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COMPLEX NONLINEAR EVOLUTION EQUATIONS IN THE CONTEXT OF OPTICAL FIBERS: NEW WAVE-FORM ANALYSIS

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Abstract In this study, the new waveforms of two nonlinear evolution models are investigated by an analytical method, namely the sigmoid function method. The considered nonlinear complex models for this are the full nonlinearity form of the Fokas-Lenells equation and the paraxial wave equation, which play an important role in the field of fiber optics by balancing the nonlinearity with the dispersion terms. Under different numeric values of the free terms, the obtained results represent varieties of wave shapes, specifically anti-kink, dark, bright, singular soliton, anti-peakon, kink, two-lump propagation during breather periodic form, single lump, two lump solutions, periodic peakon, and periodic wave solutions, which have not been obtained in the previous studies. These dynamical characteristics are discussed in detail with the help of a pictorial presentation of the derived solutions. These resultants of both the considered nonlinear equations can be useful in both fiber optics as well as in other optics-related fields.

Keywords Fokas-Lenells equation, paraxial wave equation, sigmoid function method, optical solutions, fiber optics.

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1. Introduction

The nonlinear phenomena and the complexities that are arising in the surroundings can be modeled using partial differential equations particularly nonlinear partial differential equations (NLPDEs). For this reason, it can be considered as the link between nonlinearity and its properties. To be specific, these characteristics can be understood by studying the analytical solutions in different wave shapes of various mathematical models [3]. These wave shapes include bright, dark [32], breather-type, triple solitons [29], M-lump solutions [23], and many more. Several nonlinear

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mathematical models describe distinct physical features of nature such as intelligent sensing [24], fiber optics [11], plasma physics [43], metamaterials [19], long surface gravity waves [38], fluid dynamics [4], weakly nonlocal competing nonlinear medium [40], microtubules [2], dusty plasma [41], quantum physics [34] and many more. Among these features, the Fokas-Lenells equation (FLE) and paraxial wave equation (PWE) are the models that explain the wave propagation in optical fibers as well as in other matters of optics [31, 35] under the influence of different parameters.

1.1. Fokas-Lenells equation (FLE)

Consider the complex nonlinear evolution equation as

$$\begin{aligned} &iE_t + a_1 E_{xx} + a_2 E_{xt} + |E|^2(bE + i\sigma E_x) \\ =&i[\alpha E_x + \nu(|E|^{2m}E)_x + \delta(|E|^{2m})_x E], \end{aligned} \tag{1.1}$$

which is named as the dimensionless form of the Fokas-Lenells equation (FLE) [7, 11, 22, 31]. Where, $E(x,t)$ is dependent upon x which is treated as the spatial variable, and t which is treated as the temporal variable indicating the waveform. The dual dispersion terms, a_1 and a_2 are the denotations used for the group velocity dispersion (GVD) and the spatio-temporal dispersion (STD) respectively. Parameter b is the indication for self-phase modulation while the nonlinear dispersive term, σ is the reason for additional dispersive effects. Besides, the parameters α, ν, and δ in eq.(1.1) respectively account for inter-modal dispersion, self-steepening impact, and nonlinear scattering. The inter-modal dispersion accelerates the pulse spreading while the function of the self-steepening effect is to impede the formation of the shock-type waves that arise during pulse propagation. Here, ν and δ crop up with full nonlinearity which is indicated by the parameter m.

In the field of optical telecommunication systems, the foremost issue is the propagation of pulses due to the balance between the low count of GVD and nonlinear phenomena. To maintain the balance, the nonlinear dispersive effects must be augmented. The given equation clearly describes this. Therefore, FLE has become the center of attention among researchers. And this is the main reason that motivated us to consider the FLE (1.1).

To show the functionality of FLE solution in optical fibers, there are several types of research in recent years using a number of methods. For example, sine-Gordon expansion method [7,22], $(m + (G'/G))$-expansion method [22], a modified extended direct algebraic method [8], improved $\tan(\phi(\xi)/2)$-expansion method [30], improved Bernoulli sub-ODE method [30], generalized (G'/G)-expansion method [30], ansatz function technique [1], He's variational principle [26], chirped soliton solution technique [14], traveling wave reduction [28], tanh-coth-function method [25], modified simple equation method [13], $\exp(-\phi(\xi))$-expansion method [10], generalized exponential rational function method [33], auxiliary equation method [5], Lie-symmetry analysis [12], (G'/G)-expansion method [16], extended Kudryashov method and Kudryashov method [18], and lastly complex envelope ansatz method [15].

1.2. Paraxial wave equation (PWE)

The paraxial wave equation (PWE) [6,9,17] in the form of Kerr law nonlinearity is stated as

$$iE_x + \frac{a_1}{2}E_{tt} + \frac{a_2}{2}E_{yy} + a_3|E|^2E = 0, \tag{1.2}$$

where, $E = E(y, x, t)$ is the complex envelope depending upon the transverse of the spatial parameter represented by y, the longitudinal propagation parameter given by x, and the temporal co-ordinate denoted as t. Here, a_1 is used to define dispersion, a_2 is used for the diffraction and the symbol a_3 is considered for the Kerr law nonlinearity. Additionally, for $a_1 a_2 < 0$, eq.(1.2) is converted into its hyperbolic nonlinear form of it, and for $a_1 a_2 > 0$, eq.(1.2) is converted into its elliptic nonlinear form of it.

In the field of optics, the PWE explains how diffraction and dispersion affect light beam interaction as well as light emission outside of fluctuations. These parameters must be balanced and adjusted to improve outcomes and provide an advanced understanding of their uses. And to achieve this, it is necessary to derive the model's analytical solutions, which attracted the interest of several researchers who tried to tackle the problem using various analytical methods.

Arshad et al. [9] have utilized the modified extended mapping method to eq.(1.2) and found varieties of solitary wave solutions. The extended trial equation method has been applied by Ali et al. [6] to get the elliptic function solutions of the PWE. Durur and Yokuş [17] analyzed the relations between dispersion and diffraction factors by using the modified $\left(\frac{1}{G'}\right)$-expansion method to eq.(1.2). Gao et al. [20] have studied other varieties of wave shapes of eq.(1.2) by the modified auxiliary expansion method. In addition, the fractional forms of the PWE are solved by using the sech–tanh expansion method and extended Kudryashov method [27], modified simple equation method [42], and extended sinh-Gordon expansion method [21].

Furthermore, the simplified extended tanh-function method [45], modified Khater method [44], the (G'/G)-expansion method [36] and others can be applied to find the analytical solutions of the nonlinear phenomena.

The main motivation for considering these models is to derive and show the relations between dispersion, nonlinearity, and diffraction in the field of optics. However, it is clear from the literature that much more exploration is needed to understand these physical features. In addition, this study emphasizes investigating the new exact solutions of eqs.(1.1) and (1.2) by the sigmoid function (SF) method [37]. Also, in most of the previous studies, eq.(1.1) has been considered for a specific value of m, but here we have considered the full-nonlinearity form of (1.1). Furthermore, the main novelty of this study is the resulting solutions that have not been achieved earlier by considering the relations between the physical factors which have been mentioned above. Moreover, the physical characteristics of the attained solution are presented graphically which will be useful in further study of the optical fibers [39].

The remaining sections of the paper are organized in the following order: The wave analysis of the perturbed FLE and the PWE are addressed in section 2. Section 3 covers the sequential steps and application of the SF method to the considered nonlinear models. After that, the derived solutions are pictorially presented in section 4 for different numeric values of the free parameters. The physical characteristics of these wave solutions are also discussed in detail in this section. Finally, this work's conclusion is addressed in section 5.

