FORECASTING SYSTEMIC RISK OF CHINA'S BANKING INDUSTRY BY PARTIAL DIFFERENTIAL EQUATIONS MODEL AND COMPLEX NETWORK*

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Abstract The monitoring and controlling of systemic risk have increasingly become the focus of attention in the financial field. It is important and difficult to accurately forecast systemic financial risk. In this paper, we propose a spatio-temporal partial differential equation model to describe the systemic risk of China's Banking Industry based on network, clustering, and real date of 24 China's A-share listed banks. The model considers the combined influence of local risk and transboundary contagion effects, and the prediction relative accuracy is up to 95%. Simulation results confirm that strict joint control measures, the timeliness of central bank intervention, and differences in bank strategies are efficient for reducing systemic risk. To our knowledge, this is the first paper to apply a PDE model to forecast systemic financial risk.

Keywords Systemic risk, forecast, complex network, partial differential equation, joint control.

MSC(2010) 35Q80, 35K57, 91B30, 91B84.

1. Introduction

Following the international financial crisis of 2008, the prevention of systemic financial risks has become a top priority in the economic strategies of many countries. The intricate interconnections among various markets and financial institutions serve as channels for the spread of financial risks. Given the crucial role of the banking industry within the financial system, it's essential to understand the complex correlations among banks and study the mechanisms of risk propagation. That is helpful for regulatory bodies in preventing systemic risk and maintaining the stability of the banking industry. Allen [5] studied the systemic risk contagion of the banking industry, and others researched systemic risk of areas such as the real estate

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sector, the stock market, and the foreign exchange market. Greenwood et al. [18] discussed risk contagion across these sectors.

Various methods are used to measure systemic risk. Some papers analyze the contribution of individual institutions to systemic financial risks, such as the conditional value at risk (CoVaR) in Adrian and Brunnermeier [3,39], systemic expected shortfall (SES) and marginal expected shortfall (MES) in Acharya et al. [2], and the conditional capital shortfall of a firm during a severe market decline (SRISK) in Brownlees and Engle [12]. Others measure financial risk at a systemic and overall level, such as the aggregate level of risk-taking in the financial sector (CATFIN) [6].

Systemic risk can propagate through various channels, including interbank businesses [22] and the price linkage of banks' stocks on the financial market [30]. These complex interbank relationships can generally be characterized through network analysis. Multiple methods exist to structure a banking network. For instance, some networks are formed based on the asset-liability linkage [35] to study the contagion effect of interbank business. Others are constructed by using the correlation between banks' stock prices [17] to calculate each bank's contribution to systemic risk and to evaluate the systemic importance of banks. Utilizing these networks, researchers can trace the path of systemic risk contagion within the banking network, explore the influence of network structure on the contagion effect of systemic risk, and simulate the banking system's response to various types of shocks [10]. However, existing researches often simulated the risk contagion effect of a single bank shock on the bank network, or discussed the impact of network topology on systemic risk [32]. There is few works considering the cluster effect of banks on risk contagion. In fact, risk spreads not only between closely connected banks, but also between different banking clusters [40].

To prevent systemic risks, accurate prediction and warning of such risks are crucial. This helps regulators monitor systemic risks in the banking industry and implement effective supervision to mitigate systemic risk. In recent years, significant research has been conducted on predicting systemic risks using machine learning and deep learning algorithms. Sekmen and Kurkcu [32] employed artificial neural network learning to predict financial risks. Tölö [40] utilized long and short-term memory (RNN-LSTM) and gated cycle unit (RNN-GRU) neural networks to predict systemic financial crises 1-5 years in advance. Wang and Zhu [41] used backpropagation (BP) neural networks to predict systemic risk in the three major U.S. stock indexes, achieving higher prediction accuracy than traditional models. While machine learning and deep learning methods offer reliable prediction accuracy, they rely on parameter learning and do not capture the specific process of dynamic contagion in systemic risk.

