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# A NOVEL TECHNIQUE FOR SOLVING (2+1) DIMENSIONAL SYSTEM OF NONLINEAR COUPLED PARTIAL DIFFERENTIAL EQUATION

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**Abstract** We present a highly efficient method to find numerical solutions to the system of PDEs. The method unifies the methods of collocation and Laguerre wavelet series (LWS). The system of (2+1)-dimensional PDEs is reduced to a set of equations having Laguerre wavelet coefficients (LWC). Computational examples are provided to validate the efficiency of the technique and we discussed the comparison between the present method and other methods solution with the exact solution. Computational results indicate that the present method is better than the other methods in the literature.

**Keywords** Wavelets, Laguerre wavelet, collocation method, system of partial differential equation.

**MSC(2010)** 65T60, 41A65, 35G50.

## 1. Introduction

We present a new method to acquire numerical solutions to the system of (2+1)-dimensional PDEs. Determining the numerical solutions for a system of nonlinear PDEs is a significantly useful research area. These equations appeared in many disciplines, including plasma physics, solid-state physics, fluid mechanics, chemical physics, and plasma waves. Consider,

$$\frac{\partial u}{\partial t} - \frac{\partial v}{\partial t}\frac{\partial u}{\partial y} - v\frac{\partial u}{\partial x} = P(x,y,t), \tag{1.1}$$

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$$\frac{\partial v}{\partial t} - \frac{\partial u}{\partial t}\frac{\partial v}{\partial y} - u\frac{\partial v}{\partial x} = Q(x,y,t), \tag{1.2}$$

with physical constraints,

$$\begin{aligned} u(x,y,0) &= a(x,y),\\ u(0,y,t) &= b(y,t),\\ u(x,0,t) &= c(x,t),\\ u(0,0,t) &= g(t), \end{aligned} \tag{1.3}$$

$$\begin{aligned} v(x,y,0) &= d(x,y),\\ v(0,y,t) &= e(y,t),\\ v(x,0,t) &= f(x,t),\\ v(0,0,t) &= h(t), \end{aligned} \tag{1.4}$$

where, $0 \leq x < 1, 0 \leq y < 1, 0 \leq t < 1$, and $Q(x,y,t), P(x,y,t), a(x,y), b(y,t), c(x,t), g(t), d(x,y), e(y,t), f(x,t)$ and $h(t)$ are real-valued continuous functions. Wavelets are useful and important functions having a broad range of applications including time-frequency analysis and signal processing and so on, see [23]. The use of wavelets in the solutions of the system of PDEs is still in its infancy.

Wavelets used to solve the PDEs may be listed as follows: Cardinal B-spline wavelet to the Burgers-Huxley equation (BHE) [27], some others for the nonlinear Klein-Gordon equation [9], Laguerre wavelets to the system of differential equations [28], Haar wavelets to the system of PDEs [1]. There are some other related methods to solve the system of NPDEs such as Homotopy perturbation [4], Homotopy analysis [18], solution method [32], the composite numerical scheme for the coupled Burgers system [10], Semi-analytical technique to the foam drainage equation [2], Haar wavelet operational method to the neutron point kinetics model [19], variational iteration to the BH and Huxley equations [21], Wavelet collocation to the Huxley equation [22] and two dimensional Haar wavelet collocation [20], new Homotopy perturbation method [5, 12, 13], numerical approach for drainage equation [31], efficient methods for time-dependent problems [3, 7, 8, 11, 29], Some wavelets methods applied to solve differential equations [6, 14–17, 30] etc. We employ LWS and in the view of literature, no one solved this type of problem using LWS, this impetus us to solve (2+1) dimensional system PDEs via LW.

In section 2, the fundamentals of LWS are presented. The convergence analysis is studied in the 3rd section. Sections 4 and 5 contain the solution algorithm and the applications, respectively. We complete the paper with a conclusions part in Section 6.

## 2. Laguerre wavelets

Wavelets form a family of functions generated from translation and dilation a function known as mother wavelet. LWS are described as [24]:

$$\psi_{n,m}(x) = \begin{cases} \dfrac{2^{\frac{k}{2}}}{m!} L_m(2^k x - 2n + 1), & \dfrac{n-1}{2^{k-1}} \leq x < \dfrac{n}{2^{k-1}} \\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$

in which $m = 0, 1, \ldots, M-1$ and $n = 1, 2, ..., 2^{k-1}$ with $k$ is a natural number, $L_m(x)$ are Laguerre polynomials with the weight function $W(x) = 1$ on $[0, \infty)$ satisfying $L_0(x) = 1$, $L_1(x) = 1 - x$,

$$L_{m+2}(x) = \frac{(2m+3-x)L_{m+1}(x) - (m+1)L_m(x)}{m+2} \quad \text{where } m = 0, 1, 2, \ldots.$$

# 3. Theorem on convergence analysis

**Theorem 3.1.** *Suppose that $u(x, y, t) \in L^2(R^2 \times R)$ is a bounded continuous real-valued function on $[0, 1)^2 \times [0, 1)$. Then, LW expansion of $u(x, y, t)$ uniformly converges to itself.*

**Proof.** Assume that $u(x, y, t)$ is a continuous function on $[0, 1)^2 \times [0, 1)$ and $|u(x, y, t)| \leq \kappa$, in which $\kappa$ is a natural number. Suppose that

$$u(x, y, t) = \sum_{i=1}^{\infty} \sum_{j=0}^{\infty} C_{i,j} \psi_{i,j}(x) \psi_{i,j}(y) \psi_{i,j}(t).$$

$C_{i,j} =< u(x, y, t), \psi_{i,j}(x)\psi_{i,j}(y)\psi_{i,j}(t) >$, and $<, >$ represents the inner product. Hence, LWC of $u(x, y, t)$ are described as:

$$\begin{aligned}
C_{i,j} &= \int_0^1 \int_0^1 \int_0^1 u(x, y, t) \psi_{i,j}(x) \psi_{i,j}(y) \psi_{i,j}(t) dx dy dt, \\
&= \int_0^1 \int_0^1 \psi_{i,j}(x) \psi_{i,j}(y) \int_I u(x, y, t) \frac{2^{\frac{k}{2}}}{m!} L_m(2^k t - 2n + 1) dt dx dy, \\
&\text{where} \quad I = [\frac{n-1}{2^{k-1}}, \frac{n}{2^{k-1}}).
\end{aligned}$$

Next by letting $2^k t - 2n + 1 = p$, we get

$$\begin{aligned}
C_{i,j} &= \frac{2^{\frac{k}{2}}}{m!} \int_0^1 \int_0^1 \psi_{i,j}(x) \psi_{i,j}(y) [\int_{-1}^1 u(x, y, \frac{p-1+2n}{2^k}) L_m(p) \frac{dp}{2^k}] dx dy, \\
&= \frac{2^{\frac{-k}{2}}}{m!} \int_0^1 \int_0^1 [\int_{-1}^1 u(x, y, \frac{p-1+2n}{2^k}) L_m(p) dp] \psi_{i,j}(x) \psi_{i,j}(y) dx dy.
\end{aligned}$$

Using GMVT for integrals

$$\begin{aligned}
C_{i,j} &= \frac{2^{\frac{-k}{2}}}{m!} \int_0^1 \int_0^1 u(x, y, \frac{\zeta_1 - 1 + 2n}{2^k}) \psi_{i,j}(x) \psi_{i,j}(y) dx dy [\int_{-1}^1 L_m(p) dp], \\
&\text{where } \zeta_1 \in (-1, 1).
\end{aligned}$$

