, we will have varied the parameters ζ and γ in all cases. In this case $\gamma = 0.01$ $\zeta = 0.07$, figures 1 to 3 is the representation for $\mathcal{S}(t)$, $\mathcal{M}(t)$ and $\mathcal{R}(t)$ respectively. The first three Figures 1, 2 and 3 are the simulation for case-I at different fractional order. We see that the growing of γ results the growth of $\mathcal{S}(t)$ and the decrease of $\mathcal{M}(t)$ and $\mathcal{R}(t)$. Notice that γ models the persuasion of non-consumers $\mathcal{S}(t)$ in the society interactions with alcohol takers on high level $\mathcal{R}(t)$, i.e., the social force of peoples that do not use alcohol over their contacts (friends, relatives, etc) that consume too much alcohol.

Figure 4 represents all the three agents of the proposed problem in one figure at different fractional order providing the applicability of fractional order derivatives.

Case-II In the case-II we take $\gamma = 0.15$ and $\zeta = 0.07$, while the remaining parameters does not change.

This time in figure 5 up and down showed by all the agents as compared to Case-I. $\mathcal{R}(t)$ is declines while $\mathcal{S}(t)$ is increased.

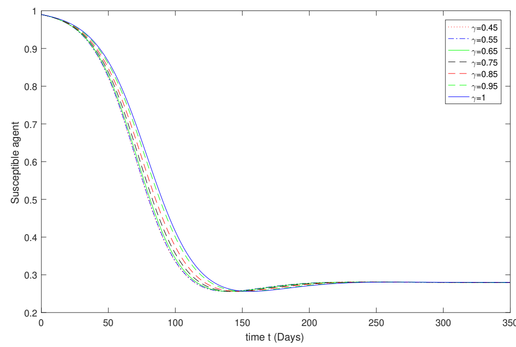

Figure 1. Plot of the approximate solution for $\mathcal{S}(t)$ of the considered system (1.2) for the non-integer order γ between 0 and 1

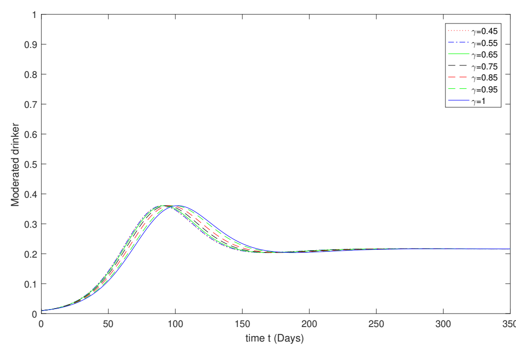

Figure 2. Plot of the approximate solution for $\mathcal{M}(t)$ of the considered system (1.2) for the non-integer order γ between 0 and 1

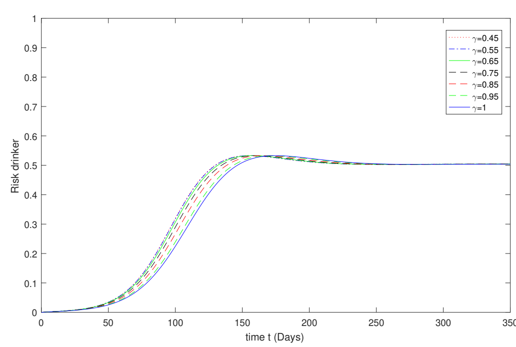

Figure 3. Plot of the approximate solution for $\mathcal{R}(t)$ of the considered system (1.2) for the non-integer order γ between 0 and 1

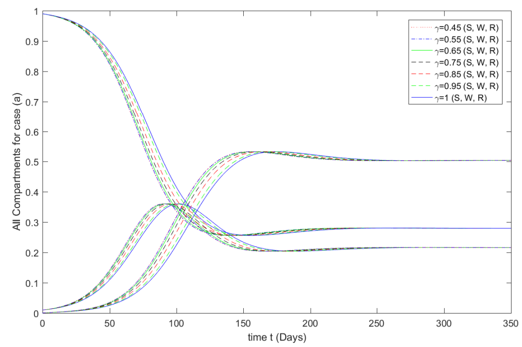

Figure 4. Combine Dynamical behavior of the approximate solution for all the three agents of the considered system (1.2) for the non-integer order γ between 0 and 1 for data of Case-I

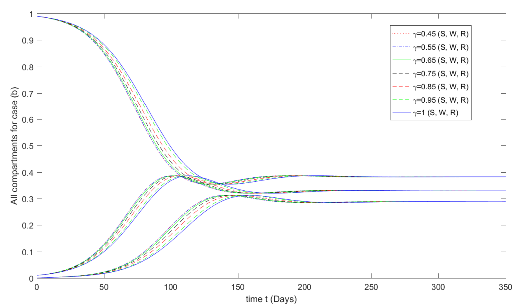

Figure 5. Dynamical representation of the approximate solution for all the three agents $\mathcal{S}(t), \mathcal{M}(t), \mathcal{R}(t)$ of the system (1.2) for the non-integer order γ between 0 and 1 having parameters values of Case-II

Case-III: In the third case $\zeta = 0.07$ and $\gamma = 0.30$, while the remaining parameters are fixed.