In recent years, data-driven differential equation models are widely used in various fields [27, 37]. Partial differential equation (PDE) is an effective tool to study the dynamic evolution of complex systems. A spatio-temporal PDEs model not only depicts the temporary dynamics, but also can describe the spatial interactions. Y. Wang and H. Wang [44] used a PDE model to forecast bitcoin price movements. Wang et al. [46] developed a specific PDE model to describe and predict the transmission of PM2.5 requiring only PM2.5 concentration data and avoiding extensive computation. Compared to machine learning and traditional statistical methods, the PDE model can effectively describe and explain the dynamic process of studied object.

In this paper, based on real stock prices date of 24 China's A-share listed banks,

we construct a banking network and propose a spatio-temporal partial differential equation model to describe the dynamic process of systemic risk generation and propagation among bank-clusters. The model considers the combined influence of local risk in each cluster and transboundary infection effects between different clusters for obtaining a high prediction accuracy. Moreover, we use the PDE model to observe the effects of different policies by simulation. To the best of our knowledge, this is the first paper to apply a PDE model to forecast systemic financial risk.

Our research framework is as follows. First, we construct a complex banking network based on the transfer entropy of the weekly stock return rate of sample banks from 2016 to 2018. Then, we divide these 24 banks into three categories using a network subgraph and spectral clustering, and embed them onto a one-dimensional axis. We calculate the weighted average of $|\Delta CoVaR|$ to measure the systemic risk of three clusters.

Next, we establish a spatio-temporal partial differential equation (PDE) model to describe the dynamic process of risk generation and propagation within and between clusters. We validate the model using actual data from 2019 to 2022, forecasting the weighted average of $|\Delta \text{CoVaR}|$ for the fifth week by using historical data of the first four weeks and achieving a prediction accuracy up to 95%. Notably, the PDE model outperforms the prediction accuracy of the BP neural network and Random Forest under the same conditions.

Furthermore, sensitivity analysis and simulations demonstrate that stricter joint control measures lead to reduced systemic risk. The timeliness of central bank interventions, the intensity of financial regulatory measures, and differences in bank strategies are identified as effective factors in controlling systemic risk.

This article contributes to several research areas. We establish a PDE model based on banking network to quantify the spatial and temporal dynamics of systemic financial risk. This model can efficiently describe the dynamic process of risk generation and propagation in the system. The spectral clustering with high-order organization reveals deeper connections between the listed banks. This makes it more effective to study the cluster effect of systemic risk contagion. Additionally, the numerical simulations give insights into controlling the spread of systemic risks and evaluating the impact of regulatory policies. Thus, this article provides a new method and insight for researching systemic financial risk.

The rest of the paper is organized as follows. In Section 2, we introduce the PDE model and bank network. The forecast process and results of systemic risk are presented in Section 3. We show sensitivity analysis and policy simulation in Section 4 and conclude the work in Section 5.

2. Method and data

2.1. Spatio-temporal partial differential equation model

Banks can form many complex network relying on various reasons. We will introduce a new bank network in Section 2.2 and 2.3. For considering cluster effect of systemic risk propagation, we divide the network into three clusters and treat the banks in the same cluster as a whole. Then we embed three clusters onto an axis in Euclidean space in Section 2.4. See Figure 1.

The dynamic process of systemic risk can be divided into two part: one generation and spread within each cluster and the other diffusion among clusters. Similar



Figure 1. Three clusters embedded in one-dimensional space

transmission processes have been wildly studied in various fields such as epidemiology [11, 25], biology [14, 26], informatics [42], and environmental science [45]. PDE models have been proved to be effective to study these processes of diffusion and infection [34, 42, 45]. Motivated by these works, we propose a spatio-temporal PDE model to describe the dynamic process of systemic risk in China's banking industry. Although the banking system is influenced by other financial sectors, for simplicity, we assume that the banking system is isolated, meaning there are no risk flows entering or leaving the boundary of this system.