2. Travelling wave analysis of the nonlinear evolution equations

2.1. Wave analysis of the perturbed FLE

The solution form of eq.(1.1) is presumed as

$$E(x,t) = V(\xi)e^{i\theta}, \tag{2.1}$$

where, $V(\xi)$ is the amplitude of the wave taking ξ as $\xi = x - qt$ and θ as the respective phase component where $\theta = \theta(x,t) = -lx + kt + \epsilon$. Here, q, l, k, and ϵ denote velocity, soliton frequency, wave number, and phase constant respectively.

Substituting eq.(2.1) into (1.1), the separated imaginary and real parts respectively become

$$V' + \sigma V^2 V' - [\nu(2m+1) + 2m\delta]V^{2m}V' = 0, \tag{2.2}$$

and

$$\left(-k - a_1 l^2 + a_2 kl - \alpha l\right)V + (b + \sigma l)V^3 + (a_1 - a_2 q)V'' - \nu l V^{2m+1} = 0. \tag{2.3}$$

Now, from the collected coefficients of the functions of eq.(2.2) and eq.(2.3), we found

$$q = \frac{a_2 k - \alpha - 2a_1 l}{1 - a_2 l}, \qquad 1 - a_2 l \neq 0, \tag{2.4}$$

$$\sigma = b = 0, \tag{2.5}$$

and

$$\nu = -\frac{2m\delta}{2m+1}. \tag{2.6}$$

Substituting eq.(2.5) and eq.(2.6) in eq.(2.3), we obtain,

$$(a_1 - a_2 q)V'' - \left(k + a_1 l^2 - a_2 kl + \alpha l\right)V + \frac{2m}{2m+1}\delta l V^{2m+1} = 0. \tag{2.7}$$

The required closed form of solutions can be attained by using the transformation

$$V = (\psi)^{\frac{1}{2m}}. \tag{2.8}$$

Accordingly, eq.(2.7) decomposes to

$$\begin{aligned} &(a_1 - a_2 q)\left[(1-2m)(\psi')^2 + 2m\psi\psi''\right] \\ &- 4m^2\left(k + a_1 l^2 - a_2 kl + \alpha l\right)\psi^2 + \frac{8m^3}{2m+1}\delta l \psi^3 \\ =&0, \end{aligned} \tag{2.9}$$

where, $m \neq 0, -\frac{1}{2}$.

2.2. Wave analysis of the PWE

Assume that the solution of eq.(1.2) can be written as

$$E(y,x,t)=\psi(\xi)e^{i\theta}, \tag{2.10}$$

where the amplitude of the waveform is $\psi(\xi)$ taken as $\xi = y + x - qt$. θ is the respective component of phase having the value $\theta = \theta(y,x,t) = -ly + mx + kt + \epsilon$. Here, q is the velocity, while l, m, k, and ϵ denote soliton frequency, wave number, and phase constant respectively.

Now, substituting eq.(2.10) into eq.(1.2) and then splitting the real and imaginary parts accordingly we get,

$$2a_3\psi^3-\left(2m+a_1k^2+a_2l^2\right)\psi+\left(a_1q^2+a_2\right)\psi''=0, \tag{2.11}$$

and

$$\begin{aligned}&(1-a_1kq-a_2l)\,\psi'=0,\\ &\Rightarrow a_2=\frac{1-a_1kq}{l}. \qquad [\because \psi'\neq 0]\,.\end{aligned} \tag{2.12}$$

Thus, putting the value of a_2 from eq.(2.12) into eq.(2.11), we have

$$2a_3l\psi^3-\left(2ml+a_1lk^2+l^2-a_1l^2kq\right)\psi+\left(1-a_1kq+a_1lq^2\right)\psi''=0. \tag{2.13}$$

3. Sigmoid function method (SFM)

In this segment, we have detailed the steps and execution of the SFM [37] to get the required resultants of eqs.(2.9) and (2.13). The main advantages of this method over other methods are that it is simple to use, straightforward, and gives more solutions as compared to others means the convergence rate is faster than others.

3.1. The procedure of the SFM

In this part, we have discussed the sequential steps of the considered method for getting FLE and PWE solutions.

Step-1:

Suppose eqs.(2.9) and (2.13)'s solution can be stated as

$$\psi(\xi)=c_0+\sum_{i=1}^{N}c_i\phi^i(\xi), \tag{3.1}$$

where, $\phi(\xi)$ satisfies

$$\phi'(\xi)=\phi(\xi)-\phi^2(\xi), \tag{3.2}$$

with the unknown parameters c_0, c_i, which will be found later. Then, by equalizing the degrees of the nonlinear and the derivative terms of eqs.(2.9) and (2.13), the value of N can be found.

Step-2:

The solution form of eq.(3.2) is

$$\phi(\xi)=\frac{e^{\xi}}{1+e^{\xi}}. \tag{3.3}$$

Also, the reason for choosing this function is to find the exact solutions of nonlinear differential equations as it is the general solution of the first order Riccati eq.(3.2), which gives more solutions to the equations. And to find the highest order singularity of the given nonlinear equations, it can be used. If N is an integer, then only this method can be applied else, by again using transformation, this method can be repeated.

Step-3:

Now, equations having the term $\phi^i(\xi)$ can be derived by plugging eqs.(3.1) and (3.2) into eqs.(2.9) and (2.13). After that, by collecting the coefficients of likely exponents of $\phi(\xi)$ and equating them to zero, a set of simultaneous equations can be achieved.

Step-4:

Consequently, by solving the obtained equations, the necessary exact solutions of eqs.(2.9) and (2.13) can be reached.

3.2. Application of the SFM

In this part, to have new varieties of solitary wave solutions, we have implemented the considered method into the reduced eqs.(2.9) and (2.13).

3.2.1. Application of the SFM to the FLE

After equalizing the degree of $\psi\psi''$ with ψ^3 of eq.(2.9), we get $N = 2$. Consequently, eq.(3.1) is transformed to,

$$\psi(\xi) = c_0 + c_1\phi(\xi) + c_2\phi^2(\xi). \tag{3.4}$$

Now putting eqs.(3.4) and (3.2) into eq.(2.9) and amounting the coefficients of equal power of $\phi(\xi)$ to zero, we find a system of equations. Then, by solving these, we get the resultants:

Set-1:

$$c_0 = -\frac{3(a_1 - qa_2)}{4l\delta},\ c_1 = \frac{3(a_1 - qa_2)}{l\delta},$$
$$c_2 = -\frac{3(a_1 - qa_2)}{l\delta},\ k = \frac{2l\alpha + a_1 + 2l^2a_1 - qa_2}{2(-1 + la_2)},\ m = 1,$$

and the corresponding solution is

$$\psi_1(\xi) = -\frac{3(a_1 - qa_2)}{4l\delta} + \frac{3(a_1 - qa_2)}{l\delta}\left(\frac{e^\xi}{1 + e^\xi}\right) - \frac{3(a_1 - qa_2)}{l\delta}\left(\frac{e^\xi}{1 + e^\xi}\right)^2. \tag{3.5}$$

Set-2:

$$c_0 = -\frac{2(a_1 - qa_2)}{l\delta},\ c_1 = \frac{12(a_1 - qa_2)}{l\delta},$$
$$c_2 = -\frac{12(a_1 - qa_2)}{l\delta},\ k = \frac{l\alpha + a_1 + l^2a_1 - qa_2}{-1 + la_2},\ m = \frac{1}{2},$$

therefore, the resultant solution is

$$\psi_2(\xi) = -\frac{2(a_1 - qa_2)}{l\delta} + \frac{12(a_1 - qa_2)}{l\delta}\left(\frac{e^\xi}{1 + e^\xi}\right) - \frac{12(a_1 - qa_2)}{l\delta}\left(\frac{e^\xi}{1 + e^\xi}\right)^2. \tag{3.6}$$