Since, $L_m(t)$ is continuous and integrable on $(-1, 1)$. Choose $\int_{-1}^1 L_m(p) dp = A$,

$$\begin{aligned}
C_{i,j} &= A \frac{2^{\frac{-k}{2}}}{m!} \int_0^1 \int_0^1 u(x, y, \frac{\zeta_1 - 1 + 2n}{2^k}) \psi_{i,j}(x) \psi_{i,j}(y) dx dy, \\
&= A \frac{2^{\frac{-k}{2}}}{m!} \int_0^1 \psi_{i,j}(x) \int_{\frac{n-1}{2^{k-1}}}^{\frac{n}{2^{k-1}}} u(x, y, \frac{\zeta_1 - 1 + 2n}{2^k}) \frac{2^{\frac{k}{2}}}{m!} L_m(2^k y - 2n + 1) dy dx.
\end{aligned}$$

Now by changing the variable $2^k y - 2n + 1 = q$, we obtain

$$C_{i,j} = \frac{A}{(m!)^2} \int_0^1 \psi_{i,j}(x) \int_{-1}^1 u(x, \frac{q-1+2n}{2^k}, \frac{\zeta_1 - 1 + 2n}{2^k}) L_m(q) \frac{dq}{2^k} dx,$$

$$C_{i,j} = \frac{A2^{-k}}{(m!)^2} \int_0^1 \psi_{i,j}(x) \int_{-1}^1 u(x, \frac{q-1+2n}{2^k}, \frac{\zeta_1 - 1 + 2n}{2^k}) L_m(q) dq dx.$$

Using GMVT for integrals

$$C_{i,j} = \frac{A2^{-k}}{(m!)^2} \int_0^1 \psi_{i,j}(x) u(x, \frac{\zeta_2-1+2n}{2^k}, \frac{\zeta_1 - 1 + 2n}{2^k}) \int_{-1}^1 L_m(q) dq dx,$$

where $\zeta_2 \in (-1, 1)$.

Since, $L_m(y)$ is continuous and integrable on $(-1, 1)$. Choose $\int_{-1}^1 L_m(q)dq = A$,

$$\begin{aligned} C_{i,j} &= \frac{A^2 2^{-k}}{(m!)^2} \int_0^1 \psi_{i,j}(x) u(x, \frac{\zeta_2-1+2n}{2^k}, \frac{\zeta_1 - 1 + 2n}{2^k}) dx, \\ &= \frac{A^2 2^{-k}}{(m!)^2} \int_I \frac{2^{\frac{k}{2}}}{m!} L_m(2^k x - 2n + 1) u(x, \frac{\zeta_2-1+2n}{2^k}, \frac{\zeta_1 - 1 + 2n}{2^k}) dx, \end{aligned}$$

now by changing the variable $2^k x - 2n + 1 = r$, we acquire

$$C_{i,j} = \frac{A^2 2^{\frac{-k}{2}}}{(m!)^3} \int_{-1}^1 L_m(r) u(\frac{r-1+2n}{2^k}, \frac{\zeta_2-1+2n}{2^k}, \frac{\zeta_1 - 1 + 2n}{2^k}) \frac{dr}{2^k}.$$

Using Generalized mean value theorem (GMVT) for integrals

$$C_{i,j} = \frac{A^2 2^{\frac{-3k}{2}}}{(m!)^3} u(\frac{\zeta_3-1+2n}{2^k}, \frac{\zeta_2-1+2n}{2^k}, \frac{\zeta_1 - 1 + 2n}{2^k}) \int_{-1}^1 L_m(r) dr.$$

Since, $L_m(x)$ is continuous and integrable on $(-1, 1)$. Choose $\int_{-1}^1 L_m(r)dr = A$,

$$C_{i,j} = \frac{A^3 2^{\frac{-3k}{2}}}{(m!)^3} u(\frac{\zeta_3-1+2n}{2^k}, \frac{\zeta_2-1+2n}{2^k}, \frac{\zeta_1 - 1 + 2n}{2^k}).$$

Therefore,

$$|C_{i,j}| = |\frac{A^3 2^{\frac{-3k}{2}}}{(m!)^3}||u(\frac{\zeta_3-1+2n}{2^k}, \frac{\zeta_2-1+2n}{2^k}, \frac{\zeta_1 - 1 + 2n}{2^k})|,$$

where $\zeta_1, \zeta_2, \zeta_3 \in (-1, 1)$.

Since $u(x, y, t)$ is bounded. That is, $|u(x, y, t)| \leq \kappa$, where $\kappa$ is real constant.

$$|C_{i,j}| = |\frac{A^3 2^{\frac{-3k}{2}}}{(m!)^3}|\kappa.$$

Therefore $\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} c_{i,j}$ is absolutely convergent. Hence the LW expansion of $u(x, y, t)$ is converges uniformly. □

# 4. Method of solution

In this section, we generate LWM to acquire numerical solution for (2+1) dimensional system of NPDE given in the equation (1.1) and (1.2) with physical conditions (1.3) and (1.4).
Consider (1.1) with constraints in (1.3) and assume:

$$\frac{\partial^3 u(x,y,t)}{\partial t \partial y \partial x} = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{\infty}\sum_{q=0}^{\infty} d_{p,q}\psi_{p.q}(y) \tag{4.1}$$

truncating the above equation,

$$\frac{\partial^3 u(x,y,t)}{\partial t \partial y \partial x} \approx \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1} d_{p,q}\psi_{p.q}(y). \tag{4.2}$$

Integrate (4.2) with respect to $t$ from 0 to $t_s$,

$$\frac{\partial^2 u(x,y,t)}{\partial y \partial x} \approx \frac{\partial^2 u(x,y,t_s)}{\partial y \partial x} + (t-t_s)\left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1} d_{p,q}\psi_{p.q}(y)\right]. \tag{4.3}$$

Integrate (4.3) with respect to $y$ from 0 to $y$,

$$\begin{aligned}\frac{\partial u(x,y,t)}{\partial x} \approx & \frac{\partial u(x,0,t)}{\partial x} + \frac{\partial u(x,y,t_s)}{\partial x} - \frac{\partial u(x,0,t_s)}{\partial x} \\ & + (t-t_s)\int_0^y \left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1} d_{p,q}\psi_{p.q}(y)\right] dy.\end{aligned} \tag{4.4}$$

Integrate (4.4) with respect to $x$ from 0 to $x$,

$$\begin{aligned} u(x,y,t) \approx & u(0,y,t) + u(x,0,t) - u(0,0,t) + u(x,y,t_s) \\ & - u(0,y,t_s) - u(x,0,t_s) + u(0,0,t_s) \\ & + (t-t_s)\int_0^x\int_0^y \left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1} d_{p,q}\psi_{p.q}(y)\right] dydx \\ u(x,y,t) \approx & b(y,t) + c(x,t) - g(t) + a(x,y,t_s) - b(y,t_s) - c(x,t_s) + g(t_s) \\ & + (t-t_s)\int_0^x\int_0^y \left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1} d_{p,q}\psi_{p.q}(y)\right] dydx \end{aligned} \tag{4.5}$$

Differentiate (4.5) with respect to $t$, $x$ and $y$, we get following equations:

$$\begin{aligned}\frac{\partial u(x,y,t)}{\partial t} \approx & \frac{\partial b(y,t)}{\partial t} + \frac{\partial c(x,t)}{\partial t} - \frac{\partial g(t)}{\partial t} \\ & + \int_0^x\int_0^y \left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1} d_{p,q}\psi_{p.q}(y)\right] dydx,\end{aligned} \tag{4.6}$$

$$\frac{\partial u(x,y,t)}{\partial x} \approx \frac{\partial c(x,t)}{\partial x} + \frac{\partial a(x,y,t_s)}{\partial x} - \frac{\partial c(x,t_s)}{\partial x} + (t-t_s)\int_0^y \left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1} d_{p,q}\psi_{p.q}(y)\right]dy, \tag{4.7}$$

$$\frac{\partial u(x,y,t)}{\partial y} \approx \frac{\partial b(y,t)}{\partial y} + \frac{\partial a(x,y,t_s)}{\partial y} - \frac{\partial b(y,t_s)}{\partial y} + (t-t_s)\int_0^x \left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1} d_{p,q}\psi_{p.q}(y)\right]dx. \tag{4.8}$$