Let u(x,t) represent the concentration of systemic risk, that is the weighted average of $|\Delta \text{CoVaR}|$ (denoted in Section 2.5), in cluster x at a given time t. The rate of change of u(x,t) depends on the generation of systemic risk within each cluster and the contagion among clusters. The following proposition gives a spatiotemporal PDE model to describe the changing process of systemic risk:

Proposition 2.1. Suppose u(x,t) represent the concentration of systemic risk, then u(x,t) satisfies

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = \frac{\partial}{\partial x} \left[d(x) \frac{\partial u(x,t)}{\partial x} \right] + r(t)u(x,t) \left[h(x) - \frac{u(x,t)}{K} \right], \\ u(x,1) = \varphi(x), 1 < x < 3, \\ \frac{\partial u}{\partial x}(1,t) = \frac{\partial u}{\partial x}(3,t) = 0, t > 1, \end{cases}$$
(2.1)

where $\frac{\partial u(x,t)}{\partial t}$ represents the rate of change in u(x,t) at time t, $\frac{\partial u(x,t)}{\partial x}$ represents the rate of change in u(x,t) at location x.

- $\frac{\partial}{\partial x} \left[d(x) \frac{\partial u(x,t)}{\partial x} \right]$ indicates the transboundary contagion of systemic risk between different clusters. Based on Fick's law [26], the term $d(x) \frac{\partial u(x,t)}{\partial x}$ measures the systemic risk quantity crossing the location x, where d(x) measures the speed at which risk flows between different bank clusters. This concept has been widely used to describe spatial contagion in infectious disease [11], spatial transport of air pollution [45] and animal movements [34]. For simplicity, we assume that $d(x) \equiv d > 0$.
- $r(t)u(x,t)\left[h(x) \frac{u(x,t)}{K}\right]$ represents the intrinsic growth and spread process of systemic risk at cluster x and time t. This type of function is commonly used to describe the growth of bacteria, tumors or social information over time.
 - 1. The function r(t) > 0 represents the growth rate of the systemic risk of each cluster at time t. A highly interconnectedness financial network not only leads to a rapid increase in systemic risks but also absorb shocks through network associations [1]. Hence, we assume that systemic risk rapidly increases initially, and reaching its peak at certain time, after which the rate of growth decreases over time. The risk growth function is expressed as $r(t) = A + Be^{-C(t-1)^2}$, where A, B, C are non-negative parameters to be determined based on actual data from 2019 to 2022.
 - 2. The location function h(x) describes the spatial heterogeneity of systemic risks and indicates the different spread rates of risks in each cluster. h(x)is constructed using cubic spline interpolation, satisfying $h(x) \equiv h_i, i =$ 1, 2, 3, where x_i represents the location of the cluster *i*. The values of h(x) depends on actual data.
 - 3. K indicates the capacity of the banking system's risk, i.e., the maximum risk that the entire banking system can bear. If the risk exceeds the maximum capacity of the system, the system will face collapse. The values of K also depends on the actual data.
- The initial function $u(x, 1) = \varphi(x)$ describes the weighted average of $|\Delta \text{CoVaR}|$ for each cluster at time t = 1, and it always satisfies $\varphi(x) \ge 0$.
- The Neumann boundary condition [15] $\frac{\partial u}{\partial x}(1,t) = \frac{\partial u}{\partial x}(3,t) = 0, t > 1$, indicates that no risk spreads across the boundaries at x = 1, and x = 3. It means that we do not consider the inflow and outflow of risks between the banking system and the outside world.

2.2. Data

To ensure the representativeness and availability of market data, we extract the closing prices of 24 China's A-share commercial banks listed before 2017 (see Appendix A) and the mainland China banking index.