Set-3:

$$
\begin{aligned}
& c_0=0,\ c_1=-\frac{3\left(a_1-q a_2\right)}{16 l \delta},\ c_2=\frac{3\left(a_1-q a_2\right)}{16 l \delta}, \\
& k=\frac{16 l \alpha-a_1+16 l^2 a_1+q a_2}{16\left(-1+l a_2\right)},\ m=-2,
\end{aligned}
$$

and hence the solution is represented as

$$
\psi_3(\xi)=-\frac{3\left(a_1-q a_2\right)}{16 l \delta}\left(\frac{e^{\xi}}{1+e^{\xi}}\right)+\frac{3\left(a_1-q a_2\right)}{16 l \delta}\left(\frac{e^{\xi}}{1+e^{\xi}}\right)^2 . \tag{3.7}
$$

Set-4:

$$
\begin{aligned}
& c_0=\frac{3\left(a_1-q a_2\right)}{16 l \delta},\ c_1=-\frac{3\left(a_1-q a_2\right)}{8 l \delta}, \\
& c_2=\frac{3\left(a_1-q a_2\right)}{16 l \delta},\ k=\frac{-4 l \alpha+a_1-4 l^2 a_1-q a_2}{4-4 l a_2},\ m=-2,
\end{aligned}
$$

with the corresponding solution structure as

$$
\psi_4(\xi)=\frac{3\left(a_1-q a_2\right)}{16 l \delta}-\frac{3\left(a_1-q a_2\right)}{8 l \delta}\left(\frac{e^{\xi}}{1+e^{\xi}}\right)+\frac{3\left(a_1-q a_2\right)}{16 l \delta}\left(\frac{e^{\xi}}{1+e^{\xi}}\right)^2 . \tag{3.8}
$$

Set-5:

$$
c_0=0,\ c_1=0,\ c_2=\frac{3\left(a_1-q a_2\right)}{16 l \delta},\ k=\frac{4 l \alpha-a_1+4 l^2 a_1+q a_2}{4\left(-1+l a_2\right)},\ m=-2,
$$

and

$$
\psi_5(\xi)=\frac{3\left(a_1-q a_2\right)}{16 l \delta}\left(\frac{e^{\xi}}{1+e^{\xi}}\right)^2 . \tag{3.9}
$$

Set-6:

$$
\begin{aligned}
& c_0=0,\ c_1=\frac{\left(1+3 m+2 m^2\right)\left(a_1-q a_2\right)}{2 l m^3 \delta}, \\
& c_2=-\frac{\left(1+3 m+2 m^2\right)\left(a_1-q a_2\right)}{2 l m^3 \delta},\ k=\frac{4 l m^2 \alpha-a_1+4 l^2 m^2 a_1+q a_2}{4 m^2\left(-1+l a_2\right)},
\end{aligned}
$$

and the retrived solution is

$$
\psi_6(\xi)=\left(\frac{\left(1+3 m+2 m^2\right)\left(a_1-q a_2\right)}{2 l m^3 \delta}\right)\left[\left(\frac{e^{\xi}}{1+e^{\xi}}\right)-\left(\frac{e^{\xi}}{1+e^{\xi}}\right)^2\right] . \tag{3.10}
$$

3.2.2. Application of the SFM to the PWE

After equalizing the degree of ψ'' with ψ^3 of eq.(2.13), we find $N=1$. Consequently, eq.(3.1) is transformed to,

$$
\psi(\xi)=c_0+c_1 \phi(\xi). \tag{3.11}
$$

Now putting eqs.(3.4) and (3.2) into eq.(2.13) and amounting the coefficients of equal power of $\phi(\xi)$ to zero, we find a system of equations. Then, solving these, we get the solutions:

Set-1:

$$c_0 = -\frac{\sqrt{\frac{-4l - 2l^2q^2a_1 + q\left(qa_1 - A\right)}{l^2}}}{4\sqrt{a_3}},\ c_1 = -2c_0,\ k = \frac{q\left(a_1 + 2l^2a_1\right) - A}{4la_1},$$

and the corresponding solution is

$$\psi_1(\xi) = \frac{\left(-1 + e^{\xi}\right)\sqrt{\frac{-4l - 2l^2q^2a_1 + q\left(qa_1 - A\right)}{l^2}}}{4\left(1 + e^{\xi}\right)\sqrt{a_3}}. \tag{3.12}$$

Set-2:

$$c_0 = \frac{\sqrt{\frac{-4l - 2l^2q^2a_1 + q\left(qa_1 - A\right)}{l^2}}}{4\sqrt{a_3}},\ c_1 = -2c_0,\ k = \frac{q\left(a_1 + 2l^2a_1\right) - A}{4la_1}.$$

Set-3:

$$c_0 = -\frac{\sqrt{\frac{-4l - 2l^2q^2a_1 + q\left(qa_1 + A\right)}{l^2}}}{4\sqrt{a_3}},\ c_1 = -2c_0,\ k = \frac{q\left(a_1 + 2l^2a_1\right) + A}{4la_1}.$$

Set-4:

$$c_0 = \frac{\sqrt{\frac{-4l - 2l^2q^2a_1 + q\left(qa_1 + A\right)}{l^2}}}{4\sqrt{a_3}},\ c_1 = -2c_0,\ k = \frac{q\left(a_1 + 2l^2a_1\right) + A}{4la_1}.$$

Set-5:

$$c_0 = -\frac{i\sqrt{\frac{(l+m)\left(1 + 6l^2 + 8lm\right)a_1 + B}{l^2\left(l+m\right)^2 a_1}}}{4\sqrt{2}\sqrt{a_3}},\ c_1 = -2c_0,$$

$$k = -\frac{-\left(1 + 2l^2\right)(l+m)\,a_1 - B}{4\sqrt{2}l\left(-\left(l+m\right)a_1\right)^{\frac{3}{2}}},\ q = -\frac{1}{\sqrt{2}\sqrt{-\left(l+m\right)a_1}},$$

and the respected solution is

$$\psi_5(\xi) = \frac{i\left(-1 + e^{\xi}\right)\sqrt{\frac{(l+m)\left(1 + 6l^2 + 8lm\right)a_1 + B}{l^2\left(l+m\right)^2 a_1}}}{4\sqrt{2}\left(1 + e^{\xi}\right)\sqrt{a_3}}. \tag{3.13}$$

Set-6:

$$c_0 = \frac{i\sqrt{\frac{(l+m)\left(1 + 6l^2 + 8lm\right)a_1 + B}{l^2\left(l+m\right)^2 a_1}}}{4\sqrt{2}\sqrt{a_3}},\ c_1 = -2c_0,$$

$$k = -\frac{-\left(1+2l^2\right)(l+m)\,a_1 - B}{4\sqrt{2}l\left(-(l+m)\,a_1\right)^{\frac{3}{2}}},\ q = -\frac{1}{\sqrt{2}\sqrt{-(l+m)\,a_1}}.$$

