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GLOBAL ASYMPTOTICAL STABILITY OF A PLANT DISEASE MODEL WITH AN ECONOMIC THRESHOLD*

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Dedicated to Professor Jibin Li on the occasion of his 80th birthday.

Abstract This paper presents a plant disease model with an economic threshold, where the replanting number of susceptible plants depends on the removing number of infective plants. Making use of Lyapunov approach and Poincaré maps, we thoroughly investigate the global dynamics. We show the global asymptotical stability of endemic equilibria as well as a pseudo equilibrium. Moreover, the convergence in finite time is also examined for the infected plants. Our theoretical results indicate that the control goal could be achieved by taking appropriate removal and replanting rates.

Keywords Filippov system, Poincaré map, economic threshold, stability, equilibrium.

MSC(2010) 34A36, 34D23, 92C80.

1. Introduction

In recent years, plant diseases have caused crop loss or yield reduction. How to control plant diseases quickly and effectively has attracted much attention [20, 28]. It is necessary to carry out effective control measures to fight against the diseases. Therefore, integrated disease management is developed [11, 12]. This strategy combines various control measures to minimize losses and maximize returns. Among these control measures, removing infected plants and replanting susceptible plants are widely used because of their little impact on ecological environment.

In order to minimize losses and maximize returns, a tolerance threshold is allowed, which is called an economic threshold [13, 37]. The control measures are only implemented once the number of infected plants exceeds the threshold. Due to the non-smoothness or discontinuity induced by the economic threshold, more and more non-smooth dynamical systems are presented to better model the disease

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dynamics and design effective control strategy, see [2, 7, 13, 22, 33, 34]. In contrast to smooth systems, the analysis of non-smooth ones are more challenging and there are distinctive behaviors, such as pseudo-equilibria [14], crossing or sliding limit cycles [29–32] and sliding heteroclinic orbits [33].

In most of the literature, the replanting measure is assumed to depend on the total number of susceptible plants, see [2, 9, 17, 33, 37]. However, due to the limit of land resources, such replanting measure might be impossible in practice when the number of susceptible plants is large, and might lead to waste of land when the number of susceptible plants is small. Therefore, it is more reasonable to assume the replanting number of susceptible plants depends on but does not exceed the removing number of the infected.

Based on this motivation, we propose a new plant disease model with an economic threshold, supposing that the replanting number of susceptible plants does not exceed the removing number of infected plants. By employing the Lyapunov approach and Poincaré maps, we discuss the global dynamics of the model. From the biological point of view, our results show that the plant diseases can be controlled by selecting appropriate replanting and removing rates.

The rest of this paper is arranged as follows. In Section 2, the plant disease model is described and some preliminaries are given. Section 3 is devoted to the analysis of the global dynamics. Finally in the last section, we discussed the biological implications of our theoretical results.

2. Model description and preliminaries

In this paper, we present the following plant disease model

$$\begin{cases} \frac{\mathrm{d}S}{\mathrm{d}t} = A - \beta SI - \eta_1 S + p\phi(I)I, \\ \frac{\mathrm{d}I}{\mathrm{d}t} = \beta SI - \eta_2 I - v\phi(I)I, \end{cases} \tag{2.1}$$

where S and I represent the numbers of susceptible and infected plants respectively; A denotes the constant planting rate of susceptible plants; $\beta > 0$ is the infection transmission rate; $\eta_1 > 0$ and $\eta_2 > 0$ are the death (or harvesting) rates of susceptible and infected plants respectively; $v > 0$ and $p \geq 0$ denote the rouging (removing) rate and the replanting rate respectively;

$$\phi(I) = \begin{cases} 0, & I < k, \\ 1, & I > k, \end{cases} \tag{2.2}$$

is the control function and $k > 0$ represents the economic threshold. When the number of infected plants is below k, no control measures are required. However, once the number of infected plants exceeds the economic threshold k, one should take measures to remove the infected and replant the susceptible to control diseases. Throughout this paper, we assume that $0 \leq p \leq v$, i.e., the replanting number of the susceptible plants is less than or equal to the removing number of the infected plants.