These 24 banks hold significant influence over the entire banking system in China. By the end of 2022, the assets of these 24 listed banks accounted for 73.78% of the total assets of all domestic commercial banks, and their liabilities accounted for 73.56% of the total liabilities of all domestic commercial banks, encompassing the majority of the actual businesses within China's entire banking system. Specially, the sample includes four globally systemically important banks (G-SIBs) of China: Bank of China, Industrial and Commercial Bank of China, Agricultural Bank of

China, and China Construction Bank. Generally, these banks are categorized into four groups based on their ownership: state-owned banks (SO), joint-stock banks (JS), urban commercial banks (UC), and rural commercial banks (RC).

To ensure that the 24 listed banks have been on the market for at least one year, we select the corresponding data from 159 trading weeks between 2016 and 2018. The data is obtained from the WIND Database.* We calculate the weekly rate of return of each bank by taking the logarithmic firstorder difference: $r_{it} = \ln (q_t^i) - \ln (q_{t-1}^i)$, where q_t^i represents the average closing stock price of stock *i* in week *t*.

2.3. Network construction

In this section, we construct banking network by using information flow between bank stock prices. The stock price of a bank, influenced by various factors such as the bank's operating results, macroeconomic environment and policy interventions, is considered a key factor when discussing bank's risk [48]. The correlation between bank stock prices can represent the relationship of banks to some extent. In recent years, information flow measurement based on transfer entropy has been widely used in the stock market [16]. It is an important tool for analyzing causal relationships in nonlinear systems [19]. Transfer entropy captures directional and dynamic information [13] without relying on any specific functional form to describe the interrelation between variables.

Let's consider X as a discrete random variable with a probability distribution p(x). The uncertainty or amount of information of X can be measured by its entropy, defined as $H(X) = -\sum_{x} p(x) \log_2 p(x)$. Suppose X and Y are stochastic processes of order k and l, the transfer entropy from Y to X is defined by formula

$$\operatorname{TE}_{Y \to X}(k,l) = \sum_{x_{n+1}, x_n^{(k)}, y_n^{(l)}} p(x_{n+1}, x_n^{(k)}, y_n^{(l)}) \log_2 \frac{p(x_{n+1} \mid x_n^{(k)}, y_n^{(l)})}{p(x_{n+1} \mid x_n^{(k)})}.$$
 (2.2)

 $\operatorname{TE}_{Y \to X}(k, l)$ indicates the reduction of uncertainty of X when Y is known, highlighting the effect of Y in the prediction of X. The notation $x_n^{(k)} = (x_n, \ldots, x_{n-k+1})$ and $y_n^{(l)} = (y_n, \ldots, y_{n-l+1})$ represent k-dimensional and l-dimensional delay embedding vectors, and $p(x_{n+1} \mid x_n^{(k)})$ represent conditional probability.

Assume W and Z represent the weekly rate of return series for each two banks, as outlined in Appendix, over 159 trading weeks from 2016 to 2018. We divide the series into five states using quantiles of 20%, 40%, 60% and 80%, denoted as $q_{0.2}$, $q_{0.4}$, $q_{0.6}$ and $q_{0.8}$ respectively. Therefore, the state set is defined as $s = \{s_1, s_2, s_3, s_4, s_5\}$, where

$$s_{x} = \begin{cases} s_{1} & x < q_{0.2}, \\ s_{2} & q_{0.2} \leqslant x < q_{0.4}, \\ s_{3} & q_{0.4} \leqslant x < q_{0.6}, \\ s_{4} & q_{0.6} \leqslant x < q_{0.8}, \\ s_{5} & x \geqslant q_{0.8}. \end{cases}$$
(2.3)

(1) (1)

Using the stochastic processes X and Y record the state W and Z belong to in every trading week. The correlation between W and Z is studied by calculating the

^{*}https://www.wind.com.cn/mobile/EDB/en.html.

transfer entropy $\text{TE}_{Y\to X}(k,l)$ and $\text{TE}_{X\to Y}(l,k)$ with k=l=1 [31]. The probabilities in formula (2.2) can be approximated by the frequency of X(and Y) located in each state $S = \{s_1, s_2, s_3, s_4, s_5\}$. In this article, we compute the transfer entropy using "RTransferEntropy" package released by Behrendt et al [7].