Set-7:

$$c_0 = -\frac{i\sqrt{\dfrac{(l+m)\left(1+6l^2+8lm\right)a_1 - B}{l^2\,(l+m)^2\,a_1}}}{4\sqrt{2}\sqrt{a_3}},\ c_1 = -2c_0,$$
$$k = -\frac{-\left(1+2l^2\right)(l+m)\,a_1 + B}{4\sqrt{2}l\left(-(l+m)\,a_1\right)^{\frac{3}{2}}},\ q = -\frac{1}{\sqrt{2}\sqrt{-(l+m)\,a_1}},$$

and the respected solution is

$$\psi_7(\xi) = \frac{i\left(-1+e^{\xi}\right)\sqrt{\dfrac{(l+m)\left(1+6l^2+8lm\right)a_1 - B}{l^2\,(l+m)^2\,a_1}}}{4\sqrt{2}\left(1+e^{\xi}\right)\sqrt{a_3}}. \tag{3.14}$$

Set-8:

$$c_0 = \frac{i\sqrt{\dfrac{(l+m)\left(1+6l^2+8lm\right)a_1 - B}{l^2\,(l+m)^2\,a_1}}}{4\sqrt{2}\sqrt{a_3}},\ c_1 = -2c_0,$$
$$k = -\frac{-\left(1+2l^2\right)(l+m)\,a_1 + B}{4\sqrt{2}l\left(-(l+m)\,a_1\right)^{\frac{3}{2}}},\ q = -\frac{1}{\sqrt{2}\sqrt{-(l+m)\,a_1}}.$$

Set-9:

$$c_0 = -\frac{i\sqrt{\dfrac{(l+m)\left(1+6l^2+8lm\right)a_1 - B}{l^2\,(l+m)^2\,a_1}}}{4\sqrt{2}\sqrt{a_3}},\ c_1 = -2c_0,$$
$$k = -\frac{\left(1+2l^2\right)(l+m)\,a_1 + B}{4\sqrt{2}l\left(-(l+m)\,a_1\right)^{\frac{3}{2}}},\ q = \frac{1}{\sqrt{2}\sqrt{-(l+m)\,a_1}}.$$

Set-10:

$$c_0 = \frac{i\sqrt{\dfrac{(l+m)\left(1+6l^2+8lm\right)a_1 - B}{l^2\,(l+m)^2\,a_1}}}{4\sqrt{2}\sqrt{a_3}},\ c_1 = -2c_0,$$
$$k = -\frac{\left(1+2l^2\right)(l+m)\,a_1 - B}{4\sqrt{2}l\left(-(l+m)\,a_1\right)^{\frac{3}{2}}},\ q = \frac{1}{\sqrt{2}\sqrt{-(l+m)\,a_1}}.$$

Set-11:

$$c_0 = -\frac{i\sqrt{\dfrac{(l+m)\left(1+6l^2+8lm\right)a_1 + B}{l^2\,(l+m)^2\,a_1}}}{4\sqrt{2}\sqrt{a_3}},\ c_1 = -2c_0,$$

$$k = -\frac{\left(1+2l^2\right)(l+m)\,a_1 + B}{4\sqrt{2}l\left(-(l+m)\,a_1\right)^{\frac{3}{2}}},\ q = \frac{1}{\sqrt{2}\sqrt{-(l+m)\,a_1}}.$$

Set-12:

$$c_0 = \frac{i\sqrt{\dfrac{(l+m)\left(1+6l^2+8lm\right)a_1 + B}{l^2\,(l+m)^2\,a_1}}}{4\sqrt{2}\sqrt{a_3}},\ c_1 = -2c_0,$$
$$k = -\frac{\left(1+2l^2\right)(l+m)\,a_1 + B}{4\sqrt{2}l\left(-(l+m)\,a_1\right)^{\frac{3}{2}}},\ q = \frac{1}{\sqrt{2}\sqrt{-(l+m)\,a_1}},$$

and the respected solution is

$$\psi_{12}(\xi) = -\frac{i\left(-1+e^{\xi}\right)\sqrt{\dfrac{(l+m)\left(1+6l^2+8lm\right)a_1 + B}{l^2\,(l+m)^2\,a_1}}}{4\sqrt{2}\left(1+e^{\xi}\right)\sqrt{a_3}}, \tag{3.15}$$

where, $A = \sqrt{a_1\left(-8l\left(1+2l\left(l+2m\right)\right) + \left(1-2l^2\right)^2 q^2 a_1\right)}$ and

$$B = \sqrt{(l+m)^2\left(1+6l^2+8lm\right)^2 a_1^2}.$$

4. Numerical simulation

It is important to address the dynamical behaviour of the FLE and PWE with the help of graphical representation because of the presence of the complex nonlinear phenomena in it, which have been discussed in this section. The two and three-dimensional figures explain the dynamics of the derived solutions.

4.1. Graphical views of the wave solutions

In this part, the wave solutions for eqs.(3.5), (3.7), (3.9), (3.10) and (3.12)-(3.15) are graphically displayed.

4.1.1. Graphical views of the wave solutions of the FLE

Here, eqs.(3.5), (3.7), (3.9) and (3.10) of FLE are presented graphically.

4.1.2. Graphical views of the wave solutions of the PWE

Here, eqs.(3.12) to (3.15) of PWE are pictorially presented.

4.2. Properties of the obtained wave solutions

In this segment, the physical features of the derived wave solutions of the FLE and the PWE are analyzed.

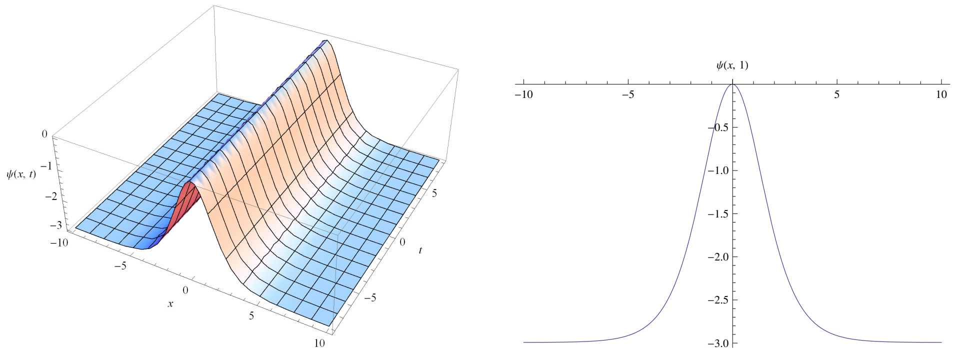


Figure 1. Wave solutions of eq.(3.5) for $a_1 = -1.2,\ a_2 = 0.3,\ \alpha = 2,\ l = 0.2,\ \delta = -1.5$ and $q = -0.01$

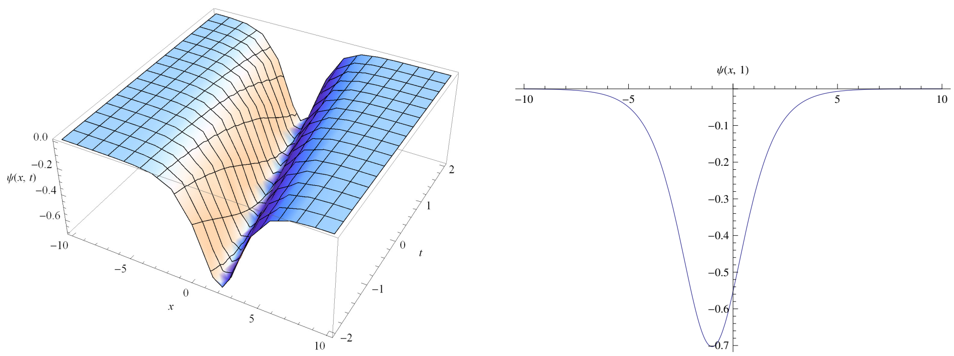


Figure 2. Wave solutions of eq.(3.7) for $a_1 = 1.2,\ a_2 = 0.3,\ \alpha = 2,\ l = 0.2,\ \delta = 0.5$ and $q = -1$