Let $R_+^2 = \{(S, I) | S > 0, I > 0\}$. Then R_+^2 is divided into two subregions

$$G_1 = \{(S, I) \in R_+^2 | I < k\}, \quad G_2 = \{(S, I) \in R_+^2 | I > k\},$$

by
$$\Sigma = \{(S, I) \in R_+^2 | I = k\},$$
which is called a switching line for the system (2.1). Clearly, the system (2.1) consists of two subsystems
$$\begin{cases} \frac{dS}{dt} = A - \beta SI - \eta_1 S, \\ \frac{dI}{dt} = \beta SI - \eta_2 I, \end{cases} \quad I < k \tag{2.3}$$
and
$$\begin{cases} \frac{dS}{dt} = A - \beta SI - \eta_1 S + pI, \\ \frac{dI}{dt} = \beta SI - \eta_2 I - vI. \end{cases} \quad I > k. \tag{2.4}$$

Because of the discontinuity of the right hand side of System (2.1), we define a solution of (2.1) in Filippov sense. Let $\overline{co}[\phi(I)]$ be the closure of the convex hull of $\phi(I)$. Then $\overline{co}[\phi(I)] = [\phi(I^-), \phi(I^+)]$ with $\phi(I^-) \leq \phi(I^+)$ by the definition (2.2) of $\phi(I)$, where $\phi(I^-)$ and $\phi(I^+)$ represent the left and the right limits of ϕ at I, respectively.

Definition 2.1. A vector function $(S(t), I(t))$ on $[0, T)$ $(0 < T \leq \infty)$, is a solution of (2.1) with initial condition $(S_0, I_0) \in R_+^2$, if $(S(t), I(t))$ is absolutely continuous on any subinterval $[t_1, t_2]$ of $[0, T)$ satisfying $S(0) = S_0$ and $I(0) = I_0$, and there exists a measurable function $\gamma = \gamma(t) \in \overline{co}[\phi(I(t))]$ for almost all (a.a.) $t \in [0, T)$ such that
$$\begin{cases} \frac{dS}{dt} = A - \beta SI - \eta_1 S + p\gamma I, \\ \frac{dI}{dt} = \beta SI - \eta_2 I - v\gamma I. \end{cases} \tag{2.5}$$

Using similar arguments as those in [33], the positiveness and boundedness of solutions can be derived for System (2.1). Moreover, each solution exists for $t \in [0, +\infty)$. For the subsystem (2.3), the possible equilibria are
$$E_0 = \left(\frac{A}{\eta_1}, 0\right), \qquad E_1 = (S_1, I_1) = \left(\frac{\eta_2}{\beta}, \frac{A\beta - \eta_1\eta_2}{\eta_2\beta}\right)$$
and its basic reproduction number is $R_1 = \frac{A\beta}{\eta_1\eta_2}$. For the subsystem (2.4), the possible equilibria are E_0 and
$$E_2 = (S_2, I_2) = \left(\frac{\eta_2 + v}{\beta}, \frac{A\beta - \eta_1(\eta_2 + v)}{(\eta_2 + v - p)\beta}\right),$$
and its basic reproduction number is $R_2 = \frac{A\beta}{(\eta_2+v)\eta_1}$.

By [33, Theorem 3.1], we have the following result.

Proposition 2.1. *For the subsystem (2.3), the disease-free equilibrium E_0 is globally asymptotically stable if $R_1 \leq 1$, while the endemic equilibrium E_1 is globally asymptotically stable if $R_1 > 1$.*

Similarly, the global dynamics can be obtained for the subsystem (2.4) by the following proposition.