Now, let's construct an interbank information flow network. The nodes in the network represent 24 sample banks. The weight of the connected edge between any two nodes is the transfer entropy of the corresponding two random processes generated from their weekly return rate series from 2016 to 2018. Considering that the transfer entropy between some banks is too small, we choose $\theta=0.0925$ [23] as the threshold value for the transfer entropy. This allows us to construct a more effective network, denoted as $\tilde{G}(V, \tilde{E})$ by filtering out redundant information, see Figure 2 (Left). The rule for determining the weight of an edge (or the length of an edge) from bank *i* to the bank *j* is shown in formula (2.4). If $\text{TE}_{i\to j}(1,1) \ge \theta$, then $e_{ij} = 1$, otherwise $e_{ij} = 0$. Similarly, the edge from bank *j* to bank *i* is determined in the same manner.

$$\tilde{E} = \begin{cases} e_{ij} = 1 & i \neq j \text{ and } \operatorname{TE}_{i \to j} \ge \theta, \\ e_{ij} = 0 & \text{otherwise.} \end{cases}$$
(2.4)

The information flow network $\tilde{G}(V, \tilde{E})$ can intuitively capture the network structure of the banking system.



Figure 2. The directed graph \tilde{G} and the undirected graph G

The network structure at the level of individual nodes and edges are considered to be lower-order connectivity patterns of complex networks. For exploring and researching complex systems, high-order organization [8] is significant. A common form of highorder organization is the small network subgraph, often referred to as triangle motifs, as shown in Figure 3.

For example, triangle motifs $M_1 - M_7$ are crucial for social networks, while twohop paths $M_8 - M_{13}$ are essential to understanding air traffic patterns. In our study, we choose the motif M_8 to analyze the network $\tilde{G}(V, \tilde{E})$ since risk typically spreads from high-risk zones (sources) to low-risk zones (targets). A bank is considered a source (target) of risk if it has more outward (inward) edges. Based on motif M_8 , the directed weighted network $\tilde{G}(V, \tilde{E})$ is transformed into an undirected weighted network G(V, E), see Figure 2 (Right). The weight of the edge between bank *i* and *j* in network G(V, E) represents the number of instances of motif M_8 that contain nodes *i* and *j*.



Figure 3. Triangle motifs

We will explore the transmission of systemic risk in China's commercial banks based on network G(V, E). It is worth noting that the network G(V, E) is constructed based on the transfer entropy. Other papers used different methods for constructing banking network, see references [9, 20]. To our best knowledge, this is the first work to apply higher-order organization of complex networks to financial risk analyzing. Network analysis not only gives a global view to system risk transmission, but also reveals hidden internal structure of bank industry in China. The basic characteristics of the network G(V, E) are presented in Table 1.

Table 1. The characteristics of the network G

Nodes	Edges	Average degree	Average path length	Average strength	Network density
24	163	7.01	1.40	10.25	0.59

Node degree represents the number of edges by which the node connects with other nodes, while the weight of every edge calls the path length. Node strength is the sum of the weight of all edges connected to it. Network density refers to the percentage of edges present in the network compared to the total number of edges. In the banking network G(V, E), banks with higher degree and strength have a greater ability for risk contagion after a crisis occurs. Since the average strength of G(V, E) is 10.25, we list all banks with strength greater than 11 in Table 2.

Table 2. Banks with strength over 11

Bank node	Strength	Bank node	Strength
3(RC)	18	13(JS)	14
$10(\mathrm{UC})$	18	16(SO)	14
$12(\mathrm{RC})$	18	18(SO)	14
$14(\mathrm{UC})$	18	$22(\mathrm{UC})$	14
7(JS)	14	24(RC)	14

From Table 2, it is evident that, although the large state-owned banks being the core of the banking system, the risk contagion capabilities of certain rural commercial banks, urban commercial banks and joint-stock banks cannot be ignored. This finding aligns with the actual situation [29]. In order to better study the generation and contagion process of systemic financial risks, we need to find an appropriate classification for the 24 China's banks based on the network G(V, E) by using a clustering method.