Now, consider (1.2) with constraints in (1.4) and assume:

$$\frac{\partial^3 v(x,y,t)}{\partial t\partial y\partial x} = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c'_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{\infty}\sum_{q=0}^{\infty} d'_{p,q}\psi_{p.q}(y) \tag{4.9}$$

truncating the above equation,

$$\frac{\partial^3 v(x,y,t)}{\partial t\partial y\partial x} \approx \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c'_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1} d'_{p,q}\psi_{p.q}(y). \tag{4.10}$$

Integrate (4.10) with respect to $t$ from 0 to $t_s$,

$$\frac{\partial^2 v(x,y,t)}{\partial y\partial x} \approx \frac{\partial^2 v(x,y,t_s)}{\partial y\partial x} + (t-t_s)\left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c'_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1} d'_{p,q}\psi_{p.q}(y)\right]. \tag{4.11}$$

Integrate (4.11) with respect to $y$ from 0 to $y$,

$$\frac{\partial v(x,y,t)}{\partial x} \approx \frac{\partial v(x,0,t)}{\partial x} + \frac{\partial v(x,y,t_s)}{\partial x} - \frac{\partial v(x,0,t_s)}{\partial x} + (t-t_s)\int_0^y \left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c'_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1} d'_{p,q}\psi_{p.q}(y)\right]dy. \tag{4.12}$$

Integrate (4.12) with respect to $x$ from 0 to $x$,

$$\begin{aligned} v(x,y,t) \approx\, & v(0,y,t) + v(x,0,t) - v(0,0,t) + v(x,y,t_s) \\ & - v(0,y,t_s) - v(x,0,t_s) + v(0,0,t_s) \\ & + (t-t_s)\int_0^x\int_0^y \left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c'_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1} d'_{p,q}\psi_{p.q}(y)\right]dydx, \\ v(x,y,t) \approx\, & e(y,t) + f(x,t) - h(t) + d(x,y,t_s) - e(y,t_s) - f(x,t_s) + h(t_s) \\ & + (t-t_s)\int_0^x\int_0^y \left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c'_{n,m}\psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1} d'_{p,q}\psi_{p.q}(y)\right]dydx. \end{aligned} \tag{4.13}$$

Differentiate (4.13) with respect to $t$, $x$ and $y$, we get following equations:

$$\frac{\partial v(x,y,t)}{\partial t} \approx \frac{\partial e(y,t)}{\partial t} + \frac{\partial f(x,t)}{\partial t} - \frac{\partial h(t)}{\partial t} + \int_0^x \int_0^y \left[ \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c'_{n,m} \psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}} \sum_{q=0}^{M-1} d'_{p,q} \psi_{p.q}(y) \right] dydx, \tag{4.14}$$

$$\frac{\partial v(x,y,t)}{\partial x} \approx \frac{\partial f(x,t)}{\partial x} + \frac{\partial d(x,y,t_s)}{\partial x} - \frac{\partial f(x,t_s)}{\partial x} + (t-t_s) \int_0^y \left[ \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c'_{n,m} \psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}} \sum_{q=0}^{M-1} d'_{p,q} \psi_{p.q}(y) \right] dy, \tag{4.15}$$

$$\frac{\partial v(x,y,t)}{\partial y} \approx \frac{\partial e(y,t)}{\partial y} + \frac{\partial d(x,y,t_s)}{\partial y} - \frac{\partial e(y,t_s)}{\partial y} + (t-t_s) \int_0^x \left[ \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c'_{n,m} \psi_{n.m}(x) + \sum_{p=1}^{2^{k-1}} \sum_{q=0}^{M-1} d'_{p,q} \psi_{p.q}(y) \right] dx. \tag{4.16}$$

Substitute the equations (4.5), (4.6), (4.7), (4.8), (4.13), (4.14), (4.15) and (4.16) in the equations (1.1) and (1.2). Then descritize the equations (1.1) and (1.2) with following collocation points,

$$x_i = y_i = \frac{i - \frac{1}{2}}{2M-1}, \;\; i = 1, 2, 3, ..., 2M-1, \;\; t \in [0, 1).$$

Which yields a system containing $4M-2$ number of nonlinear algebraic equations as follows:

$$\begin{aligned}
&P(x_i, y_i, t) \\
=&\frac{\partial b(y_i,t)}{\partial t} + \frac{\partial c(x_i,t)}{\partial t} + \int_0^x \int_0^y \left[ \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \psi_{n.m}(x_i) + \sum_{p=1}^{2^{k-1}} \sum_{q=0}^{M-1} d_{p,q} \psi_{p.q}(y_i) \right] dydx \\
&- \frac{\partial g(t)}{\partial t} - [e(y_i,t) + f(x_i,t) - h(t) + d(x_i,y_i,t_s) - e(y_i,t_s) - f(x_i,t_s) + h(t_s) \\
&+ (t-t_s) \int_0^x \int_0^y \left[ \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c'_{n,m} \psi_{n.m}(x_i) + \sum_{p=1}^{2^{k-1}} \sum_{q=0}^{M-1} d'_{p,q} \psi_{p.q}(y_i) \right] dydx] \\
&\times [\frac{\partial c(x_i,t)}{\partial x} + \frac{\partial a(x_i,y_i,t_s)}{\partial x} - \frac{\partial c(x_i,t_s)}{\partial x} \\
&+ (t-t_s) \int_0^y \left[ \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \psi_{n.m}(x_i) + \sum_{p=1}^{2^{k-1}} \sum_{q=0}^{M-1} d_{p,q} \psi_{p.q}(y_i) \right] dy] \\
&- [\frac{\partial e(y_i,t)}{\partial t} + \frac{\partial f(x_i,t)}{\partial t} - \frac{\partial h(t)}{\partial t}
\end{aligned}$$

$$+\int_0^x\int_0^y\left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}c'_{n,m}\psi_{n.m}(x_i)+\sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1}d'_{p,q}\psi_{p.q}(y_i)\right]dydx][\frac{\partial b(y_i,t)}{\partial y}-\frac{\partial b(y_i,t_s)}{\partial y}$$
$$+(t-t_s)\int_0^x\left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}c_{n,m}\psi_{n.m}(x_i)+\sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1}d_{p,q}\psi_{p.q}(y_i)\right]dx+\frac{\partial a(x_i,y_i,t_s)}{\partial y}] \tag{4.17}$$

$$Q(x_i,y_i,t)$$
$$=\frac{\partial e(y_i,t)}{\partial t}+\frac{\partial f(x_i,t)}{\partial t}+\int_0^x\int_0^y\left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}c'_{n,m}\psi_{n.m}(x_i)+\sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1}d'_{p,q}\psi_{p.q}(y_i)\right]dydx$$
$$-\frac{\partial h(t)}{\partial t}-[b(y_i,t)+c(x_i,t)-g(t)+a(x_i,y_i,t_s)-b(y_i,t_s)-c(x_i,t_s)+g(t_s)$$
$$+(t-t_s)\int_0^x\int_0^y\left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}c_{n,m}\psi_{n.m}(x_i)+\sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1}d_{p,q}\psi_{p.q}(y_i)\right]dydx]$$
$$\times[\frac{\partial f(x_i,t)}{\partial x}+\frac{\partial d(x_i,y_i,t_s)}{\partial x}-\frac{\partial f(x_i,t_s)}{\partial x}$$
$$+(t-t_s)\int_0^y\left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}c'_{n,m}\psi_{n.m}(x_i)+\sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1}d'_{p,q}\psi_{p.q}(y_i)\right]dy]$$
$$-[\frac{\partial b(y_i,t)}{\partial t}+\frac{\partial c(x_i,t)}{\partial t}-\frac{\partial g(t)}{\partial t}$$
$$+\int_0^x\int_0^y\left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}c_{n,m}\psi_{n.m}(x_i)+\sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1}d_{p,q}\psi_{p.q}(y_i)\right]dydx][\frac{\partial e(y_i,t)}{\partial y}-\frac{\partial e(y_i,t_s)}{\partial y}$$
$$+(t-t_s)\int_0^x\left[\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}c'_{n,m}\psi_{n.m}(x_i)+\sum_{p=1}^{2^{k-1}}\sum_{q=0}^{M-1}d'_{p,q}\psi_{p.q}(y_i)\right]dx+\frac{\partial d(x_i,y_i,t_s)}{\partial y}]. \tag{4.18}$$

On solving above system by Newton-Raphson algorithm yields $4M-2$ LWCs. substitute these coefficients in (4.5) and (4.13) will contribute numerical solution for given system of PDEs.

