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INFLUENCE OF INITIAL RAMP ON CONVOLUTIONAL NONVISCOUS DAMPING MATERIALS*

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Abstract In this paper, taking the stress relaxation test of viscoelastic material as an example, the viscoelastic materials used in the test are characterized by the convolutional nonviscous damping model. When the kernel function of the convolutional nonviscous damping model is taken as the power exponential function and the exponential function respectively, the influence of the initial ramp on the stress change is proved theoretically and numerically. This will affect the accuracy of parameter determination of fitting the convolutional nonviscous damping model.

Keywords Stress relaxation tests, initial ramp, convolutional nonviscous damping model, kernel function.

MSC(2010) O29.

1. Introduction

In recent years, more and more scholars describe that new materials, biomimetic materials, polymer materials, and polymer viscoelastic materials adopt integral constitutive model, that is, convolutional nonviscous damping model (cf. T. Abbasi, F. Faraz and S. Abbas [1], M. Lázaro and L. García-Raffi [11] and R. Shen, X. Qian and J. Zhou [18]). The convolutional nonviscous damping model is represented by the following Volterra type integro-differential equation

$$\sigma(t) = \int_{-\infty}^{t} G(t-\tau)\mathrm{d}\epsilon(\tau) = G(t) \otimes \mathrm{d}\epsilon(\tau),$$

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$$G(t) = \sum_{k=1}^{n} C_k g_k(t),$$

where $\sigma(t)$ and $\epsilon(\tau)$ represent the stress and strain of the convolutional nonviscous damping model respectively, $\otimes$ is called the Stieltjes convolution, $G(t)$ is the kernel function, C_k is a damping coefficient matrix, $g_k(t)$ is the damping function, and n is the number of relaxation parameters describing different damping mechanisms.

When the kernel function $G(t) = C\delta(t)$, that is, when $G(t)$ takes the Dirac delta function, the damping model is the traditional viscous damping model $C\dot{x}(t)$. Therefore, it can be said that the convolutional nonviscous damping model is the general form of viscous damping model.

When the damping function takes the exponential function, that is $g(t) = \mu e^{-\mu t}$, where μ is the relaxation factor of the convolutional nonviscous damping model. This model is called exponential damping model.

When the kernel function $G(t)$ takes the form of Abel kernel function of the decay power function, the convolutional nonviscous damping model is the fractional derivative model.

This model has many advantages: (1) It can fit the experimental data in a wide frequency range with fewer parameters (usually 3-5) to describe the mechanical properties of the material more accurately; (2) It not only conforms to the principles of thermodynamics and molecular theory, but also can describe the memory decay effect of materials and record the load time history acting on materials; (3) From the mathematical point of view, the form of the differential equation of the damped vibration system formed by this model is concise, which is more conducive to theoretical analysis.

In recent years, with the wide application of modern industrial materials, more and more new materials, biomaterials and biomimetic materials appear in large quantities. The traditional viscous damping model can not reflect the actual structural characteristics and complex damping energy dissipation characteristics of materials. Convolutional nonviscous damping model is a new damping model developed in the past 30 years. More and more researchers at home and abroad use the convolutional nonviscous damping model to model the constitutive relationship of viscoelastic materials. Therefore, the model has been developed rapidly and gradually become a powerful modeling tool to describe the mechanical properties of viscoelastic materials. S. Adhikari team has been doing more in-depth research and discussion on the model in the past two decades (cf. S. Adhikari [3, 4] and S. Adhikari, D. Karličič and X. Liu [5]), including the dynamic analysis, modal identification and other aspects of the model. Mario Lázaro, a famous scholar at the Universitàt Politecnica de València in Spain, has studied the model for nearly two decades and published a large number of academic monographs and high-quality papers (cf. M. Lázaro [11] and M. Lázaro and L. García-Raffi [12, 13]). There are also many scholars, such as Li Li's team from Huazhong University of Science and Technology and Ding Zhe's team from Chinese University of Hong Kong, who have conducted extensive research on the model, and have achieved rich research results (cf. Z. Ding, L. Zhang and Q. Gao [7], X. Du, W. Guo and H. Xia [10], L. Li and Y. Hu [14], L. Li, R. Lin and T. Y. Ng [15] and G. Xiu, B. Shi and F. Qian [22]).