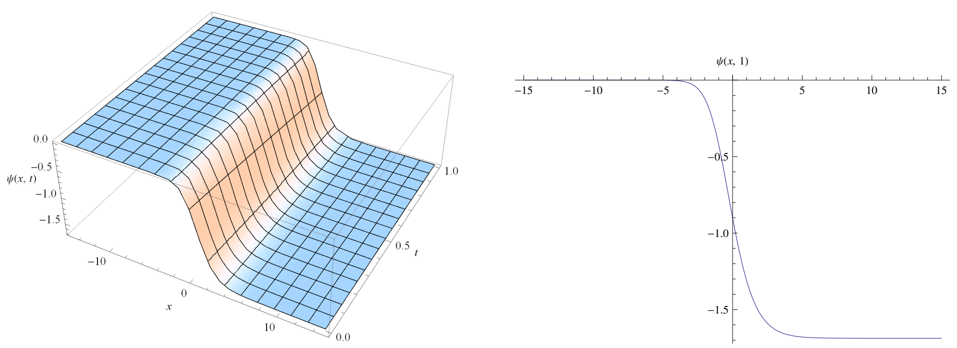


Figure 3. Wave solutions of eq.(3.9) for $a_1 = -1.2,\ a_2 = 0.3,\ \alpha = 2,\ l = 0.2,\ \delta = 0.5$ and $q = -1$

4.2.1. Analysis of the wave solutions of the FLE

From the graphs presented in sec.4.1.1, the bright soliton of eq.(3.5) has been depicted in fig.1 while the dark soliton of eq.(3.7) has been shown in fig.2. Here, the bright soliton of eq.(3.5) is propagating in the anomalous GVD and normal STD regions with $q = -0.01$ within the limits $-10 \leq x \leq 10$ and $-8 \leq t \leq 8$. The dark soliton of eq.(3.7) is propagating in the normal GVD and STD regions with $q = -1$ within the range $-10 \leq x \leq 10$ and $-2 \leq t \leq 2$.

In fig.3, the anti-kink waveform of eq.(3.9) has been presented. Here, this wave

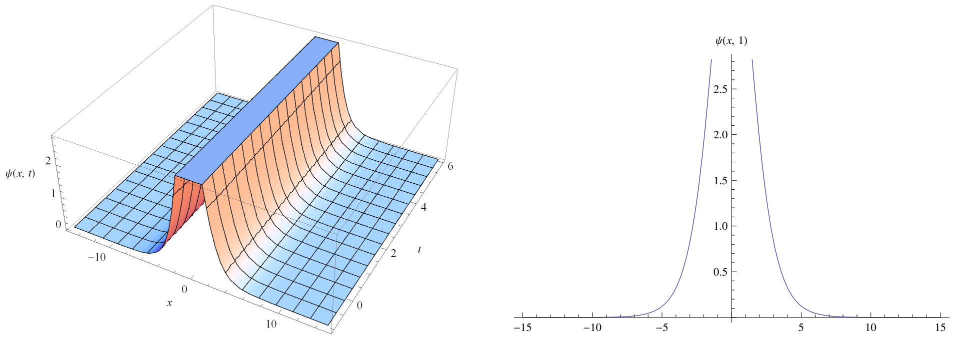


Figure 4. Wave solutions of eq.(3.10) for $a_1 = -0.2,\ a_2 = -0.3,\ m = 2,\ \alpha = -2,\ l = -0.2,\ \delta = 0.05$ and $q = 0.01$

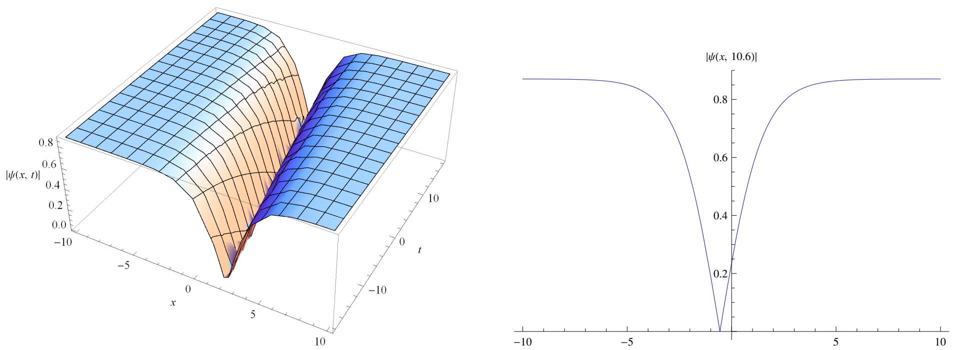


Figure 5. Wave solutions of eq.(3.12) for the values of $a_1 = 1.2,\ m = -0.3,\ a_3 = -2,\ l = -0.2,\ q = -0.1$ and $y = -0.5$

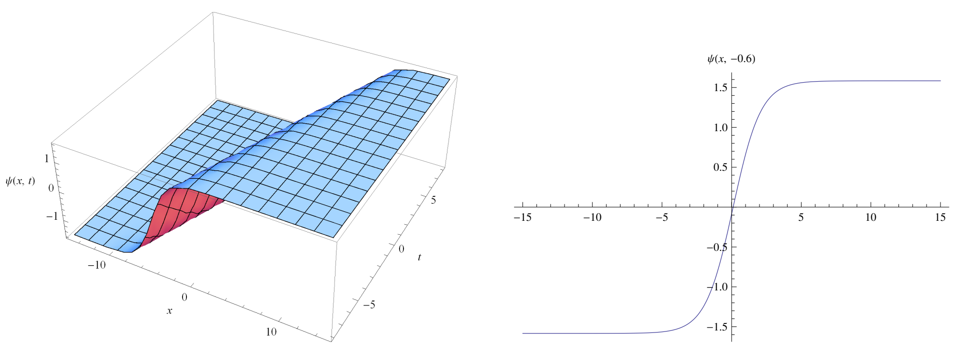


Figure 6. Wave solutions of eq.(3.12) for the values of $a_1 = -1.2,\ m = -0.3,\ a_3 = -1.38,\ l = 0.2,\ q = 0.65$ and $y = -0.5$

shape is generated in the anomalous GVD and normal STD regions with $q = -1$ between the range $-15 \leq x \leq 15$ and $0 \leq t \leq 1$. Fig.4 shows the pattern of singular soliton solution of eq.(3.10). Here, it is propagating in the anomalous GVD and STD regions with $q = 0.01$ between the limits $-15 \leq x \leq 15$ and $-1 \leq t \leq 6$. Singular solutions are another type of solitary wave having singularity that means infinite discontinuity which can be taken as the imaginary space in the middle connected to solitary waves so as fig.4 does.