# 5. Numerical Results

**Example 5.1.** Consider the system of (2+1)-D PDE is of the form [1]:

$$\frac{\partial u}{\partial t}-\frac{\partial v}{\partial t}\frac{\partial u}{\partial y}-v\frac{\partial u}{\partial x}=1-x+y+t \tag{5.1}$$
$$\frac{\partial v}{\partial t}-\frac{\partial u}{\partial t}\frac{\partial v}{\partial y}-u\frac{\partial v}{\partial x}=x-y-t+1 \tag{5.2}$$

with following physical constaints,

$$\begin{aligned} u(x,y,0) &= x-1+y, \\ u(0,y,t) &= y-1+t, \\ u(x,0,t) &= x-1+t, \\ u(0,0,t) &= t-1, \end{aligned} \tag{5.3}$$

$$\begin{aligned} v(x,y,0) &= x+1-y, \\ v(0,y,t) &= -y+1-t, \\ v(x,0,t) &= x+1-t, \\ v(0,0,t) &= 1-t. \end{aligned} \tag{5.4}$$

The analytic solutions are of the form [4],

$$u(x,y,t) = x+y+t-1, \;\; v(x,y,t) = x-y-t+1.$$

On solving this problem by proposed LWM at $k=1$ and $M=2$ we get a system containing six nonlinear algebraic equations, by solving this system with a suitable solver, we acquire six LWCs as follows:

$$\begin{aligned} c_{1,0} &= 0.383372579736753, \\ c_{1,1} &= -2.823876970375000, \\ c_{1,2} &= 2.614109221441289, \\ c_{1,3} &= -0.221177735008325, \\ c_{1,4} &= 0.098831394776984, \\ c_{1,5} &= -0.690688027006323. \end{aligned}$$

Substitute these coefficients in $u(x,y,t)$ and $v(x,y,t)$ which yields numerical solutions (ns) for given equation as,

$$\begin{aligned} u_{app} &= t+x+y-1-\sqrt{2}txy(2.6141y-2.8239x+0.036163), \\ v_{app} &= x-t-y+1-\sqrt{2}txy(0.098831x-0.69069y+1.4049). \end{aligned}$$

Again, we solved this problem by increasing the size of $M$ and these results are compared numerically with Haar wavelet method (HWM) [1], Homotopy perturbation method (HPM) [4], and Homotopy analysis method (HAM) [18] in tables 1 to 6. These tables reveal that the present method is better than methods in the literature such as HWM, HPM, and HAM. Figures 1 to 6 represent the graphical behavior of the exact solution (es) with the numerical solutions at different values of $M$ and its absolute errors. In MATLAB 2013 version, CPU time for the proposed method is 7.34 seconds at M=2 and 12.05 seconds at M=5. For the Haar wavelet method CPU time is 18.27 seconds.

**Example 5.2.** Consider the system of (1+1)-D PDEs is of the form [5]:

$$\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial t} + u\frac{\partial v}{\partial t} = -1 + e^{x}\sin(t), \tag{5.5}$$

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial t}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial x}\frac{\partial v}{\partial t} = -1 - e^{-x}\cos(t) \tag{5.6}$$

**Table 1.** Comparison of exact and numerical values (env) of $u(x, y, t)$ at $t = 0.01$, $k = 1$.

| (x,y) | $u_{Exact}$ | $u_{LWM}$ at M=2 | $u_{LWM}$ at M=5 | $u_{HWM}$ [1] | $u_{HAM}$ [18] | $u_{HPM}$ [4] |
|---|---|---|---|---|---|---|
| (0.125,0.125) | -0.74 | -0.74000 | -0.74000 | -0.73984 | -0.73000 | -0.73000 |
| (0.125,0.375) | -0.49 | -0.49043 | -0.49000 | -0.48982 | -0.48000 | -0.48000 |
| (0.125,0.625) | -0.24 | -0.24145 | -0.24012 | -0.23985 | -0.23000 | -0.23000 |
| (0.125,0.875) | 0.01 | 0.00695 | 0.00990 | 0.01016 | 0.01999 | 0.01999 |
| (0.375,0.125) | -0.49 | -0.48953 | -0.49001 | -0.48958 | -0.48000 | -0.48000 |
| (0.375,0.375) | -0.24 | -0.23991 | -0.24124 | -0.23952 | -0.23000 | -0.23000 |
| (0.375,0.625) | 0.01 | 0.00797 | 0.00997 | 0.01039 | 0.01999 | 0.01999 |
| (0.375,0.875) | 0.26 | 0.25413 | 0.26032 | 0.26044 | 0.26999 | 0.26999 |
| (0.625,0.125) | -0.24 | -0.23845 | -0.24145 | -0.23939 | -0.23000 | -0.23000 |
| (0.625,0.375) | 0.01 | 0.01248 | 0.01004 | 0.01069 | 0.01999 | 0.01999 |
| (0.625,0.625) | 0.26 | 0.26052 | 0.26002 | 0.26058 | 0.26999 | 0.26999 |
| (0.625,0.875) | 0.51 | 0.50567 | 0.51007 | 0.51064 | 0.51999 | 0.51999 |
| (0.875,0.125) | 0.01 | 0.01326 | 0.01006 | 0.01074 | 0.01999 | 0.01999 |
| (0.875,0.375) | 0.26 | 0.26674 | 0.26004 | 0.26084 | 0.26999 | 0.26999 |
| (0.875,0.625) | 0.51 | 0.51619 | 0.51010 | 0.51070 | 0.51999 | 0.51999 |
| (0.875,0.875) | 0.76 | 0.76159 | 0.76019 | 0.76078 | 0.76999 | 0.76999 |

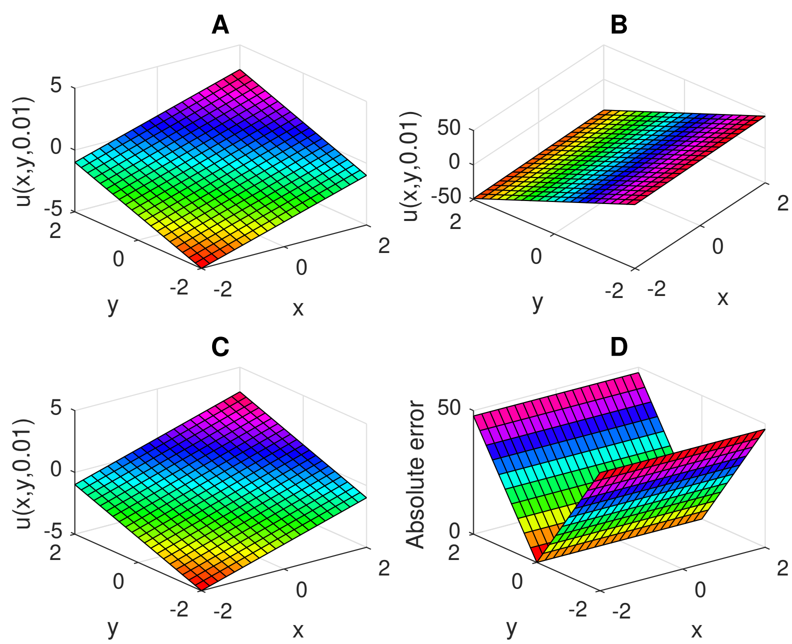


**Figure 1.** Graphical representation of ns of $u(x, y, 0.01)$. (**A**) es of $u(x, y, 0.01)$, (**B**) ns of $u(x, y, 0.01)$ at $M = 2$, (**C**) ns of $u(x, y, 0.01)$ at $M = 5$, (**D**) absolute error (ae) between ns of $u(x, y, 0.01)$ at $M = 2$ with es.