2.4. Clustering and embedding

Previous research shown that contagion of systemic financial risks has cluster effects. Murugan and Sree Kala analysed financial risk using machine learning strategies and cluster to predict loan defaults [27]. Since K-means clustering relies on strong assumptions about cluster shape and is sensitive to outliers, the clustering results can be vary based on the initial configuration of the centroids. To overcome these limitations, spectral clustering has been proposed which is more advantageous in capturing the primary features of the data. The essence of spectral clustering lies in grouping similar data together as much as possible while separating different data as much as possible. Several studies have demonstrated the effectiveness of spectral clustering in various applications, such as image segmentation, social network analysis, and bioinformatics. For further research on spectral clustering, please refer to Sharma and Seal [33] and Karlekar et al. [21]. Here, we apply spectral clustering to cluster the bank network G(V, E).

Proposition 2.2. Let G(V, E) be above bank network. Then it can be divided three parts by spectral clustering. Moreover, these clusters can be embed onto a onedimensional axis with corresponding positions at x = 1, 2, 3, as shown in Figure 1.

Method: Using the algorithm based on high-order organization, we divide the 24 banks into 3 clusters. The main steps of spectral clustering are:

Step 1: Computing the adjacency matrix W_M of the network G(V, E) based on the motif M_8 .

Step 2: Apply spectral clustering to the the adjacency matrix W_M .

- Calculating a normalized Laplace matrix $L_M = D_M^{-\frac{1}{2}} (D_M W_M) D_M^{-\frac{1}{2}}$, where D_M is a diagonal matrix with $(D_M)_{ii} = \sum_{j=1}^n (W_M)_{ij}$.
- Calculating the eigenvectors z_2 and z_3 corresponding to the second smallest eigenvalue λ_2 and the third smallest eigenvalue λ_3 of the Laplacian matrix L_M .
- Combining the eigenvectors z_2 and z_3 a matrix whose dimension is $n \times 2$.
- Perform the K-means clustering algorithm is on the $n \times 2$ matrix, grouping it into k classes. The clustering results are shown in Table 3.

By comparing Table 3 with Table 2, we observe that Cluster 1 includes six out of the ten banks from Table 2, while Cluster 2 covers three banks, and Cluster 3 includes only one bank. The average strength of nodes in Cluster 1, Cluster 2, and Cluster 3 is 11.4, 11.1, and 7.7, respectively. This implies that the banks in Cluster 1 have the highest level of interconnectedness, followed by Cluster 2 and Cluster 3.

Ownership	Cluster 1	Cluster 2	Cluster 3
SO	16 21	18 20	17
$_{ m JS}$	47	$5\ 13\ 19$	1 6 23
UC	$10 \ 11 \ 22$	9	$2 \ 14 \ 15$
RC	$8\ 12\ 24$	3	

Table 3. The results of three clusters in 24 listed banks

The greater the interconnection, the stronger the ability for risk contagion within the cluster.

Next, we calculate the sum of the weights of all edges connecting two clusters, which can be interpreted as the distance between these two clusters. A higher level of interconnectedness indicates a smaller distance between clusters. The sum of the weights of edges between Cluster 1 and Cluster 2 is 53, that between Cluster 2 and Cluster 3 is 21, and that between Cluster 1 and Cluster 3 is 39. Based on the average strength and the distances among clusters, we embed Cluster 1, Cluster 2, and Cluster 3 onto a one-dimensional axis with corresponding positions at x = 1, 2, 3, as shown in Figure 1. Then we can use PDE model (2.1) to study the contagion of systemic risk of China's Banking Industry.