The mechanical properties of convolutional nonviscous damping materials are obtained by stress relaxation test or creep test. In the stress relaxation test, it is usually assumed that the strain reaches the set value through the step function at the initial time under the ideal state. However, in the actual test, it takes a time process to reach the set strain value. As for the length of time, it is closely related to the tester, equipment and other factors. we call this process from the initial moment to the fixed strain the initial ramp. Because of the existence of this time process, there must be an initial ramp. We assume that the initial ramp is linear. Since the convolutional nonviscous damping material has a long historical memory performance, applying prestress to the sample will affect the fitting of the data in the test, thus affecting the accuracy of the parameters of the convolutional nonviscous damping model.

In this paper, the kernel function of the convolutional nonviscous damping model is divided into two forms for research and discussion. One is the form of power exponential function, and the other is the form of exponential function. Taking the stress relaxation test as an example, it can be proved theoretically and experimentally that the initial ramp has an impact on the stress change, thus affecting the accuracy of the parameter determination of the convolutional nonviscous damping model.

2. Influence of initial ramp when the kernel function is a power exponential function

For the full mechanical properties of convolutional nonviscous damping materials, it has to be taken into account the time evolution of stress and strain history, that is, the stress history $\sigma(t)$ and strain history $\epsilon(t)$. For this reason, the classical tensile test is not able to describe the time-dependent stress-strain relation and we need another type of experimental test. Exist two fundamental tests to characterize the viscoelastic material, such tests are known as creep and relaxation test. The creep test aims to evaluate the time evolution of the strain response due to an imposed stress history which follows a unit-step function. Conversely in the relaxation test is carried out the measurement of the response in terms of stress history duo to an imposed strain history which follows a unit-step function. Both the aforementioned tests are idealizations. For this reason, the real creep and relaxation tests are different respect to the theoretical mentioned description because the test machines using for these tests are not able to reproduce a unit step function both in imposed stress and or strain history, as shown in Fig. 1.

On the other hand the reference standard (ASTM E328-02 Standard, 2002), for stress relaxation tests for materials and structures give not specific indication regarding to the initial ramp for the initial stress (cf. W. Shen, C. Zhang and L. Zhang [19] and M. Taneco-Hernández, V. Morales-Delgado and J. Gómez-Aguilar [20]). The stress application rate in either case should be reasonably rapid, without impact or vibration, so that any relaxation during the stress application period will be small. Then the rate of the initial ramp is selected on the basis of the material hand as well as the total strain selected for the relaxation test. The time t_0 at which the deformation takes the constant value ϵ_0 is strictly related the test machine as well as from the people making the test. Usually the rate of deformation is very high and consequently t_0 is very small. It follows that usually this influence is neglected and it is assumed that $t_0 = 0$ so the unit step deformation history is present. However, for the case that the kernel function is a power law, infinite values have been generated in the corresponding stress history, which causes significant errors

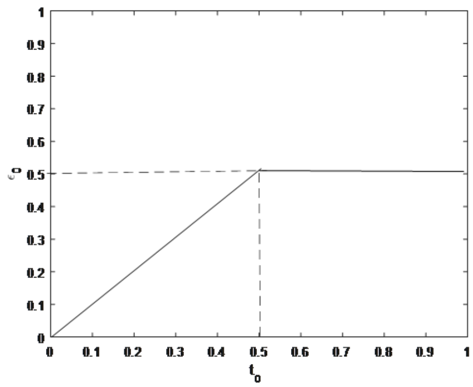

Figure 1. Real deformation history during the relaxation test.

in parameter evaluation.

In the stress relaxation test without considering the influence of initial ramp, the strain is to reach the specified strain value $\epsilon(t) = \epsilon_0 U(t)$ under the action of unit step function, where $U(t)$ is the unit step function.