Proposition 2.2. *For the subsystem (2.4), the disease-free equilibrium E_0 is globally asymptotically stable if $R_2 < 1$, while the endemic equilibrium E_2 is globally asymptotically stable if $R_2 > 1$.*

Proof. When $R_2 < 1$, it is seen that E_0 is the unique equilibrium which is a locally stable node. Let $N = S + I$. Then

$$\frac{dN}{\mathrm{d}t}\bigg|_{(2.4)} = A - \eta_1 S - (\eta_2 + v - p)I \le A - \mu N,$$

where $\mu = \min\{\eta_1, \eta_2+v-p\} > 0$. This means that any solution of (2.4) is bounded. Taking a Dulac function $B(S,I) = \frac{1}{SI}$, we have

$$\frac{\partial B(S,I)(A - \beta SI - \eta_1 S + pI)}{\partial S} + \frac{\partial B(S,I)(\beta SI - \eta_2 I - vI)}{\partial I} = -\frac{A}{S^2 I} - \frac{p}{S^2} < 0,$$

which implies that System (2.4) does not have limit cycles. Thus E_0 is globally asymptotically stable.

Now suppose $R_2 > 1$. Then E_2 is a positive equilibrium for System (2.4). Notice that the system (2.4) can be rewritten as

$$\begin{cases} \frac{\mathrm{d}S}{\mathrm{d}t} = -\eta_1(S - S_2) - \beta(S - S_2)I - (\eta_2 + v - p)(I - I_2), \\ \frac{\mathrm{d}I}{\mathrm{d}t} = \beta(S - S_2)I. \end{cases} \tag{2.6}$$

Let

$$V(S,I) = \frac{1}{2}(S - S_2)^2 + \frac{\eta_2 + v - p}{\beta}\left(I - I_2 - I_2 \ln\frac{I}{I_2}\right).$$

Then

$$\frac{dV(S,I)}{\mathrm{d}t}\bigg|_{(2.4)} = -(\eta_1 + \beta I)(S - S_2)^2,$$

which implies that E_2 is a globally asymptotically stable equilibrium of System (2.4) by the LaSalle Invariance Principle. □

For the system (2.1), E_1 is called a real (virtual) equilibrium if $I_1 < k$ ($I_1 > k$), and E_2 is called a real (virtual) equilibrium if $I_2 > k$ ($I_2 < k$). Next, let us discuss the dynamics on the switching line Σ by using some concepts from [5]. The crossing region is $\Sigma_c = \{(S,I) \in R_+^2 \mid S \in (0, S_1) \cup (S_2, +\infty), I = k\}$. The sliding region is

$$\Sigma_s = \{(S,I) \in R_+^2 | S_1 < S < S_2, I = k\},$$

whose closure is denoted by $\overline{\Sigma}_s$. Let $T_1 = (S_1, k)$ and $T_2 = (S_2, k)$, then both T_1 and T_2 are tangent points [5]. In order to get the sliding equation, let $I = k$ in the second equation of (2.5), we have $\gamma = \frac{\beta S - \eta_2}{v} \in \overline{co}[\phi(k)] = [0,1]$. This means that the sliding dynamics is determined by the equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} = f(S) \tag{2.7}$$

for $(S,I) \in \Sigma_s$, where

$$f(S) = \frac{1}{v}\left[-(kv\beta + v\eta_1 - kp\beta)S - kp\eta_2 + Av\right]. \tag{2.8}$$

Note that the function $f(S)$ has a unique zero S_p, where

$$S_p = \frac{Av - kp\eta_2}{k\beta(v-p) + v\eta_1}.$$

Hence the possible pseudo-equilibrium is $E_p = (S_p, k)$ for System (2.1). According to [5], E_p is a pseudo-equilibrium if and only if $S_1 < S_p < S_2$.