2.5. Computing weighted average of $|\Delta CoVaR|$

The most commonly measure for assessing the risk of an individual financial institution is value-at-risk (VaR). The maximum loss $\operatorname{VaR}_{\alpha,t}^{i}$ of the institution *i* at time *t* at the confidence level $1 - \alpha$ (e.g., 95%) is defined as:

$$\Pr(r_{it} \le \operatorname{VaR}^{i}_{\alpha,t}) = \alpha, \tag{2.5}$$

where r_{it} is the return rate of the institution *i* at time *t*. To measure the contribution of each institutional asset to systemic risk, Adrian and Brunnermeier [3] proposed the concept of CoVaR, defined as follows:

$$\Pr(r^s \le \operatorname{CoVaR}_{\alpha,t}^{s|i} \mid r^i = \operatorname{VaR}_{\alpha,t}^i) = \alpha.$$
(2.6)

Here r^s represents the return rate of the market, $\text{CoVaR}_{\alpha,t}^{s|i}$ represents the conditional risk to which the system is exposed when the financial institution *i* has suffered an extreme loss $\text{VaR}_{\alpha,t}^i$ at time *t*.

We employ quantile regression techniques to estimate CoVaR. The selection of state variables is primarily based on the approaches adopted by Adrian and Brunnermeier [3]. Taking into account the actual conditions of China's financial market, we have chosen the following lagged variables: (1) the weekly market return of the HS300 index[†]; (2) the volatility (the 22-day rolling standard deviation of the daily HS300 return); (3) the change in the 3-month Treasury bill rate, referred to as the short term "liquid spread" (which is defined as the difference between the

[†]The HS300 Index is composed of 300 most representative stocks with large scale and good liquidity in the Shanghai and Shenzhen Stock Exchange. It was officially released on April 8, 2005, to reflect the overall performance of the stocks of listed companies in the Shanghai and Shenzhen Stock Exchange.

3-month repo rate and the 3-month Treasury bill rate); (4) the term spread; (5) the weekly real estate sector return; (6) the change in the 3-month Treasury bill rate.[‡]

 $\operatorname{CoVaR}_{\alpha,t}^{s|i}$ and $\operatorname{CoVaR}_{\alpha,t}^{s|i,50\%}$ respectively represent the systemic risk of the banking system during the period where the bank *i* is in crisis and when bank *i* is in the mediate state (with the return of bank *i* as the median). The spillover of risk from bank *i* to the banking system can be measured in terms of:

$$\Delta \text{CoVaR}_{\alpha,t}^{s|i} = \text{CoVaR}_{\alpha,t}^{s|i} - \text{CoVaR}_{\alpha,t}^{s|i,50\%}.$$
(2.7)

 Δ CoVaR has proven to be a valuable tool for assessing systemic risk in financial markets.

Proposition 2.3. Based on formula (2.2) and stock return rates of 208 weeks from January 2019 to December 2020, we obtain the weekly weighted average $|\Delta CoVaR|$ for the three Clusters. We find that Cluster 1 has the highest systemic risk and Cluster 3 has the lowest systemic risk for most periods, as shown in Figure 4.

Method: We calculate the weekly Δ CoVaR for 24 banks using the stock return rates of 208 weeks from January 2019 to December 2020. Since the value of Δ CoVaR are all negative, we use the absolut value $|\Delta$ CoVaR| to represent systemic risk for convenient. The top ten banks with the highest average weekly $|\Delta$ CoVaR| are listed in Table 4. The average weekly $|\Delta$ CoVaR| for the three Clusters are 1.9945, 1.3316, 1.0707, respectively. This indicates that the Cluster 1 has the highest average systemic risk, then Cluster 2 and Cluster 3.