The convolutional stress-strain relationship is obtained by Boltzmann superposition principle

$$\sigma(t) = \int_0^t G(t-\tau)\mathrm{d}\epsilon(\tau) = \int_0^t G(t-\tau)\dot{\epsilon}(\tau)\mathrm{d}\tau.$$

The condition that the equation holds is $\epsilon(0) = 0$. That is, without considering the influence of initial ramp, we can get

$$\epsilon(t) = \begin{cases} 0, & t \leqslant 0, \\ \epsilon_0, & t > 0. \end{cases} \tag{2.1}$$

Considering the influence of initial ramp, the strain history of the convolutional nonviscous damping model is

$$\epsilon(t) = \begin{cases} 0, & t < 0, \\ \dfrac{\epsilon_0 t}{t_0}, & 0 \leqslant t < t_0, \\ \epsilon_0, & t \geqslant t_0. \end{cases} \tag{2.2}$$

According to Blotzman superposition principle, when $\epsilon(0) = 0$, we can get

$$\sigma(t) = \int_0^t G(t-\tau)\mathrm{d}\epsilon(\tau) = G(t)\epsilon\left(0_+\right) + \int_0^t G(t-\tau)\dot{\epsilon}(\tau)\mathrm{d}\tau. \tag{2.3}$$

Through the above calculation, the stress-strain relationship of the convolutional nonviscous damping model is obtained, which is general. Because the material has the property of decay heredity, $G(t)$ is a continuous monotonic decreasing function. When the kernel function $G(t)$ takes the Abel kernel form of the decay power function, the convolutional nonviscous damping model is the fractional derivative model. The simplest fractional Scott-Blair's model discussed by Mario Di Paola, a famous professor at the University of Italy degli Studi di Palermo, in M. Di Paola [8] and by Professor Maolin Du of the National University of Defense Technology in M. Du, Y. Wang and Z. Wang [9], is different in that Mario Di Paola uses the Caputo fractional derivative, while Professor Maolin Du uses the Riemann-Liouville's derivative, and the stress-strain relationship obtained is the same. In this section, taking as the form of power exponential function (cf. M. Abu-Shady, M. K. A. Kaabar [2], W. Chen [6] and G. Teodoro and J. Machado [21]), we can get more general results

$$G(t) = \frac{a}{t^\alpha} + b$$

or

$$G(t) = \sum_{i=1}^{n} \left(\frac{a_i}{t^{\alpha_i}} + b_i \right), \quad 0 < \alpha_i < 0.2.$$

In particular, when $G(t) = \frac{a}{t^\alpha} + b$, without considering the initial ramp, that is, the strain is shown in Equation (2.1), then it can be obtained from Equation (2.3)

$$\sigma(t) = \begin{cases} 0, & t \leqslant 0, \\ G(t)\epsilon_0, & t > 0, \end{cases}$$

that is

$$\sigma(t) = \begin{cases} 0, & t \leqslant 0, \\ \left(\dfrac{a}{t^\alpha} + b \right) \epsilon_0, & t > 0. \end{cases} \tag{2.4}$$

During the relaxation test, considering the initial ramp, that is, the strain history is shown in Equation (2.2), then it can be obtained from Equation (2.3).

When $0 \leqslant t < t_0$,

$$\begin{aligned} \sigma(t) &= G(t)\epsilon\left(0_+\right) + \int_0^t G(t-\tau)\dot{\epsilon}(\tau)\mathrm{d}\tau = \frac{\epsilon_0}{t_0} \int_0^t \left[\frac{a}{(t-\tau)^\alpha} + b \right] \mathrm{d}\tau \\ &= \frac{a\epsilon_0}{t_0(1-\alpha)} t^{1-\alpha} + \frac{\epsilon_0 t b}{t_0}; \end{aligned}$$