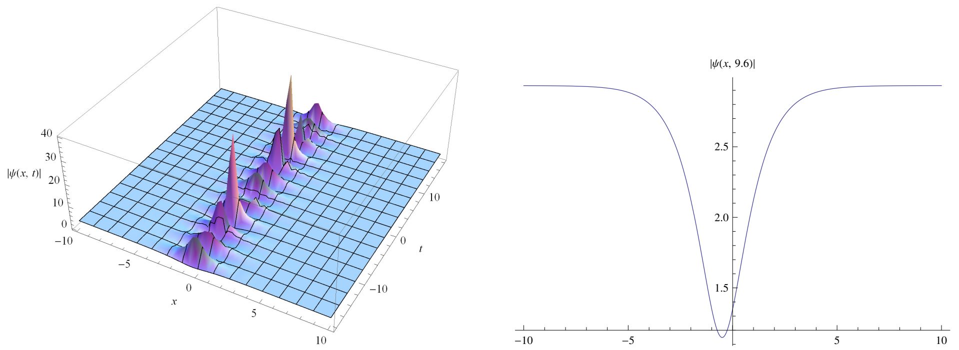


Figure 7. Wave solutions of eq.(3.13) for the values of $a_1 = -1.2,\ m = -0.3,\ a_3 = 1.38,\ l = 0.2$ and $y = 0.5$

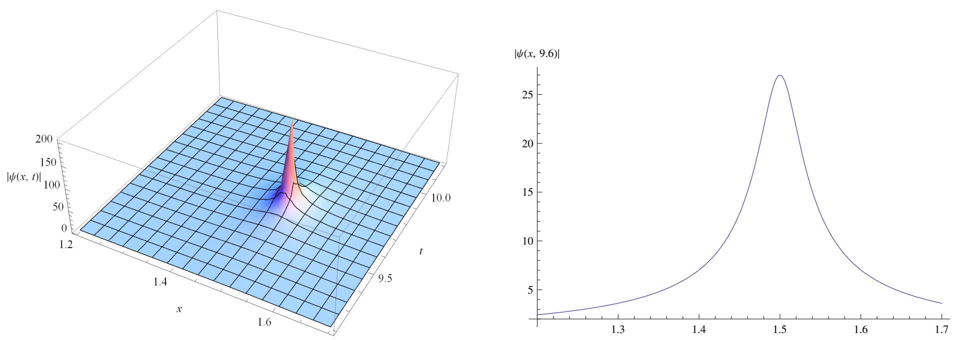


Figure 8. Wave solutions of eq.(3.13) for the values of $a_1 = -1.9,\ m = -0.3,\ a_3 = -1.38,\ l = -2.2$ and $y = -1.5$

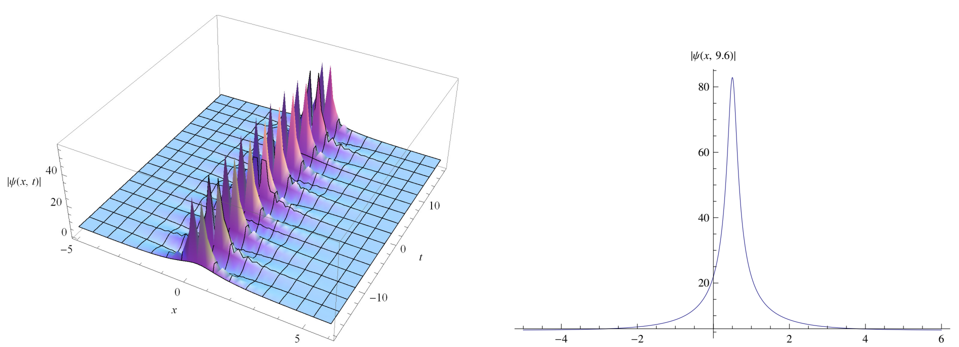


Figure 9. Wave solutions of eq.(3.13) for the values of $a_1 = -1.9,\ m = -0.3,\ a_3 = -0.38,\ l = 0.2$ and $y = -0.5$

4.2.2. Analysis of the wave solutions of the PWE

The dynamical properties of the PWE that have been pictorially presented in sec.4.1.2 are discussed in detail here. The distinct behaviours of eq.(3.12) under different free parameters have been delineated graphically in fig.s 5 and 6. The anti-peakon behaviour of eq.(3.12) has been displayed in fig.5 while the kink behaviour of eq.(3.12) has been shown in fig.6. Here, the anti-peakon wave is propagating for $q = -0.1$ and $y = -0.5$ within the limits $-10 \le x \le 10$ and $-18 \le t \le 18$ while

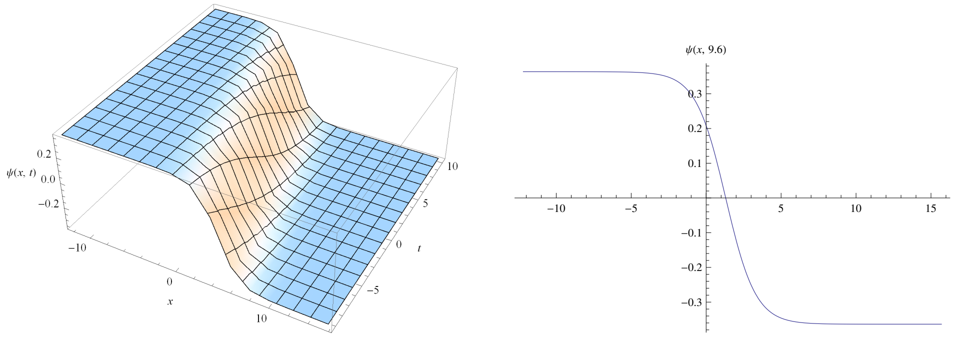


Figure 10. Wave solutions of eq.(3.14) for the values of $a_1 = 1.9,\ m = -0.3,\ a_3 = 1.38,\ l = -2.2$ and $y = -1.5$

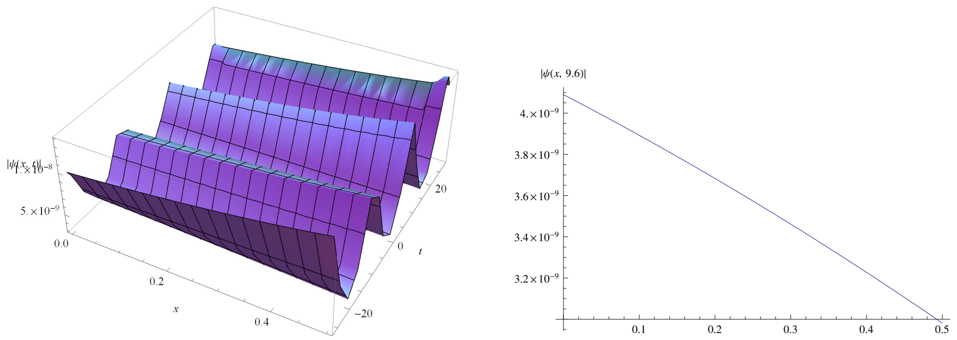


Figure 11. Wave solutions of eq.(3.14) for the values of $a_1 = -1.9,\ m = -0.3,\ a_3 = -0.38,\ l = -1.82$ and $y = -1.5$

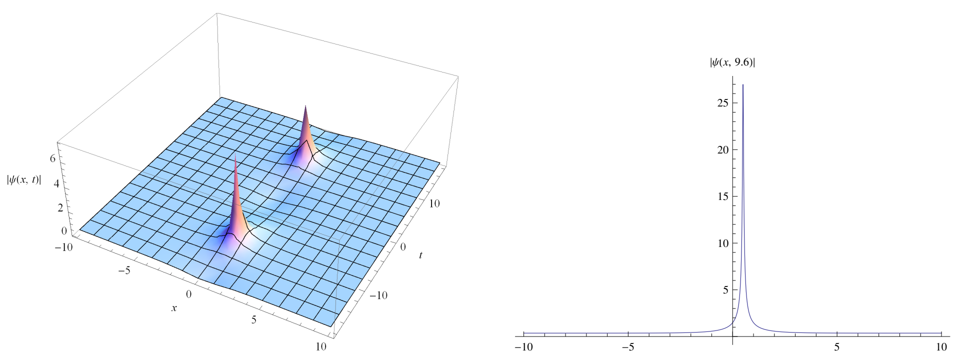
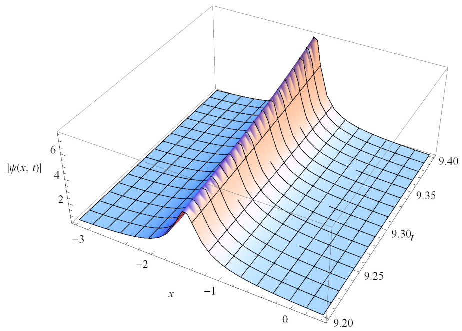


Figure 12. Wave solutions of eq.(3.15) for the values of $a_1 = -1.9,\ m = -0.3,\ a_3 = -1.38,\ l = -2.2$ and $y = -0.5$

the kink wave is propagating for the values $q = 0.65$ and $y = -0.5$ within the range $-15 \leq x \leq 15$ and $-8 \leq t \leq 8$.