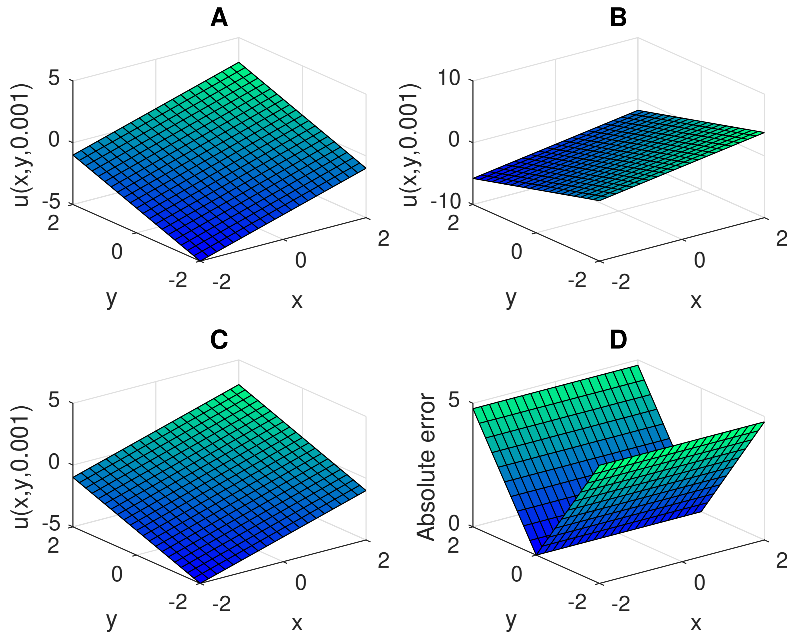


**Figure 2.** Graphical representation of ns of $u(x, y, 0.01)$. (**A**) es of $u(x, y, 0.01)$, (**B**) ns of $u(x, y, 0.01)$ at $M = 2$, (**C**) ns of $u(x, y, 0.01)$ at $M = 5$, (**D**) absolute error (ae) between ns of $u(x, y, 0.01)$ at $M = 2$ with es.

**Table 2.** Comparison of env of $u(x, y, t)$ at $t = 0.001$, $k = 1$.

| (x,y) | $u_{Exact}$ | $u_{LWM}$ at M=2 | $u_{LWM}$ at M=5 | $u_{HWM}$ [1] | $u_{HAM}$ [18] | $u_{HPM}$ [4] |
|---|---|---|---|---|---|---|
| (0.125,0.125) | -0.749 | -0.74900 | -0.74900 | -0.74899 | -0.74800 | -0.74800 |
| (0.125,0.375) | -0.499 | -0.49904 | -0.49900 | -0.49899 | -0.49800 | -0.49800 |
| (0.125,0.625) | -0.249 | -0.24914 | -0.24901 | -0.24899 | -0.24800 | -0.24800 |
| (0.125,0.875) | 0.001 | 0.00069 | 0.00099 | 0.00100 | 0.00199 | 0.00199 |
| (0.375,0.125) | -0.499 | -0.49895 | -0.49915 | -0.49899 | -0.49800 | -0.49800 |
| (0.375,0.375) | -0.249 | -0.24899 | -0.24910 | -0.24899 | -0.24800 | -0.24800 |
| (0.375,0.625) | 0.001 | 0.00079 | 0.00100 | 0.00100 | 0.00199 | 0.00199 |
| (0.375,0.875) | 0.251 | 0.25041 | 0.25100 | 0.25100 | 0.25199 | 0.25199 |
| (0.625,0.125) | -0.249 | -0.24884 | -0.24910 | -0.24899 | -0.24800 | -0.24800 |
| (0.625,0.375) | 0.001 | 0.00124 | 0.00100 | 0.00100 | 0.00199 | 0.00199 |
| (0.625,0.625) | 0.251 | 0.25105 | 0.25100 | 0.25100 | 0.25199 | 0.25199 |
| (0.625,0.875) | 0.501 | 0.50056 | 0.50100 | 0.50100 | 0.50199 | 0.50199 |
| (0.875,0.125) | 0.001 | 0.00132 | 0.00100 | 0.00100 | 0.00199 | 0.00199 |
| (0.875,0.375) | 0.251 | 0.25167 | 0.25100 | 0.25100 | 0.25199 | 0.25199 |
| (0.875,0.625) | 0.501 | 0.50161 | 0.50100 | 0.50100 | 0.50199 | 0.50199 |
| (0.875,0.875) | 0.751 | 0.75115 | 0.75100 | 0.75100 | 0.75199 | 0.75199 |

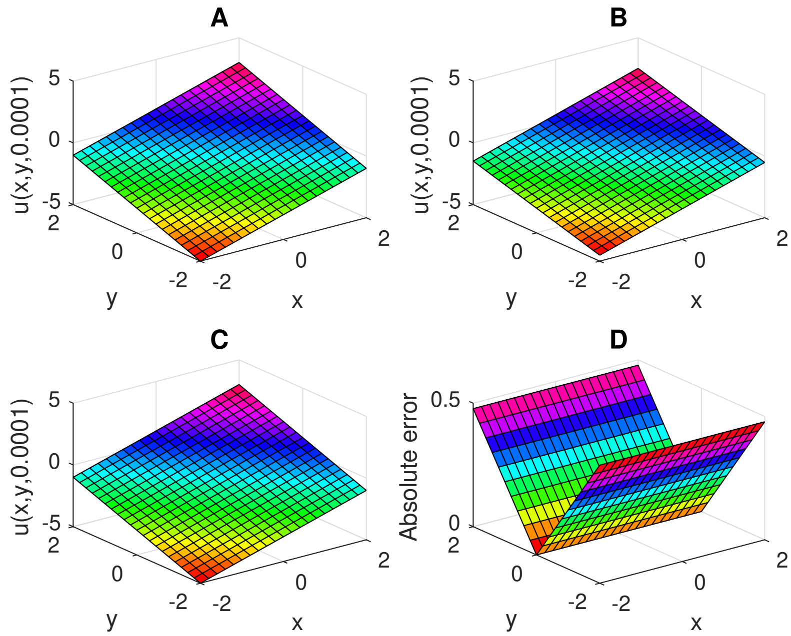


**Figure 3.** Graphical representation of ns of $u(x, y, 0.0001)$. (**A**) es of $u(x, y, 0.0001)$, (**B**) ns of $u(x, y, 0.0001)$ at $M = 2$, (**C**) ns of $u(x, y, 0.0001)$ at $M = 5$, (**D**) ae between ns of $u(x, y, 0.0001)$ at $M = 2$ with es.