when $t > t_0$,

$$\begin{aligned} \sigma(t) &= \int_0^t G(t-\tau)\mathrm{d}\epsilon(\tau) = \int_0^{t_0} G(t-\tau)\mathrm{d}\epsilon(\tau) + \int_{t_0}^t G(t-\tau)\mathrm{d}\epsilon(\tau) \\ &= \frac{\epsilon_0}{t_0} \int_0^{t_0} G(t-\tau)\mathrm{d}\epsilon(\tau) = \frac{\epsilon_0}{t_0} \int_0^{t_0} \left[\frac{a}{(t-\tau)^\alpha} + b \right] \mathrm{d}\tau \\ &= \frac{\epsilon_0}{t_0} \frac{-a}{1-\alpha} \left[(t-t_0)^{1-\alpha} - t^{1-\alpha} \right] + b\epsilon_0 \\ &= \frac{a\epsilon_0}{t_0(1-\alpha)} \left[t^{1-\alpha} - (t-t_0)^{1-\alpha} \right] + b\epsilon_0. \end{aligned}$$

Therefore

$$\sigma(t) = \begin{cases} 0, & t \leqslant 0, \\ \dfrac{a\epsilon_0}{t_0(1-\alpha)} t^{1-\alpha} + \dfrac{\epsilon_0 t b}{t_0}, & 0 < t \leqslant t_0, \\ \dfrac{a\epsilon_0}{t_0(1-\alpha)} \left[t^{1-\alpha} - (t-t_0)^{1-\alpha} \right] + b\epsilon_0, & t > t_0. \end{cases} \tag{2.5}$$

Especially when $b = 0$, $a = \frac{1}{\mu\Gamma(1-\alpha)}$, this conclusion is consistent with the result that only Scott-Blair model is considered in literature (cf. M. Di Paola [8] and M. Du, Y. Wang and Z. Wang [9]). Therefore, the conclusions obtained are more general, covering and extending the conclusions of literature (cf. M. Di Paola [8] and M. Du, Y. Wang and Z. Wang [9]).

It can be seen from the comparison between formula (2.4) and formula (2.5) that the stress change formula of the model is different with and without the initial ramp, which shows that the existence of the initial ramp has an impact on the stress change of the model.

3. Influence of initial ramp when kernel function is exponential function

If the kernel function $G(t)$ takes the form of exponential function, that is, the convolutional nonviscous damping model is an exponential damping model, the research of exponential damping model has also made a lot of achievements (cf. J. Richter, F. Jin and L. Knipschild [16], R. Shen, X. Qian and J. Zhou [17] and G. Xiu, J. Yuan and B. Shi [23]), but no relevant literature has been found on the influence of initial ramp on the exponential damping model

$$G(t) = c_0\delta(t) + \sum_{k=1}^{n} a_k \mathrm{e}^{-b_k t} c_k.$$

Without considering the initial ramp, that is, the strain is shown in Equation (2.1), then it can be obtained from Equation (2.3)

$$\epsilon(t) = \begin{cases} 0, & t \leqslant 0, \\ G(t)\epsilon_0, & t > 0, \end{cases}$$

that is

$$\epsilon(t) = \begin{cases} 0, & t \leqslant 0, \\ \left[c_0\delta(t) + \sum_{k=1}^{n} a_k \mathrm{e}^{-b_k t} c_k \right] \epsilon_0, & t > 0. \end{cases} \tag{3.1}$$

During the relaxation test, considering the initial ramp, that is, the strain history is shown in Equation (2.2), then it can be obtained from Equation (2.3).

When $0 < t \leqslant t_0$

$$\begin{aligned} \sigma(t) &= G(t)\epsilon\left(0_+\right) + \int_0^t G(t-\tau)\dot{\epsilon}(\tau)\mathrm{d}\tau \\ &= 0 + \frac{\epsilon_0}{t_0} \int_0^t \left[c_0\delta(t-\tau) + \sum_{k=1}^{n} a_k \mathrm{e}^{-b_k(t-\tau)} c_k \right] \mathrm{d}\tau \\ &= \frac{\epsilon_0}{t_0} \int_0^t c_0\delta(t-\tau)\mathrm{d}\tau + \frac{\epsilon_0}{t_0} \int_0^t \sum_{k=1}^{n} a_k \mathrm{e}^{-b_k(t-\tau)} c_k \mathrm{d}\tau \\ &= \frac{\epsilon_0 c_0}{t_0} \int_0^t \delta(t-\tau)\mathrm{d}\tau + \frac{\epsilon_0}{t_0} \sum_{k=1}^{n} a_k \int_0^t \mathrm{e}^{-b_k(t-\tau)} c_k \mathrm{d}\tau \end{aligned}$$