The different dynamical behaviour of eq.(3.13) is depicted in fig.s 7 to 9. The two lump waves arising during the propagation of the breather periodic solution have been displayed in fig.7 while the single lump solution has depicted in fig.8. The two lump wave propagation is generated for $y = 0.5$ between the limits $-10 \leq x \leq 10$ and $-18 \leq t \leq 18$ while the single lump propagation is generated for $y = -1.5$ between the limits $1.2 \leq x \leq 1.7$ and $9.2 \leq t \leq 10.2$. The periodic peakon pattern



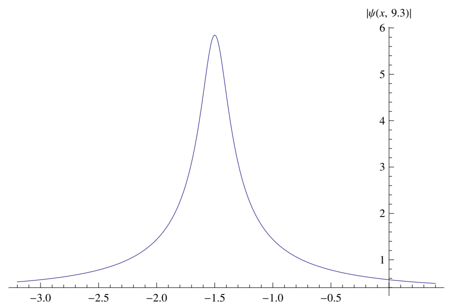


Figure 13. Wave solutions of eq.(3.15) for the values of $a_1 = -1.9$, $m = -0.3$, $a_3 = 1.38$, $l = -2.2$ and $y = 1.5$

of eq.(3.13) has been delineated in fig.9 for the y value -0.5 within the limits $-5 \leq x \leq 6$ and $-18 \leq t \leq 18$.

Fig. 10 shows the anti-kink solution of eq.(3.14) while fig.11 shows the periodic wave solution of eq.(3.14) in the time domain. Here, the anti-kink solution is transmitting for $y = -1.5$ between the range $-12.2 \leq x \leq 15.7$ and $-9.2 \leq t \leq 10.2$ while the periodic wave is transmitting for $y = -1.5$ between the limits $0 \leq x \leq 0.5$ and $-26 \leq t \leq 28$. Fig.12 delineates the two lump propagation of eq.(3.15) while the bright wave propagation of eq.(3.15) has been viewed in fig.13. The two lump solution is for $y = -0.5$ within the range $-10 \leq x \leq 10$ and $-18 \leq t \leq 18$ while the bright wave is for $y = 1.5$ within the range $-3.2 \leq x \leq 0.4$ and $9.2 \leq t \leq 9.4$.

The bright-shaped soliton shows an increase in amplitude, while the dark-shaped soliton shows a decrease in amplitude with a uniform-intensity background. These waveforms are responsible for the data transferring from one end to another without any loss. The kink and anti-kink solutions transfer energy from one core to another during propagation. The periodic solutions transmit with a uniform-intensity background. These types of wave solutions have been derived by properly balancing the physical factors with nonlinear terms which are responsible for data transmission without causing any harm to them.

5. Conclusion

In this work, new waveform solutions of two nonlinear evolution equations are derived by an analytical method, namely the sigmoid function method. The considered nonlinear models for this work are the full nonlinearity form of the perturbed FLE and the PWE. Under different numerical values of the free parameters, the obtained solutions depict varieties of wave shapes such as anti-kink, dark, bright, singular soliton, anti-peakon, kink, two lump propagation during breather periodic form, single lump, two lump solutions, periodic peakon, and periodic wave solutions. These physical characteristics of the derived solutions are discussed in detail with the help of three- and two-dimensional graphical views. These solutions show how the physical factors are properly balanced with each other, along with nonlinearity terms that have not been achieved before. These resultants of the FLE and PWE can be useful in the field of fiber optics as well as in other matters of optics.

From the future scope of this study, one can determine the analytical solution to the different fractional derivative forms of the given nonlinear evolution equations

(NLEEs) and analyze their solutions. Other physical elements that can affect the system can also be considered. Furthermore, the modulation instability analysis of various kinds of the given NLEEs with various parameters may be found.

Conflicts of interest and funding

Conflict of interest: There is no conflict of interest from the author's side.

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Data availability statement

The current manuscript's developed and/or used datasets are available upon justifiable request from the corresponding author.

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References

[1] I. Ahmed, A. R. Seadawy and D. Lu, *M-shaped rational solitons and their interaction with kink waves in the Fokas-lenells equation*, Physica Scripta, 2019, 94(5), 055205.

[2] M. A. Akbar, A. M. Wazwaz, F. Mahmud, et al., *Dynamical behavior of solitons of the perturbed nonlinear Schrödinger equation and microtubules through the generalized Kudryashov scheme*, Results in Physics, 2022, 43, 106079.

[3] L. Akinyemi and E. Morazara, *Integrability, multi-solitons, breathers, lumps and wave interactions for generalized extended Kadomtsev–Petviashvili equation*, Nonlinear Dynamics, 2023, 111(5), 4683–4707.

[4] L. Akinyemi, M. Şenol and O. S. Iyiola, *Exact solutions of the generalized multidimensional mathematical physics models via sub-equation method*, Mathematics and Computers in Simulation, 2021, 182, 211–233.

[5] K. S. Al-Ghafri, E. V. Krishnan and A. Biswas, *Chirped optical soliton perturbation of Fokas-Lenells equation with full nonlinearity*, Advances in Difference Equations, 2020, 2020(1).

[6] K. Ali, S. T. R. Rizvi, B. Nawaz and M. Younis, *Optical solitons for paraxial wave equation in Kerr media*, Modern Physics Letters B, 2019, 33(3), 1–9.

[7] K. K. Ali, M. S. Osman and M. Abdel-Aty, *New optical solitary wave solutions of Fokas-Lenells equation in optical fiber via Sine-Gordon expansion method*, Alexandria Engineering Journal, 2020, 59(3), 1191–1196.

[8] M. Arshad, D. Lu, M. U. Rehman, et al., *Optical solitary wave and elliptic function solutions of Fokas-Lenells equation in presence of perturbation terms and its modulation instability*, Physica Scripta, 2019, 94(10), 105202.

[9] M. Arshad, A. R. Seadawy, D. Lu and M. S. Saleem, *Elliptic function solutions, modulation instability and optical solitons analysis of the paraxial wave dynamical model with Kerr media*, Optical and Quantum Electronics, 2021, 53(1), 1–20.

[10] S. Arshed, A. Biswas, Q. Zhou, et al., *Optical solitons pertutabation with Fokas-Lenells equation by* $exp(-\phi(\xi))$*-expansion method*, Optik, 2019, 179, 341–345.

[11] S. Arshed and N. Raza, *Optical solitons perturbation of Fokas-Lenells equation with full nonlinearity and dual dispersion*, Chinese Journal of Physics, 2020, 63, 314–324.

[12] A. Bansal, A. H. Kara, A. Biswas, et al., *Optical soliton perturbation, group invariants and conservation laws of perturbed Fokas-Lenells equation*, Chaos, Solitons and Fractals, 2018, 114, 275–280.