In figure 6 all the three agents are graphed at different fractional order. Here the fractional representation is an one graph. In the graph we can observe that $\mathcal{S}(t)$ and $\mathcal{M}(t)$ intersect each other at the same convergent point on the 100^{th} day. $\mathcal{R}(t)$ in the said case declines up to 0.1.

Case-IV: In the fourth case $\zeta = 0.20$, $\gamma = 0.15$ while the rest of the parameters are unchanged.

Figure 7 shows combine graph for all the three compartmental agents at different fractional order. In the said situation, we can observe that $\mathcal{S}(t)$ is rapidly declining while $\mathcal{R}(t)$ is rapidly growing. This time $\mathcal{M}(t)$ increases and then become stable.

5. Conclusion

The analyzed manuscript has been treated for the drinking behaviors of various individuals through the fractional drinking problem. On applying Picard's iterative tool along with fixed point results, we have developed the theoretical analysis of

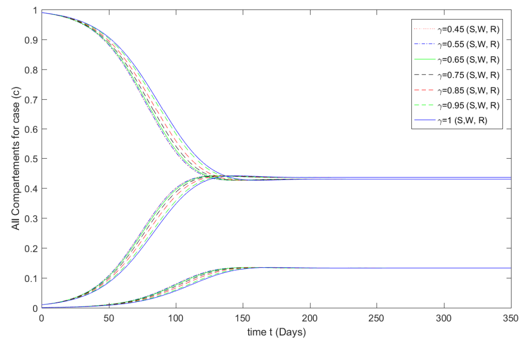

Figure 6. Dynamical behavior of the approximate solution for all the three compartments $\mathcal{S}(t), \mathcal{M}(t), \mathcal{R}(t)$ of the considered system (1.2) for the non-integer order γ between 0 and 1 having parameters values of Case-III

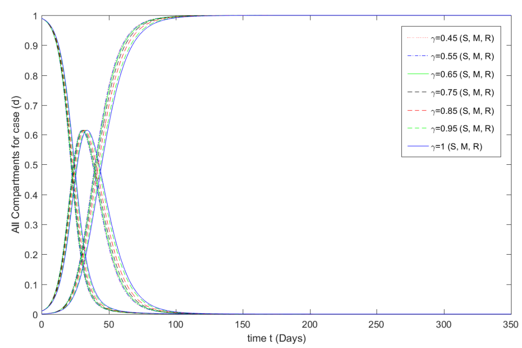

Figure 7. Dynamical behavior of the approximate solution for all the three agents $\mathcal{S}(t), \mathcal{M}(t), \mathcal{R}(t)$ of the considered system (1.2) for the non-integer order γ between 0 and 1 having parameters values of Case-IV

a solution for a model addressing the drinking behavior. Also, numerical results have been derived via ABM of the numerical side. The concerned model has been investigated under CFD with fractional order. The whole analysis has been demonstrated via plots against various fractional-order values. We have chosen different fractional orders for the simulation of the proposed problem to test the behavior of each compartmental agent lying between 0 and 1. The advantage of choosing such various fractional orders is to find the whole continuous spectrum and density from 0 to 1 for every agent of the said problem. Hence CFD of fractional order can also be nicely applied to study dynamical analysis of many real-world problems.

Conflict of interest. There are no conflict of interest.

Acknowledgment. This work was supported by the Natural Science Foundation of Hunan Province (No. 2019JJ50024), Hunan Philosophy and Social Science Foundation (No. 17YBQ020), and the Key Laboratory of Key Technologies of Digital Urban-Rural Spatial Planning of Hunan Province (No. 2018TP1042).

References

[1] A. Atangana and D. Baleanu, *Caputo-Fabrizio derivative applied to groundwater flow within confined aquifer*, J. of Eng. Mech., 2017, 143(5), D4016005.

[2] M. ur Rahman, M. Arfan and S. Ahmad, *On the analysis of semi-analytical solutions of Hepatitis B epidemic model under the Caputo-Fabrizio operator*, Cha., Sol. & Frac., 2021, 146, 110892.

[3] A. Atangana, *On the new fractional derivative and application to nonlinear Fisher's reaction-diffusion equation*, Appl. Math. Comput., 2016 273, 948–956.