Proposition 2.3. *Suppose that $R_1 \geq 1$. Then the following assertions hold:*

(i) *if $R_1 < 1 + \frac{\beta k}{\eta_1}$, System (2.1) has no pseudo-equilibria and solutions starting from $\overline{\Sigma}_s \backslash \{T_1\}$ will slide along Σ_s to the tangent point T_1 in finite time;*

(ii) *if $1 + \frac{\beta k}{\eta_1} < R_1 < 1 + \frac{\beta k(\eta_2+v-p)+\eta_1 v}{\eta_1\eta_2}$, System (2.1) has a unique pseudo-equilibrium E_p and solutions starting from $\overline{\Sigma}_s \backslash \{E_p\}$ will slide along Σ_s and converge to E_p as $t \to +\infty$;*

(iii) *if $R_1 > 1+\frac{\beta k(\eta_2+v-p)+\eta_1 v}{\eta_1\eta_2}$, System (2.1) has no pseudo-equilibria and solutions starting from $\overline{\Sigma}_s \backslash \{T_2\}$ will slide along Σ_s to the tangent point T_2 in finite time.*

Proof. Notice that $R_1 < 1 + \frac{\beta k}{\eta_1}$ is equivalent to $I_1 < k$ while $R_1 > 1 + \frac{\beta k(\eta_2+v-p)+\eta_1 v}{\eta_1\eta_2}$ is equivalent to $I_2 > k$. Furthermore,

$$f(S_1) = \eta_2(I_1 - k) \quad \text{and} \quad f(S_2) = (\eta_2 + v - p)(I_2 - k).$$

Thus, when $R_1 < 1 + \frac{\beta k}{\eta_1}$, we have $f(S_1) < 0$ and $f(S_2) < 0$, which means the assertion (i) follows. Similarly, the assertion (iii) holds. Now we assume $1 + \frac{\beta k}{\eta_1} < R_1 < 1 + \frac{\beta k(\eta_2+v-p)+\eta_1 v}{\eta_1\eta_2}$. Then $f(S_1)f(S_2) < 0$. As a result, $f(S)$ has a unique zero $S_p \in (S_1, S_2)$. Consequently, System (2.1) has a unique pseudo-equilibrium E_p. Since $f'(S_p) < 0$, we have that solutions starting from $\overline{\Sigma}_s \backslash \{E_p\}$ will slide along Σ_s and converge to E_p as $t \to +\infty$. □

3. Global asymptotical stability

In this section, we will investigate the global asymptotical stability for equilibria of System (2.1). At first, by employing the Lyapunov approach, we show the disease-free equilibrium E_0 is globally asymptotically stable if $R_1 \leq 1$.

Theorem 3.1. *If $R_1 \leq 1$, then the disease-free equilibrium E_0 is globally asymptotically stable.*

Proof. By Definition 2.1, System (2.1) can be rewritten as

$$\begin{cases} \dfrac{\mathrm{d}S}{\mathrm{d}t} = -\eta_1(S - S^*) - \beta I(S - S^*) - (\beta S^* - p\gamma)I, \\ \dfrac{\mathrm{d}I}{\mathrm{d}t} = \beta I(S - S^*) + (\beta S^* - \eta_2 - v\gamma)I, \end{cases}$$

where $S^* = \frac{A}{\eta_1}$ and $\gamma \in [0, 1]$. Consider the Lyapunov function

$$V_0(S, I) = S - S^* - S^* \ln \frac{S}{S^*} + I.$$

Then

$$\begin{aligned} \left.\frac{dV_0(S, I)}{\mathrm{d}t}\right|_{(2.1)} &= \frac{(S - S^*)}{S}\frac{\mathrm{d}S}{\mathrm{d}t} + \frac{\mathrm{d}I}{\mathrm{d}t} \\ &= -\frac{\eta_1}{S}(S - S^*)^2 + \frac{1}{S}\left[-\beta I(S - S^*)^2 - (\beta S^* - p\gamma)(S - S^*)I + \beta S(S - S^*)I\right] \end{aligned}$$

$$
\begin{aligned}
&\quad + (\beta S^* - \eta_2 - v\gamma)I \\
&= -\frac{\eta_1}{S}(S - S^*)^2 + \frac{p\gamma(S - S^*)I}{S} + (\beta S^* - \eta_2 - v\gamma)I \\
&\le -\frac{\eta_1}{S}(S - S^*)^2 + (\beta S^* - \eta_2 - v\gamma + p\gamma)I \\
&= -\frac{\eta_1}{S}(S - S^*)^2 + \eta_2(R_1 - 1)I - \gamma(v - p)I \\
&\le -\frac{\eta_1}{S}(S - S^*)^2,
\end{aligned}
$$

which means that E_0 is globally asymptotically stable by the LaSalle Invariance Principle. □