[13] A. Biswas, Y. Yildirim, E. Yaşar, et al., *Optical soliton solutions to Fokas-lenells equation using some different methods*, Optik, 2018, 173, 21–31.

[14] G. Dieu-donne, M. B. Hubert, A. R. Seadawy, et al., *Chirped soliton solutions of Fokas–Lenells equation with perturbation terms and the effect of spatio-temporal dispersion in the modulational instability analysis*, European Physical Journal Plus, 2020, 135(2).

[15] G. Dieu-donne, C. G. L. Tiofack, A. R. Seadawy, et al., *Propagation of W-shaped, M-shaped and other exotic optical solitons in the perturbed Fokas-Lenells equation*, European Physical Journal Plus, 2020, 135(4).

[16] Y. Ding, M. S. Osman and A. M. Wazwaz, *Abundant complex wave solutions for the nonautonomous Fokas-Lenells equation in presence of perturbation terms*, Optik, 2019, 181, 503–513.

[17] H. Durur and A. Yokuş, *Discussions on diffraction and the dispersion for traveling wave solutions of the (2+1)-dimensional paraxial wave equation*, Mathematical Sciences, 2022, 16(3), 269–279.

[18] A. Ebaid, E. R. El-Zahar, A. F. Aljohani, et al., *Exact solutions of the generalized nonlinear Fokas-Lennells equation*, Results in Physics, 2019, 14, 102472.

[19] M. Ekici, *Exact solitons in optical metamaterials with quadratic-cubic nonlinearity using two integration approaches*, Optik, 2018, 156, 351–355.

[20] W. Gao, H. F. Ismael, H. Bulut and H. M. Baskonus, *Instability modulation for the (2+1)-dimension paraxial wave equation and its new optical soliton solutions in Kerr media*, Physica Scripta, 2020, 95(3), 035207.

[21] W. Gao, H. F. Ismael, S. A. Mohammed, et al., *Complex and real optical soliton properties of the paraxial non-linear Schrödinger equation in kerr media with M-fractional*, Frontiers in Physics, 2019, 7, 1–8.

[22] H. F. Ismael, H. Bulut and H. M. Baskonus, *Optical soliton solutions to the Fokas-Lenells equation via sine-Gordon expansion method and* $(m + (G'/G))$*-expansion method*, Pramana-Journal of Physics, 2020, 94(1), 1–9.

[23] H. F. Ismael, H. Bulut, C. Park and M. S. Osman, *M-lump, N-soliton solutions, and the collision phenomena for the (2+1)-dimensional Date-Jimbo-Kashiwara-Miwa equation*, Results in Physics, 2020, 19, 103329.

[24] Z. P. Izgi, F. N. Saglam, S. Sahoo, et al., *A partial offloading algorithm based on intelligent sensing*, International Journal of Modern Physics B, 2022, 36(17), 2250097.

[25] A. J. M. Jawad, A. Biswas, Q. Zhou, et al., *Optical soliton perturbation of Fokas-Lenells equation with two integration schemes*, Optik, 2018, 165, 111–116.

[26] Y. Khan, *A novel type of soliton solutions for the Fokas-Lenells equation arising in the application of optical fibers*, Modern Physics Letters B, 2020, 2150058, 1–10.

[27] M. M. A. Khater, A. M. Alabdali, A. Mashat and S. A. Salama, *Optical soliton wave solutions of the fractional complex paraxial wave dynamical model along with kerr media*, Fractals, 2022, 30(5), 1–17.

[28] N. A. Kudryashov, *First integrals and general solution of the Fokas-Lenells equation*, Optik, 2019, 195, 163135.

[29] S. Kumar, M. Niwas, M. S. Osman and M. A. Abdou, *Abundant different types of exact soliton solution to the (4+1)-dimensional Fokas and (2+1)-dimensional breaking soliton equations*, Communications in Theoretical Physics, 2021, 73(10), 105007.

[30] M. Lakestani and J. Manafian, *Novel dark, singular and combo optical solitons for Fokas-Lenells equation*, Acta Physica Polonica A, 2019, 136(1), 214–224.

[31] Z. Li, C. Huang and B. Wang, *Phase portrait, bifurcation, chaotic pattern and optical soliton solutions of the Fokas-Lenells equation with cubic-quartic dispersion in optical fibers*, Physics Letters A, 2023, 465, 128714.

[32] S. Malik, H. Almusawa, S. Kumar, et al., *A (2+1)-dimensional Kadomtsev–Petviashvili equation with competing dispersion effect: Painlevé analysis, dynamical behavior and invariant solutions*, Results in Physics, 2021, 23, 104043.

[33] M. S. Osman and B. Ghanbari, *New optical solitary wave solutions of Fokas-Lenells equation in presence of perturbation terms by a novel approach*, Optik, 2018, 175, 328–333.

[34] R. U. Rahman, M. M. M. Qousini, A. Alshehri, et al., *Evaluation of the performance of fractional evolution equations based on fractional operators and sensitivity assessment*, Results in Physics, 2023, 49, 106537.

[35] T. Rasool, R. Hussain, M. A. Al Sharif, et al., *A variety of optical soliton solutions for the M-truncated Paraxial wave equation using Sardar-subequation technique*, Optical and Quantum Electronics, 2023, 55(5), 396.

[36] S. S. Ray and S. Sahoo, *New exact solutions of time fractional modified kawahara equations in modelling surface tension in shallow-water and capillary gravity water waves*, The European Physical Journal Plus, 2017, 132(1), 1–11.

[37] S. Sahoo and S. S. Ray, *A new method for exact solutions of variant types of time-fractional Korteweg-de Vries equations in shallow water waves*, Math. Methods in the Appl. Sci., 2016, 40(1), 106–114.

[38] S. Sahoo and S. Saha Ray, *A novel approach for stochastic solutions of wick-type stochastic time-fractional Benjamin–Bona–Mahony equation for modeling long surface gravity waves of small amplitude*, Stochastic Analysis and Applications, 2019, 37(3), 377–387.

[39] M. Savescu, Q. Zhou, L. Moraru, et al., *Singular optical solitons in birefringent nano-fibers*, Optik, 2016, 127(20), 8995–9000.

[40] A. Souleymanou, A. Houwe, A. H. Kara, et al., *Explicit exact solutions and conservation laws in a medium with competing weakly nonlocal nonlinearity and parabolic law nonlinearity*, Optical and Quantum Electronics, 2023, 55(5), 464.

[41] K. U. Tariq, H. Rezazadeh, M. Zubair et al., *New Exact and Solitary Wave Solutions of Nonlinear Schamel–KdV Equation*, International Journal of Applied and Computational Mathematics, 2022, 8(3), 114.

[42] K. U. Tariq, H. Zainab, A. R. Seadawy, et al., *On some novel optical wave solutions to the paraxial M-fractional nonlinear Schrödinger dynamical equation*, Optical and Quantum Electronics, 2021, 53(5), 1–14.

[43] A. Tripathy and S. Sahoo, *A novel analytical method for solving (2+1)-dimensional extended Calogero-Bogoyavlenskii-Schiff equation in plasma physics*, Journal of Ocean Engineering and Science, 2021, 6(4), 405–409.

[44] A. Tripathy and S. Sahoo, *New optical behaviours of the time–fractional Radhakrishnan–Kundu–Lakshmanan model with Kerr law nonlinearity arise in optical fibers*, Optical and Quantum Electronics, 2022, 54(4), 1–16.

[45] K. J. Wang and G. D. Wang, *Exact traveling wave solutions for the system of the ion sound and Langmuir waves by using three effective methods*, Results in Physics, 2022, 35, 105390.