**Table 3.** Comparison of env of $u(x, y, t)$ at $t = 0.0001$, $k = 1$.

| (x,y) | $u_{Exact}$ | $u_{LWM}$ at M=2 | $u_{LWM}$ at M=5 | $u_{HWM}$ [1] | $u_{HAM}$ [18] | $u_{HPM}$ [4] |
|---|---|---|---|---|---|---|
| (0.125,0.125) | -0.7499 | -0.74990 | -0.74990 | -0.74989 | -0.74980 | -0.74980 |
| (0.125,0.375) | -0.4999 | -0.49990 | -0.49990 | -0.49989 | -0.49980 | -0.49980 |
| (0.125,0.625) | -0.2499 | -0.24991 | -0.24990 | -0.24989 | -0.24980 | -0.24980 |
| (0.125,0.875) | 0.0001 | 0.00007 | 0.00010 | 0.00010 | 0.00019 | 0.00019 |
| (0.375,0.125) | -0.4999 | -0.49989 | -0.49991 | -0.49989 | -0.49980 | -0.49980 |
| (0.375,0.375) | -0.2499 | -0.24989 | -0.24990 | -0.24989 | -0.24980 | -0.24980 |
| (0.375,0.625) | 0.0001 | 0.00008 | 0.00010 | 0.00010 | 0.00019 | 0.00019 |
| (0.375,0.875) | 0.2501 | 0.25004 | 0.25010 | 0.25010 | 0.25020 | 0.25020 |
| (0.625,0.125) | -0.2499 | -0.24988 | -0.24990 | -0.24989 | -0.24980 | -0.24980 |
| (0.625,0.375) | 0.0001 | 0.00012 | 0.00010 | 0.00010 | 0.00019 | 0.00019 |
| (0.625,0.625) | 0.2501 | 0.25010 | 0.25010 | 0.25010 | 0.25020 | 0.25020 |
| (0.625,0.875) | 0.5001 | 0.50005 | 0.50010 | 0.50010 | 0.50020 | 0.50020 |
| (0.875,0.125) | 0.0001 | 0.00013 | 0.00010 | 0.00010 | 0.00019 | 0.00019 |
| (0.875,0.375) | 0.2501 | 0.25016 | 0.25010 | 0.25010 | 0.25020 | 0.25020 |
| (0.875,0.625) | 0.5001 | 0.50016 | 0.50010 | 0.50010 | 0.50020 | 0.50020 |
| (0.875,0.875) | 0.7501 | 0.75011 | 0.75010 | 0.75010 | 0.75020 | 0.75020 |

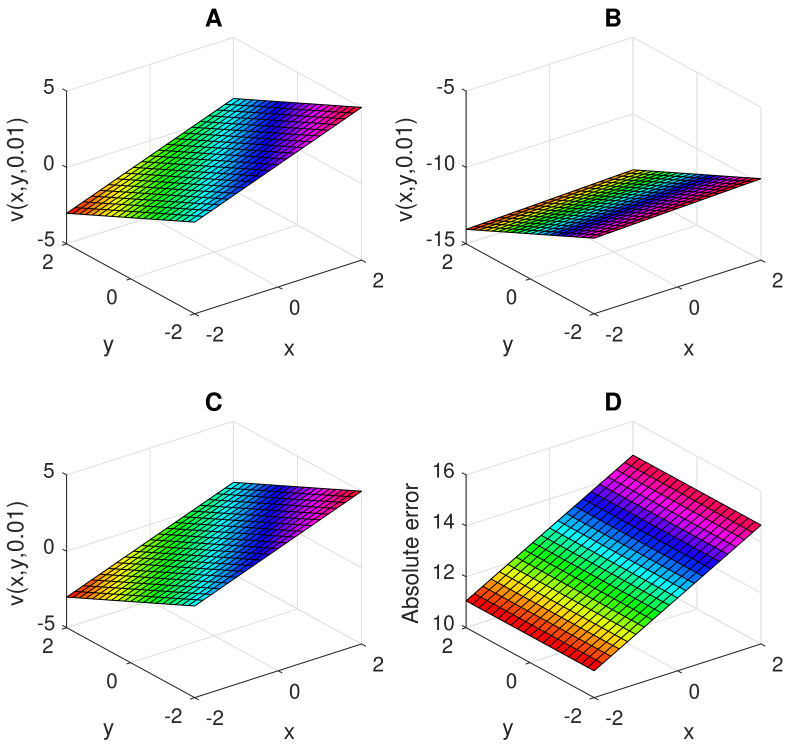


**Figure 4.** Graphical representation of ns of $v(x, y, 0.01)$. (**A**) es of $v(x, y, 0.01)$, (**B**) ns of $v(x, y, 0.01)$ at $M = 2$, (**C**) ns of $v(x, y, 0.01)$ at $M = 5$, (**D**) ae between ns of $v(x, y, 0.01)$ at $M = 2$ with es.

**Table 4.** Comparison of env of $v(x, y, t)$ at $t = 0.01$, $k = 1$.

| (x,y) | $v_{Exact}$ | $v_{LWM}$ at $M = 2$ | $v_{LWM}$ at $M = 5$ | $v_{HWM}$ [1] | $v_{HAM}$ [18] | $v_{HPM}$ [4] |
|---|---|---|---|---|---|---|
| (0.125,0.125) | 0.99 | 0.98970 | 0.99102 | 0.98984 | 1.02000 | 1.02000 |
| (0.125,0.375) | 0.74 | 0.73923 | 0.74013 | 0.73986 | 0.77000 | 0.77000 |
| (0.125,0.625) | 0.49 | 0.48891 | 0.49145 | 0.48983 | 0.52000 | 0.52000 |
| (0.125,0.875) | 0.24 | 0.23874 | 0.24001 | 0.23987 | 0.27000 | 0.27000 |
| (0.375,0.125) | 1.24 | 1.23910 | 1.24045 | 1.23956 | 1.27000 | 1.27000 |
| (0.375,0.375) | 0.99 | 0.98764 | 0.99001 | 0.98963 | 1.02000 | 1.02000 |
| (0.375,0.625) | 0.74 | 0.73665 | 0.74010 | 0.73953 | 0.77000 | 0.77000 |
| (0.375,0.875) | 0.49 | 0.48611 | 0.49002 | 0.48954 | 0.52000 | 0.52000 |
| (0.625,0.125) | 1.49 | 1.48847 | 1.49125 | 1.48936 | 1.52000 | 1.52000 |
| (0.625,0.375) | 1.24 | 1.23599 | 1.24189 | 1.23946 | 1.27000 | 1.27000 |
| (0.625,0.625) | 0.99 | 0.98428 | 0.99245 | 0.98932 | 1.02000 | 1.02000 |
| (0.625,0.875) | 0.74 | 0.73333 | 0.74000 | 0.73932 | 0.77000 | 0.77000 |
| (0.875,0.125) | 1.74 | 1.73782 | 1.74102 | 1.73923 | 1.77000 | 1.77000 |
| (0.875,0.375) | 1.49 | 1.48428 | 1.49201 | 1.48934 | 1.52000 | 1.52000 |
| (0.875,0.625) | 1.24 | 1.23180 | 1.24001 | 1.23918 | 1.27000 | 1.27000 |
| (0.875,0.875) | 0.99 | 0.98039 | 0.99031 | 0.98917 | 1.02000 | 1.02000 |

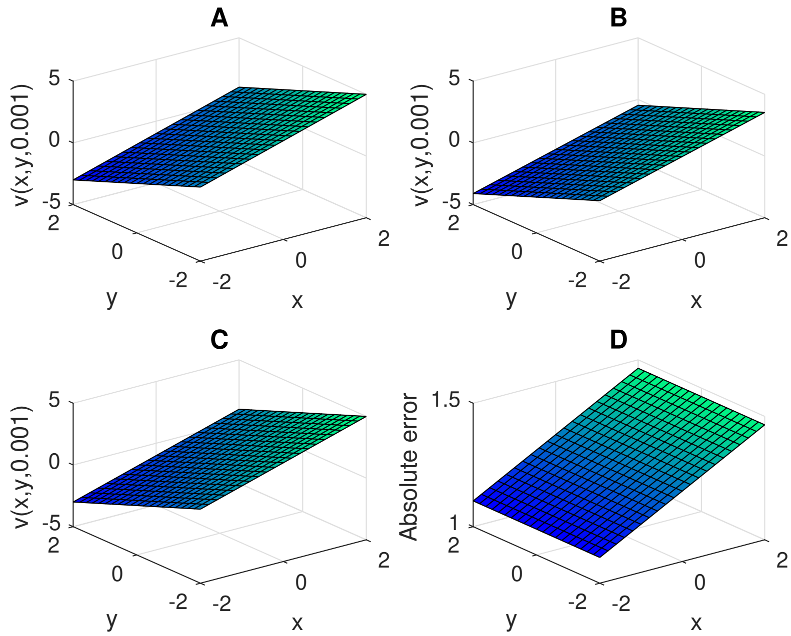


**Figure 5.** Graphical representation of ns of $v(x, y, 0.001)$. (**A**) es of $v(x, y, 0.001)$, (**B**) ns of $v(x, y, 0.001)$ at $M = 2$, (**C**) ns of $v(x, y, 0.001)$ at $M = 5$, (**D**) ae between ns of $v(x, y, 0.001)$ at $M = 2$ with es.