In the sequel, we resort to Poincaré maps to study the global asymptotical stability of an endemic equilibrium or a pseudo-equilibrium. For System (2.1), suppose that there is an orbit starting from (S_0, k) with $S_0 \in (0, S_1)$ such that it will enter the region G_1 firstly, reach Σ at $(\tilde{S}, k)$ with $\tilde{S} > S_2$ in finite time secondly, go into G_2 thirdly, and reach Σ again at $(\hat{S}, k)$ with $\hat{S} \le S_1$ finally. Then a Poincaré map $P(\cdot)$ can be defined as

$$\hat{S} = P(S_0),$$

see Fig. 1 for illustration. Some properties are observed by the following lemma.

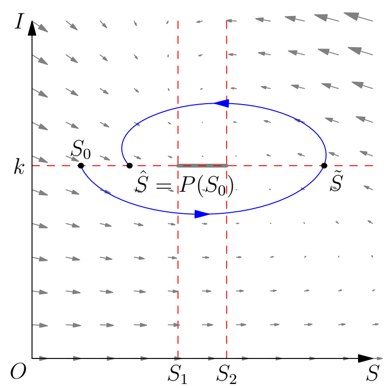

Figure 1. Illustration for the definition of the Poincaré map $P(\cdot)$.

Lemma 3.1. *Suppose* $P^n(\cdot) = \underbrace{P \circ P \circ \cdots \circ P}_{n}(\cdot)$ *is well defined on a subinterval* $\mathcal{I}$ *of* $(0, S_1)$ *for a positive integer* n. *Then* $S_0 + \frac{2nv}{\beta} < P^n(S_0) \le S_1$ *for* $S_0 \in \mathcal{I}$.

Proof. By the definition of $P(\cdot)$, it is obvious that $P^{(}S_0) \le S_1$ and thus $P^n(S_0) \le S_1$ if $P^n(S_0)$ is well defined. Rewrite the subsystem (2.3) as

$$
\begin{cases}
\frac{\mathrm{d}S}{\mathrm{d}t} = -\eta_1(S - S_1) - \beta(S - S_1)I - \beta S_1(I - I_1), \\
\frac{\mathrm{d}I}{\mathrm{d}t} = \beta(S - S_1)I
\end{cases}
\tag{3.1}
$$

and consider

$$V_1(S, I) = \frac{1}{2}(S - S_1)^2 + S_1\left(I - I_1 - I_1 \ln \frac{I}{I_1}\right),$$

we have

$$\left.\frac{\mathrm{d}V_1(S,I)}{\mathrm{d}t}\right|_{(3.1)} = -(\eta_1+\beta I)(S-S_1)^2 \le 0. \tag{3.2}$$

Consequently, $V_1(S_0,k) \ge V_1(\tilde{S}_0,k)$, i.e.,

$$S_0+\tilde{S}_0 \le 2S_1. \tag{3.3}$$

Similarly, it comes that

$$\tilde{S}_0+P(S_0) \ge 2S_2. \tag{3.4}$$

Therefore, inequalities (3.3) and (3.4) imply that

$$P(S_0)-S_0 \ge 2(S_2-S_1) = \frac{2v}{\beta}.$$

Thus

$$P^n(S_0) = P(P^{n-1}(S_0)) \ge P^{n-1}(S_0)+\frac{2v}{\beta} \ge \cdots \ge S_0+\frac{2nv}{\beta},$$

which completes the proof. □

When $1 < R_1 < 1+\frac{\beta}{\eta_1}k$, we have $I_2 < I_1 < k$, which implies that E_1 is real and E_2 is virtual. The global asymptotical stability of E_1 is gained by the following theorem.