$$= \frac{\epsilon_0 c_0}{t_0} + \frac{\epsilon_0}{b_k} \sum_{k=1}^{n} \frac{a_k c_k}{t_0} \left(1 - \mathrm{e}^{-b_k t}\right);$$

when $t > t_0$,

$$\begin{aligned}
\sigma(t) &= \int_0^t G(t-\tau)\mathrm{d}\epsilon(\tau) \\
&= \int_0^{t_0} G(t-\tau)\mathrm{d}\epsilon(\tau) + \int_{t_0}^{t} G(t-\tau)\mathrm{d}\epsilon(\tau) \\
&= \frac{\epsilon_0}{t_0} \int_0^{t_0} G(t-\tau)\mathrm{d}\tau \\
&= \frac{\epsilon_0}{t_0} \int_0^{t_0} \left[c_0 \delta(t-\tau) + \sum_{k=1}^{n} a_k \mathrm{e}^{-b_k(t-\tau)} c_k \right] \mathrm{d}\tau \\
&= \frac{\epsilon_0}{t_0} \int_0^{t_0} c_0 \delta(t-\tau)\mathrm{d}\tau + \frac{\epsilon_0}{t_0} \int_0^{t_0} \sum_{k=1}^{n} a_k \mathrm{e}^{-b_k(t-\tau)} c_k \mathrm{d}\tau \\
&= \frac{\epsilon_0 c_0}{t_0} + \frac{\epsilon_0}{b_k} \sum_{k=1}^{n} \frac{a_k c_k}{t_0} \left[\mathrm{e}^{-b_k(t-t_0)} - \mathrm{e}^{-b_k t}\right].
\end{aligned}$$

So we can get

$$\sigma(t) = \begin{cases} 0, & t \leqslant 0, \\ \dfrac{\epsilon_0 c_0}{t_0} + \dfrac{\epsilon_0}{b_k} \displaystyle\sum_{k=1}^{n} \dfrac{a_k c_k}{t_0} \left(1 - \mathrm{e}^{-b_k t}\right), & 0 < t \leqslant t_0, \\ \dfrac{\epsilon_0 c_0}{t_0} + \dfrac{\epsilon_0}{b_k} \displaystyle\sum_{k=1}^{n} \dfrac{a_k c_k}{t_0} \left[\mathrm{e}^{-b_k(t-t_0)} - \mathrm{e}^{-b_k t}\right], & t > t_0. \end{cases} \tag{3.2}$$

It can be seen from the comparison between formula (3.1) and formula (3.2) that the stress change formula of exponential damping model is different when the initial ramp is considered and not considered, which indicates that the existence of initial ramp has an influence on the stress change of exponential damping model.

4. Numerical simulation

When the kernel function $G(t)$ takes the form of a power exponential function, the numerical simulation of stress relaxation test has been done for the Scott Blair's model with the simplest fractional order in the literature (cf. M. Di Paola [8] and M. Du, Y. Wang and Z. Wang [9]), and we will not repeat the discussion here. Now this paper only considers that the kernel function is an exponential function, which is not involved in the relevant literature. When $c_0 = 0$, $a_1 = c_1 = 1$, $n = 1$, take different values of b_1 and t_0, and considering the influence of initial ramp, that is, the change of strain is shown in Fig. 1, and the change of relative stress is shown in Fig. 2, Fig. 3 and Fig. 4. When $t = t_0$, the stress reaches the maximum value

$$y = \frac{\sigma(t)}{\epsilon_0} = \frac{1}{b_1 t_0} \left(1 - \mathrm{e}^{-b_1 t_0}\right).$$