[4] M. Al-Refai and T. Abdeljawad, *Analysis of the fractional diffusion equations with fractional derivative of non-singular kernel*, Adv. Differ. Equ., 2017, 2017(1), 1–12.

[5] D. Baleanu and T. Abdeljawad, *On fractional derivatives with exponential kernel and their discrete versions*, Rep. Math. Phys., 2017, 80(1), 11–27.

[6] A. E. Hassanien, I. Arpaci, I. Mahariq, M. Al-Emran, M. Khasawneh, S. Alshehabi and T. Abdeljawad, *Analysis of twitter data using evolutionary clustering during the COVID-19 pandemic*, Comp., Mate. and Cont., 2020, 65(1), 193–204.

[7] A. Atangana and R. T. Alqahtani, *Numerical approximation of the space-time Caputo-Fabrizio fractional derivative and application to groundwater pollution equation*, Adv. Differ. Equ., 2016, 2016(1), 1–13.

[8] A. Jajarmi, D. Baleanu, H. Mohammadi and S. Rezapour, *A new study on the mathematical modeling of human liverwith Caputo-Fabrizio fractional derivative*, Cha. Sol. & Frac., 2020, 134, 109705.

[9] A. Mousalou, D. Baleanu, and S. Rezapour, *The extended fractional Caputo–Fabrizio derivative of order* $0 = s < 1, 0 \le \sigma < 1$ *on* $CR[0,1]C_{\mathfrak{R}}[0,1]$ *and the existence of solutions for two higher-order series-type differential equations*, Adv. Differ. Equ., 2018, 2018, 1–11.

[10] A. Mousalou, D. Baleanu, and S. Rezapour, *On the existence of solutions for some infinite coefficient-symmetric Caputo-Fabrizio fractional integro-differential equations*, Bound. Value Probl., 2017, 2017, 145.

[11] D. Baleanu, S. Rezapour and Z. Saberpour, *On fractional integro-differential inclusions via the extended fractional Caputo-Fabrizio derivation*, Bound. Value Probl., 2019, 2019, 79.

[12] N. T. J. Bailey, *The mathematical theory of infectious diseases and its applications*, Charles Griffin & Company Ltd, 5a Crendon Street, High Wycombe, Bucks HP13 6LE., 1975.

[13] N. Crokidakis, R. M. Brum, *Dynamics of tax evasion through an epidemic-like model*, Int. J. Mod. Phy. C, 2017, 28, 1750023.

[14] D. Baleanu, H. Mohammadi, R. P. Agarwal and S. Rezapour, *Some existence results for a nonlinear fractional differential equation on partially ordered Banach spaces*, Bound. Val. Probl., 2013, 2013, 112.

[15] M. Caputo and M. Fabrizio, *A new definition of fractional derivative without singular kernel*, Prog. Fract. Differ. Appl., 2015, 1(2), 73–85.

[16] J. S. Sa Martins and N. Crokidakis, *Can honesty survive in a corrupt parliament?*, Int. J. Mod. Phy. C, 2018, 29(10), 1850094.

[17] J. Chodkiewicz, J. Miniszewska, M. Talarowska, N. Nawrocka and P. Bilin-ski, *Alcohol consumption reported during the covid-19 pandemic: The initial stage*, Int. J. Envir. Res. Pub. Hea., 2020, 17, 4677.

[18] J. M. Clay and M. O. Parker, *Alcohol use and misuse during the covid-19 pandemic: a potential public health crisis?*, The Lan. Pub. Hea., 2020, 5, e259.

[19] L. Sigaud and N. Crokidakis, *Modeling the evolution of drinking behavior: A Statistical Physics perspective*, Phy. A: Stat. Mec. and its Appl., 2021, 570, 125814.

[20] M. Caputo and M. Fabrizio, *On the singular kernels for fractional derivatives. some applications to partial differential equations*, Progr. Fract. Differ. Appl., 2021, 7(2), 1–4.

[21] D. J. Daley and D. G. Kendall, *Epidemics and rumours*, Nature, 1964, 204(4963), 1118–1118.

[22] H. Nishiura, K. Ejima, K. Aihara, *Modeling the obesity epidemic: social contagion and its implications for control*, Theo. Bio. and Med. Mode., 2013, 10(1), 17.

[23] M. A. Javarone, S. Galam, *Modeling radicalization phenomena in heterogeneous populations*, PloS One, 2016, 11(5), e0155407.

[24] C. Hall, G. A. Kaplan and S. Galea, *Social epidemiology and complex system dynamic modelling as applied to health behaviour and drug use research*, Int. J. Drug Pol., 2009, 20, 209–216.

[25] A. Jajarmi and D. Baleanu, *On the fractional optimal control problems with a general derivative operator*, Asi. Jour. of Cont., 2021, 23(2), 1062–1071.