Theorem 3.2. *If $1 < R_1 < 1+\frac{\beta k}{\eta_1}$, then the endemic equilibrium E_1 is globally asymptotically stable for System* (2.1).

Proof. Since E_2 is a global asymptotically stable equilibrium of the subsystem (2.4) by Proposition 2.2, any orbit starting from G_2 will reach the switching line Σ in finite time. Furthermore, any orbit from Σ_s slides along Σ_s to the tangent point T_1 in finite time by the assertion (i) of Proposition 2.3, and the orbit from T_1 stays in G_1 for $\bar{t} > 0$ and converges to E_1 by the fact that E_1 is a global asymptotically stable equilibrium of the subsystem (2.3) from Proposition 2.1. Based on these observations, we claim that any orbit of System (2.1) will always stay in G_1 after some time $\tilde{t} \ge 0$. We prove this claim by contradiction. If it is not true, then there is $S_0 \in (0,S_1)$ such that $P^n(S_0)$ is well defined for $n=1,2,\cdots$. It follows from Lemma 3.1 that $S_0+\frac{2nv}{\beta} < P^n(S_0) \le S_1$ for $n=1,2,\cdots$, which leads to a contradiction since $\lim\limits_{n\to+\infty} S_0+\frac{2nv}{\beta} = +\infty$. Therefore, the claim is true and E_1 is globally asymptotically stable again by Proposition 2.1. □

Now assume

$$1+\frac{\beta k}{\eta_1} < R_1 < 1+\frac{(\eta_2+v-p)\beta k+\eta_1 v}{\eta_1\eta_2}. \tag{3.5}$$

In this case, we have $I_2 < k < I_1$. Thus both E_1 and E_2 are virtual. It follows from Proposition 2.3 that there is a unique pseudo-equilibrium E_p. Moreover, we will show E_p is globally asymptotically stable.

Theorem 3.3. *Suppose that* (3.5) *holds. Then the pseudo-equilibrium E_p is globally asymptotically stable for System* (2.1)*. Moreover, the number of the infected plants converges to k in finite time, i.e., for any orbit $(S(t),I(t))$ of System* (2.1) *there is $\tilde{t} > 0$ such that $I(t)=k$ for $t \ge \tilde{t}$.*

Proof. It follows from Proposition 2.3 that orbits starting from $\overline{\Sigma}_s\backslash\{E_p\}$ will slide along Σ_s to E_p as $t \to +\infty$. Notice that both E_1 and E_2 are virtual under the condition (3.5). Then orbits starting from G_1 or G_2 will reach the switching line Σ in finite time by Proposition 2.1 and Proposition 2.2. Next by the way of contradiction, we show that any orbit of System (2.1) will reach Σ in finite time, which implies that the number of the infected plants converges to k in finite time and E_p is globally asymptotically stable. Assume this conclusion is not true. Then there is an orbit of System (2.1) spiraling Σ_s. Hence there exists $S_0 \in (0, S_1)$ such that $P^n(S_0)$ is well defined for $n = 1, 2, \cdots$. Consequently, $P^n(S_0) \leq S_1$ by Lemma 3.1. However, $\lim\limits_{n\to+\infty} P^n(S_0) = +\infty$ again by Lemma 3.1, which is a contradiction. □

When $R_1 > 1 + \frac{(\eta_2+v-p)\beta k+\eta_1 v}{\eta_1\eta_2}$, we get $I_1 > I_2 > k$. This means E_1 is virtual and E_2 is real.

Theorem 3.4. *If $R_1 > 1 + \frac{(\eta_2+v-p)\beta k+\eta_1 v}{\eta_1\eta_2}$, then the endemic equilibrium E_2 is globally asymptotically stable for System* (2.1).