When $b_i = 0.5, 1, 2$ and $t_0 = 0.5, 1, 2$ are taken respectively, the stress changes are shown in Fig. 2, Fig. 3 and Fig. 4. From the numerical simulation image, it can be seen that for a given ϵ_0, when $b_1 < 0.5$ and $t_0 < 0.5$, the stress is close to linear, and the convolutional nonviscous damping material shows elastic properties; When $b_1 \geqslant 1$, $t > t_0$, the convolutional nonviscous damping material obviously shows viscous property, and the curve presents exponential function attenuation. It can also be seen from the simulation image that with the gradual increase of the total observation time t^*, the influence of the initial ramp will gradually weaken and the stress curve will tend to coincide, gradually in a robust stable state. Moreover, the maximum stress $\sigma(t) = \frac{\epsilon_0}{b_1 t_0}\left(1 - \mathrm{e}^{-b_1 t_0}\right)$ depends on the value of t_0 and b_1. When the value of t_0 and b_1 is larger, the maximum value of stress is smaller. According to the numerical simulation results, we can easily draw the following conclusions:

1. The existence of the initial ramp has a great influence on the determination of the parameter b_k of the convolutional nonviscous damping material;

2. t_0 must be measured in the actual test and cannot be ignored;

3. The longer the total observation time t^*, the smaller the influence of the initial ramp, and the better the robust stability.

The above is the influence of the selection of t_0 and the total observation time t^* on the stress change when we consider the initial ramp.

Now let's compare the stress changes in the relaxation test by numerical simulation under the two conditions of considering and not considering the initial ramp. In the stress relaxation test, without considering the initial ramp, the stress change is shown in the dotted line in Fig. 5. In the case of considering the initial ramp, the change of stress affected by the initial ramp is shown in the solid line in Fig. 5. Obviously, the influence of initial ramp is not considered, and the stress shows exponential attenuation. However, under the influence of the initial ramp, the stress changes relatively slowly, and there is still a large change between the two. Therefore, the initial ramp can not be ignored in the actual test. If it is ignored, it will have a great influence on the determination of parameters.

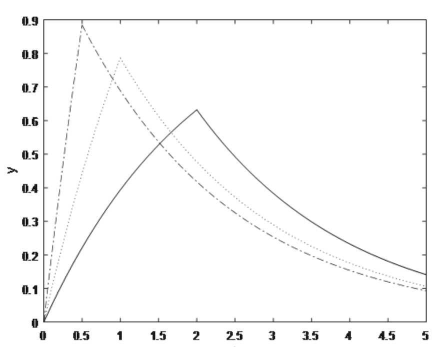

Figure 2. Change of stress for different value of $b_1 = 0.5$ and $t_0 = 0.5, 1, 2$.

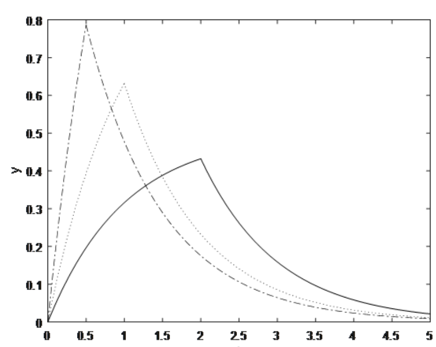

Figure 3. Change of stress for different value of $b_1 = 1$ and $t_0 = 0.5, 1, 2$.

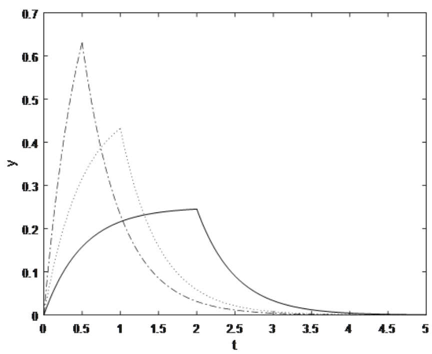

Figure 4. Change of stress for different value of $b_1 = 2$ and $t_0 = 0.5, 1, 2$.