**Table 5.** Comparison of env of $v(x, y, t)$ at $t = 0.001$, $k = 1$.

| (x,y) | $v_{Exact}$ | $v_{LWM}$ at M=2 | $v_{LWM}$ at M=5 | $v_{HWM}$ [1] | $v_{HAM}$ [18] | $v_{HPM}$ [4] |
|---|---|---|---|---|---|---|
| (0.125,0.125) | 0.999 | 0.99897 | 0.99902 | 0.99899 | 1.00200 | 1.00200 |
| (0.125,0.375) | 0.749 | 0.74892 | 0.74913 | 0.74899 | 0.75200 | 0.75200 |
| (0.125,0.625) | 0.499 | 0.49889 | 0.49905 | 0.49899 | 0.50200 | 0.50200 |
| (0.125,0.875) | 0.249 | 0.24887 | 0.24901 | 0.24899 | 0.25200 | 0.25200 |
| (0.375,0.125) | 1.249 | 1.24891 | 1.24905 | 1.24899 | 1.25200 | 1.25200 |
| (0.375,0.375) | 0.999 | 0.99876 | 0.99901 | 0.99899 | 1.00200 | 1.00200 |
| (0.375,0.625) | 0.749 | 0.74866 | 0.74910 | 0.74899 | 0.75200 | 0.75200 |
| (0.375,0.875) | 0.499 | 0.49861 | 0.49902 | 0.49899 | 0.50200 | 0.50200 |
| (0.625,0.125) | 1.499 | 1.49884 | 1.49900 | 1.49899 | 1.50200 | 1.50200 |
| (0.625,0.375) | 1.249 | 1.24859 | 1.24949 | 1.24899 | 1.25200 | 1.25200 |
| (0.625,0.625) | 0.999 | 0.99842 | 0.99945 | 0.99899 | 1.00200 | 1.00200 |
| (0.625,0.875) | 0.749 | 0.74833 | 0.74910 | 0.74899 | 0.75200 | 0.75200 |
| (0.875,0.125) | 1.749 | 1.73782 | 1.74902 | 1.74899 | 1.75200 | 1.75200 |
| (0.875,0.375) | 1.499 | 1.49842 | 1.49901 | 1.49899 | 1.50200 | 1.50200 |
| (0.875,0.625) | 1.249 | 1.24818 | 1.24901 | 1.24899 | 1.12520 | 1.12520 |
| (0.875,0.875) | 0.999 | 0.99803 | 0.99911 | 0.99899 | 1.00200 | 1.00200 |

with following physical constaints,

$$\begin{aligned} u(0,t) &= \sin(t), \\ u(x,0) &= 0, \end{aligned} \tag{5.7}$$

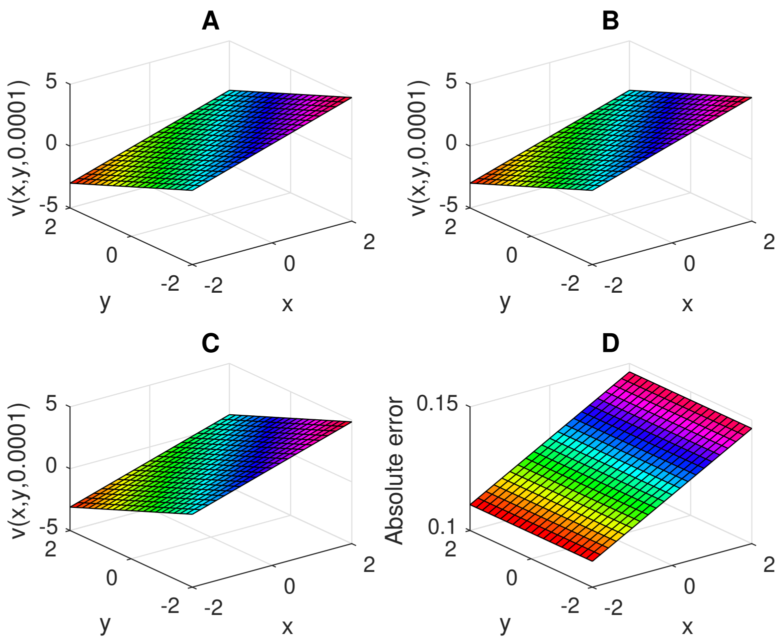

**Figure 6.** Graphical representation of ns of $v(x, y, 0.0001)$. (**A**) es of $v(x, y, 0.0001)$, (**B**) ns of $v(x, y, 0.0001)$ at $M = 2$, (**C**) ns of $v(x, y, 0.0001)$ at $M = 5$, (**D**) ae between ns of $v(x, y, 0.0001)$ at $M = 2$ with es.

**Table 6.** Comparison of env of $v(x, y, t)$ at $t = 0.0001$, $k = 1$.

| (x,y) | $v_{Exact}$ | $v_{LWM}$ at M=2 | $v_{LWM}$ at M=5 | $v_{HWM}$ [1] | $v_{HAM}$ [18] | $v_{HPM}$ [4] |
|---|---|---|---|---|---|---|
| (0.125,0.125) | 0.9999 | 0.99989 | 0.99990 | 0.99989 | 1.00020 | 1.00020 |
| (0.125,0.375) | 0.7499 | 0.74989 | 0.74991 | 0.74989 | 0.75020 | 0.75020 |
| (0.125,0.625) | 0.4999 | 0.49988 | 0.49990 | 0.49989 | 0.50020 | 0.50020 |
| (0.125,0.875) | 0.2499 | 0.24988 | 0.24990 | 0.24989 | 0.25020 | 0.25020 |
| (0.375,0.125) | 1.2499 | 1.24989 | 1.24990 | 1.24989 | 1.25020 | 1.25020 |
| (0.375,0.375) | 0.9999 | 0.99987 | 0.99990 | 0.99989 | 1.00020 | 1.00020 |
| (0.375,0.625) | 0.7499 | 0.74986 | 0.74991 | 0.74989 | 0.75020 | 0.75020 |
| (0.375,0.875) | 0.4999 | 0.49986 | 0.49990 | 0.49989 | 0.50020 | 0.50020 |
| (0.625,0.125) | 1.4999 | 1.49988 | 1.49990 | 1.49989 | 1.50020 | 1.50020 |
| (0.625,0.375) | 1.2499 | 1.24985 | 1.24994 | 1.24989 | 1.25020 | 1.25020 |
| (0.625,0.625) | 0.9999 | 0.99984 | 0.99994 | 0.99989 | 1.00020 | 1.00020 |
| (0.625,0.875) | 0.7499 | 0.74983 | 0.74991 | 0.74989 | 0.75020 | 0.75020 |
| (0.875,0.125) | 1.7499 | 1.74987 | 1.74990 | 1.74989 | 1.75020 | 1.75020 |
| (0.875,0.375) | 1.4999 | 1.49984 | 1.49990 | 1.49989 | 1.50020 | 1.50020 |
| (0.875,0.625) | 1.2499 | 1.24981 | 1.24990 | 1.24989 | 1.12502 | 1.12502 |
| (0.875,0.875) | 0.9999 | 0.99980 | 0.99991 | 0.99989 | 1.00020 | 1.00020 |