Proof. Since E_1 is a global asymptotically stable equilibrium of the subsystem (2.3) by Proposition 2.1, any orbit starting from G_1 will reach the switching line Σ in finite time. Moreover, any orbit starting from Σ_s slides along Σ_s to the tangent point T_2 in finite time by the assertion (iii) of Proposition 2.3, and the orbit starting from T_2 stays in G_2 afterwards and converges to E_2 by Proposition 2.1. These facts allow us to prove that any orbit of System (2.1) will always stay in G_2 after some time. By the way of contradiction, assume it is not true, then there is $S_0 \in (0, S_1)$ such that $P^n(S_0)$ is well defined for $n = 1, 2, \cdots$. On one hand, $P^n(S_0) \leq S_1$, which means $\{P^n(S_0)\}$ is bounded by S_1. On the other hand, $P^n(S_0) > S_0 + \frac{2nv}{\beta}$ and thus $\lim\limits_{n\to+\infty} P^n(s_0) = +\infty$. This is a contradiction to the boundedness. Therefore, E_2 is globally asymptotically stable for System (2.1) by Proposition 2.2. □

4. Biological implications

In this section, we will reveal possible biological implications of our theoretical results and discuss the effectiveness of the replanting and removing measures. It is shown that the control goal, namely maintaining the infected plants not to exceed the economic threshold k eventually, could be achieved by taking appropriate replanting rate p and removing rate v.

When $R_1 \leq 1$, Theorem 3.1 tells us that the disease-free equilibrium E_0 is globally asymptotically stable, see Fig. 2. This means that the disease will die out eventually, and thus the control goal could be reached. Without the control measures, our goal can also be achieved, i.e., $p = v = 0$. However, it should be pointed out that the control measures will help us to speed up to achieve the goal, see Fig. 3

When $1 < R_1 < 1 + \frac{\beta k}{\eta_1}$, it follows from Theorem 3.2 that E_1 is globally asymptotically stable, see Fig. 4. This means that the number of the infected plants is below k eventually and the control goal could be achieved. Although the control goal can also be reached without the measures, it could be got rapidly by taking appropriate p and v, see Fig. 5.

When $R_1 > 1 + \frac{\beta k}{\eta_1}$, the control goal fails to be reached without the control measures as well as with small removing rate v, since E_2 is globally asymptotically

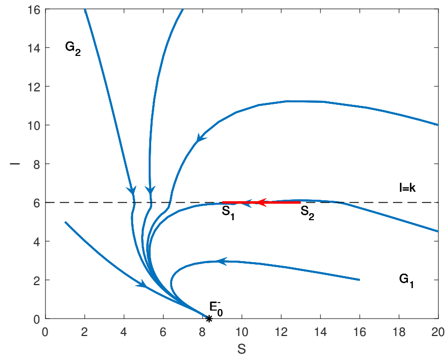

Figure 2. Global asymptotical stability of E_0 for System (2.1) with $A = 5$, $\beta = 0.1$, $\eta_1 = 0.6$, $\eta_2 = 0.9$, $v = 0.4$, $p = 0.3$, $k = 6$.

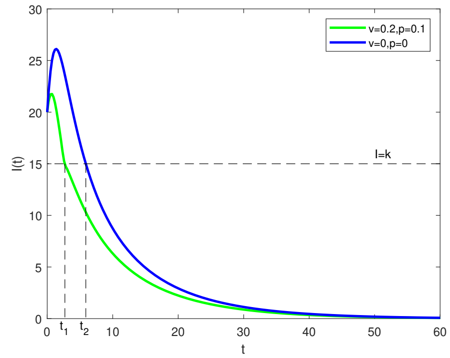

Figure 3. Time series of $I(t)$ by taking different p and v, where the initial condition is $(S(0), I(0)) = (60, 20)$ and the other parameters are fixed as: $A = 8$, $\beta = 0.04$, $\eta_1 = 0.3$, $\eta_2 = 0.8$, $k = 10$.