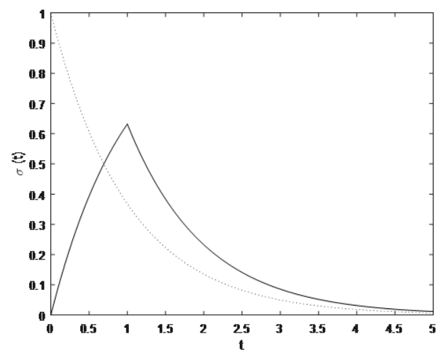

Figure 5. Influences of initial ramps on relaxation experimental test.

5. Conclusion

In this paper, taking the stress relaxation test of viscoelastic material as an example, the viscoelastic materials used in the test are characterized by the convolutional nonviscous damping model. When the kernel function in the convolutional nonviscous damping model takes the form of power exponential function, without considering the influence of initial ramp, the stress change formula is obtained as follows:

$$\sigma(t) = \begin{cases} 0, & t \leqslant 0, \\ \left(\dfrac{a}{t^{\alpha}} + b\right)\epsilon_0, & t > 0. \end{cases}$$

Considering the influence of initial ramp, the stress change formula is obtained as follows:

$$\sigma(t) = \begin{cases} 0, & t \leqslant 0, \\ \dfrac{a\epsilon_0}{t_0(1-\alpha)} t^{1-\alpha} + \dfrac{\epsilon_0 t b}{t_0}, & 0 < t \leqslant t_0, \\ \dfrac{a\epsilon_0}{t_0(1-\alpha)} \left[t^{1-\alpha} - (t-t_0)^{1-\alpha}\right] + b\epsilon_0, & t > t_0. \end{cases}$$

Through comparison and numerical calculation, it is easy to conclude that the stress change formula is different in the two cases of considering and not considering the influence of the initial ramp, which shows that the existence of the initial ramp has an influence on the stress change of the model. This conclusion covers and extends the results of Scott-Blair model, which is the simplest model of fractional order discussed in references (cf. M. Di Paola [8] and M. Du, Y. Wang and Z. Wang [9]), and is more general.

When the kernel function in the convolutional nonviscous damping model takes the form of exponential function, that is, the convolutional nonviscous damping model is exponential damping model. Without considering the initial ramp, the stress change formula is obtained as follows:

$$\epsilon(t) = \begin{cases} 0, & t \leqslant 0, \\ \left[c_0\delta(t) + \displaystyle\sum_{k=1}^{n} a_k \mathrm{e}^{-b_k t} c_k\right]\epsilon_0, & t > 0. \end{cases}$$

Considering the influence of initial ramp, the stress change formula is obtained as follows:

$$\sigma(t)=\begin{cases}0, & t\leqslant 0,\\ \dfrac{\epsilon_0 c_0}{t_0}+\dfrac{\epsilon_0}{b_k}\displaystyle\sum_{k=1}^{n}\dfrac{a_k c_k}{t_0}\left(1-\mathrm{e}^{-b_k t}\right), & 0<t\leqslant t_0,\\ \dfrac{\epsilon_0 c_0}{t_0}+\dfrac{\epsilon_0}{b_k}\displaystyle\sum_{k=1}^{n}\dfrac{a_k c_k}{t_0}\left[\mathrm{e}^{-b_k(t-t_0)}-\mathrm{e}^{-b_k t}\right], & t>t_0.\end{cases}$$

Similarly, through comparison and numerical calculation, it is easy to conclude that the stress change formula is different in the two cases of considering and not considering the influence of the initial ramp, which shows that the existence of the initial ramp has an influence on the stress change of the exponential damping model. The following conclusions can also be obtained through numerical simulation: under the condition of considering and not considering the initial ramp, it has a significant influence on the determination of the parameter b_k of the convolutional nonviscous damping material; In the test, t_0 must be determined and cannot be ignored; The longer the total observation time t^*, the smaller the influence of the initial ramp, and the better the robust stability.

The influence of initial ramp will also occur in creep test, and we will study and discuss it in future work.

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