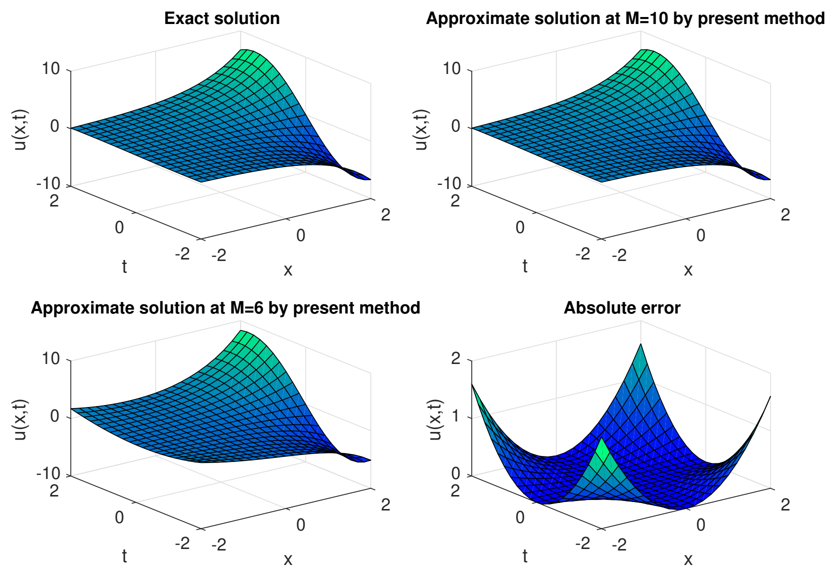


**Figure 7.** Graphical representation of es $u(x,t)$, ns at $k=1$ and $M=10,6$ and its ae.

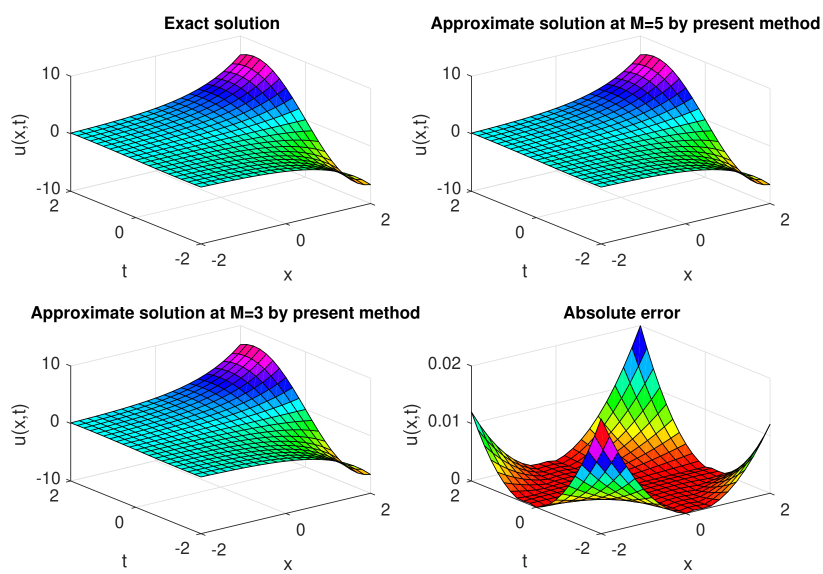


**Figure 8.** Graphical representation of es $u(x,t)$, ns at $k=2$ and $M=5,3$ and its ae.

$$\begin{aligned} v(x,0) &= e^{-x}, \\ v(0,t) &= \cos(t). \end{aligned} \tag{5.8}$$

The analytic solutions is of the form [5],

$$u(x,t) = e^{x}\sin(t), \;\; v(x,t) = e^{-x}\cos(t).$$

We solved this problem with the proposed algorithm at different values of $k$ and $M$.

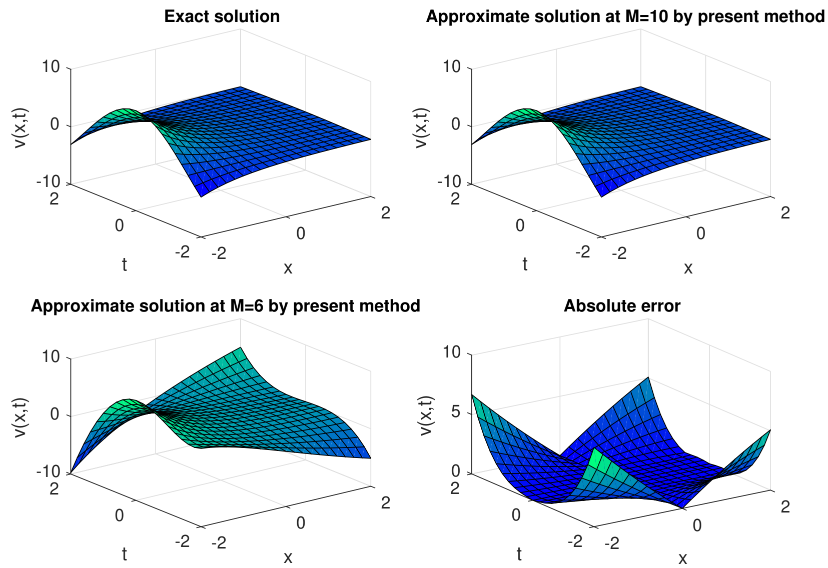


**Figure 9.** Graphical representation of es $v(x,t)$, ns at $k = 1$ and $M = 10, 6$ and its ae.

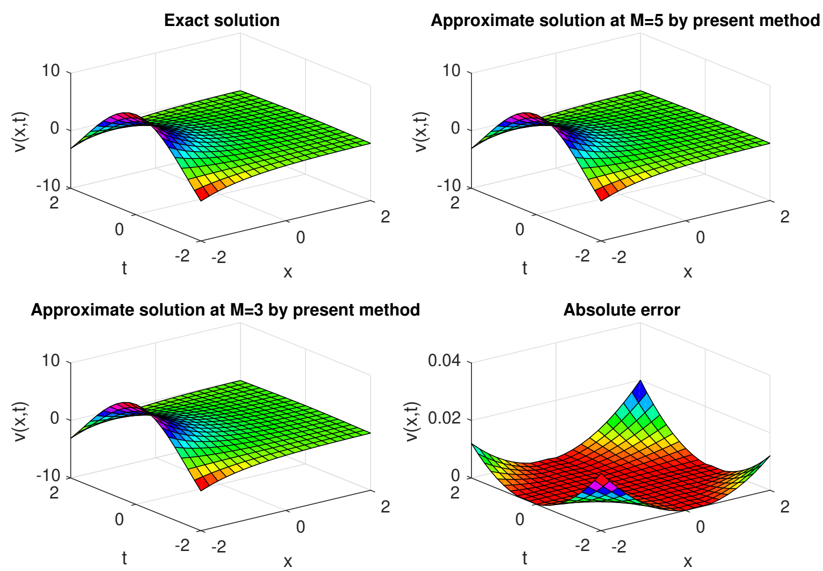


**Figure 10.** Graphical representation of es $v(x,t)$, ns at $k = 2$ and $M = 5, 3$ and its ae.

Figs. 7,9 reveals that accuracy in solution on varying $M$ values by fixing $k = 1$ with absolute errors. Figs. 8,10 shows that accuracy in solution on varying both $M$ and $k$ values with absolute errors. From these figures, we can observe that increasing $k$ values is directly proportional to the accuracy of the solution. In MATLAB 2013 version, CPU time for the proposed method is 5.98 seconds at $M = 2$ and 9.53 seconds at $M = 5$.

# 6. Conclusion

We developed an effective numerical technique to solve some (2+1)-D nonlinear coupled system of PDEs. This technique is based on the LW with the collocation method and applied to solve the (2+1)-D nonlinear coupled system of PDEs. This technique is a simple and new approach to solve such equations also, convergence analysis is discussed in the form of a theorem. Proposed technique yields better results than the HWM, HPM, and HAM which are represented in tables 1 to 6. As increasing the size of $M$ accuracy goes on increases this can be observed in figures 1 to 6 as well as in the tables. This technique also works for higher-order PDEs with slight modifications in the method. From these merits, this method might become a promising technique in solving a (2+1)-D NPDEs and systems.

**Conflict of interest.** The authors declare that they have no conflict of interest.
**Data Availibility.** All data generated or analysed during this study are included in this article.

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