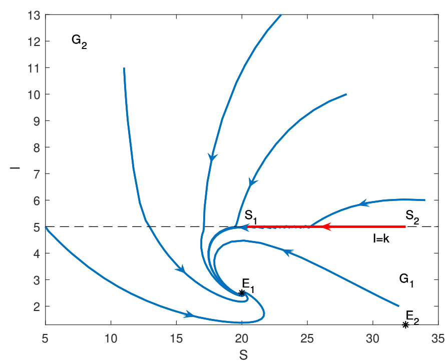

Figure 4. Global asymptotical stability of E_1 for System (2.1) with $A = 8$, $\beta = 0.04$, $\eta_1 = 0.3$, $\eta_2 = 0.8$, $v = 0.5$, $p = 0.1$, $k = 5$.

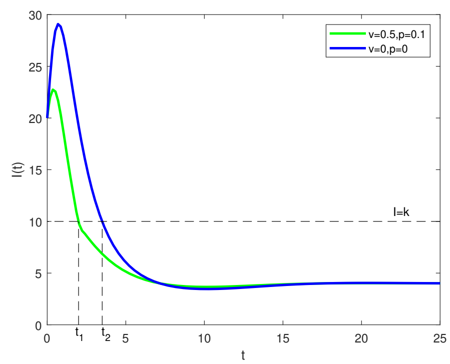

Figure 5. Time series of $I(t)$ by taking different p and v, where the initial condition is $(S(0), I(0)) = (60, 20)$ and the other parameters are fixed as: $A = 8$, $\beta = 0.04$, $\eta_1 = 0.3$, $\eta_2 = 0.8$, $k = 10$.

stable by Theorem 3.4, see Fig. 6. However, according to Theorem 3.3, E_p is globally asymptotically stable if (3.5) holds, see Fig. 7. Thus one can maintain the number of the infected plants not to exceed k as long as p and v are picked up to satisfy $R_1 < 1 + \frac{(\eta_2+v-p)\beta k+\eta_1 v}{\eta_1\eta_2}$.

In summary, some remarks are given in the following to show the novelty and new phenomenon in our model compared with the model without the economic threshold. It is clear that some new dynamics such as sliding solutions and pseudo-equilibria have been induced by the economic threshold in our model. These dynamics lead to some difficulties in investigating the global dynamics, which cannot appear in the model without the economic threshold. Moreover, when $R_1 > 1 + \frac{\beta k}{\eta_1}$, the control goal will not be achieved without the economic threshold and the control measures, while it can be reached if the disease dynamics can be modeled by our model.

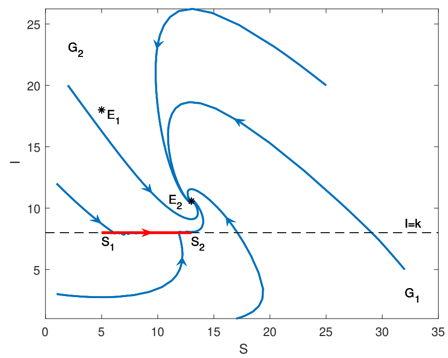

Figure 6. Global asymptotical stability of E_2 for System (2.1), where the parameters are chosen as: $A = 10$, $\beta = 0.1$, $\eta_1 = 0.2$, $\eta_2 = 0.5$, $v = 0.8$, $p = 0.6$, $k = 8$.

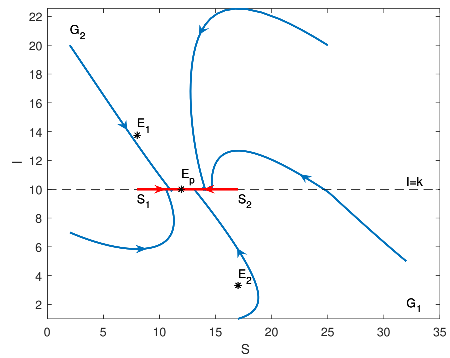

Figure 7. Global asymptotical stability of $E_p$for System (2.1), where the parameters are chosen as: $A = 15$, $\beta = 0.1$, $\eta_1 = 0.55$, $\eta_2 = 0.8$, $v = 0.9$, $p = 0.8$, $k = 10$.

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