[26] E. A. Kojaba and S. Rezapour, *Approximate solutions of a sum-type fractional integro-differential equation by using Chebyshev and Legendre polynomials*, Adv. Differ. Equ., 2017, 2017, 1–18.

[27] A. Atangana and I. Koca, *Solutions of Cattaneo-Hristov model of elastic heat diffusion with Caputo-Fabrizio and Atangana-Baleanu fractional derivatives*, Therm. Scie., 2017, 21(6 Part A), 2299–2305.

[28] A. Seadawy, G. Zaman, K. Shah, P. Kumam, S. A. Khan and Z. Shah, *Study of mathematical model of Hepatitis B under Caputo-Fabrizo derivative*, AIMS Math., 2020, 6, 195.

[29] D. Baleanu, M. A. Khan and Z. Hammouch, *Modeling the dynamics of hepatitis E via the Caputo-Fabrizio derivative*, Math. Model. Nat. Phenom., 2019, 14(3), 311.

[30] J. Losada and J. Nieto, *Fractional integral associted to fractional derivatives with nonsingular kernels*, Progr. Fract. Differ. Appl., 2021, 7(3), 1–7.

[31] D. Bui, I. Mahariq, M. Kavyanpoor, M. Ghalandari and M. A. Nazari, *Identification of nonlinear model for rotary high aspect ratio flexible blade using free vibration response*, Alex. Eng. J., 2020, 59(4), 2131–2139.

[32] A. Atangana and K. M. Owolabi, *On the formulation of Adams-Bashforth scheme with Atangana-Baleanu-Caputo fractional derivative to model chaotic problems*, Cha.: An Int. J. of Non. Scie., 2019, 29(2), 023111.

[33] A. Yusuf and S. Qureshi, *Fractional derivatives applied to MSEIR problems: Comparative study with real world data*, The Eur. Phy. J. Plu., 2019, 134(4), 171.

[34] A. Ali, F. Rahman, and S. Saifullah, *Analysis of Time-Fractional ϕ^4-Equation with Singular and Non-Singular Kernels*, Int. J. of App. and Comp. Math., 2021, 7(5), 1–17.

[35] F. Jarad, I. Mahariq, K. Shah and T. Abdeljawad, *Qualitative analysis of a mathematical model in the time of COVID-19*, Bio. Res. Inte., 2020, 2020.

[36] A. Ali, K. Shah, M. Irfan and S. Saifullah, *Time-Fractional Klein–Gordon Equation with Solitary/Shock Waves Solutions*, Math. Prob. in the Eng., 2021, 2021, Article ID: 6858592..

[37] D. Baleanu, M. Shoaib, M. A. Z. Raja and Z. Sabir, *Design of stochastic numerical solver for the solution of singular three-point second-order boundary value problems*, Neu. Comp. and Appl., 2021, 33(7), 2427–2443.

[38] F. Jarad, K. Shah and T. Abdeljawad, *On a nonlinear fractional order model of dengue fever disease under Caputo-Fabrizio derivative*, Alex. Eng. J., 2020, 59(4), 2305–2313.

[39] E. Śanchez, F. J. Santonja, M. Rubio and R. J. Villanueva, *Predicting cocaine consumption in spain: A mathematical modelling approach*, Drugs: Edu. Prev. Pol., 2011, 18(2), 108–115.

[40] D. A. Fiellin, L. E. Sullivan and P. G. O'Connor, *The prevalence and impact of alcohol problems in major depression: a systematic review*, The Ame. J. Med., 2005, 118, 330–341.

[41] E. Śanchez, F. J. Santonja, J. L. Morera and M. Rubio, *Alcohol consumption in spain and its economic cost: a mathematical modeling approach*, Math. Com. Mod., 2010, 52, 999–1003.

[42] A. Gilani, M. Talaee, M. Shabibi and S. Rezapour, *On the existence of solutions for a pointwise defined multi-singular integro-differential equation with integral boundary condition*, Adv. Differ. Equ., 2020, 2020, 1–16.

[43] M. ur Rahman, N. A. Alshehri, R. T. Matoog, S. Ahmad and T. Khan, *Study on the mathematical modelling of COVID-19 with Caputo-Fabrizio operator*, Cha., Sol. & Frac., 2021, 111121.

[44] I. Ullah, M. ur Rahman, M. Arfan and S. Ahmad, *Investigation of fractional order tuberculosis (TB) model via Caputo derivative*, Cha., Sol. & Frac., 2020, 138, 110479.

[45] WHO, World health statistics 2018: monitoring health for the sdgs sustainable development goals, Geneva: World Health Organization CC BY-NC-SA, 2018, 3.0 IGO.