Table 4. Top ten banks with the average weekly $|\Delta CoVaR|$ from 2019 to 2022

Bank node	$ \Delta \mathrm{CoVaR} $	Bank node	$ \Delta \mathrm{CoVaR} $
5(JS)	6.0606	$15(\mathrm{UC})$	5.4092
6(JS)	5.7446	4(JS)	5.3424
7(JS)	5.7336	$14(\mathrm{UC})$	4.7438
19(SO)	5.6093	1(JS)	4.7319
$2(\mathrm{UC})$	5.4326	$22(\mathrm{UC})$	4.7063

Based on the discussion above, we find that Cluster 1 has the highest interconnectedness and contributes the most to overall risk, followed by Cluster 2 and Cluster 3. Furthermore, the risk tends to spill over from high-risk clusters to lowrisk clusters. Taking this into consideration, we calculate the weighted average value of $|\Delta \text{CoVaR}|$ for each cluster at every trading week as a measure of the cluster's systemic financial risk.

The weight of each bank *i* is given by $w_i = \frac{(D_M)_{ii}}{\sum_{i=1}^n (D_M)_{ii}}$, where D_M is the diagonal matrix generated by the adjacency matrix W_M of the network G(V, E). Figure 4 illustrates the weekly weighted average of $|\Delta \text{CoVaR}|$ of three clusters from 2019 to 2022. It represents a dynamic change of systemic risk of each cluster. It can be observed that the systemic risk of Cluster 1 is higher than that of Cluster 2, and Cluster 2 is higher than that of Cluster 3 for most periods. This result is consistent with the order in which the clusters are embedded onto the Euclidean space.

 $^{^{\}ddagger} In$ the absence of readily available data on credit spread change in China, we have opted to use these six state variables as proxies.



Figure 4. The weighted average of $|\Delta CoVaR|$ of three clusters from 2019 to 2022. The Date-axis represents the 208 trading weeks from 2019 to 2022, and the $|\Delta CoVaR|$ -axis represents the value of the weighted average of $|\Delta CoVaR|$ for each cluster.

3. Forecast and analysis

3.1. Forecast process and result

The prediction is based on the PDE model (2.1). To predict the weighted average $|\Delta \text{CoVaR}|$ of three clusters, the model parameters are trained using historical data fom 2019 to 2022. The PDE is then solved for prediction.

Proposition 3.1. Using PDE model (2.1) and historical data, we forecast the weekly $|\Delta CoVaR|$ of three Clusters. The prediction relative accuracy is up to 95%, which is higher than two common machine learning methods under the same conditions, as shown in Figure 5, Figure 6, Figure 7 and Table 5.

Method: The forecast is conducted by using a rolling window approach, where historical data from every 4 weeks is used to forecast the weighted $|\Delta \text{CoVaR}|$ for the following week. That is, the real stock data of weeks 1-4 are used to predict the weighted $|\Delta \text{CoVaR}|$ for the fifth week, weeks 2-5 for week 6, and so on. The forecast covers the period from 2019 to December 2022, totaling 204 weeks.

The parameter space is explored using a tensor train global optimization method [28], followed by the Nelder-Mead simplex local optimization method [24] to finetune the weighted $|\Delta \text{CoVaR}|$ modeling problem. The local optimization is implemented using the fminsearch function in MATLAB. Once the model parameters are determined, the fourth-order Runge-Kutta method is employed to numerically solve the partial differential equations for one-step forward pediction.

The forecast parameters vary for each step. For example, the parameters for the last week (from December 26 to December 30, 2022) are as follows: d = 4.37, K = 4.17, A = 0.35, B = 0.25, C = 0.59 and $h_1 = 0.54$, $h_2 = 0.12$ and $h_3 = 0.47$. Figure 5 illustrates the forecast of systemic risk for the three clusters from 2019 to 2022.

Next, we assess the prediction accuracy using relative accuracy (RA) for each



Figure 5. The weighted average of $|\Delta CoVaR|$ of three clusters from 2019 to 2022. The Date-axis represents the 208 trading weeks from 2019 to 2022, and the $|\Delta CoVaR|$ -axis represents the value of the weighted average of $|\Delta CoVaR|$ for each cluster.

week, defined as:

$$RA_t = 1 - \left| \frac{u_{t,real} - x_{t,predict}}{u_{t,real}} \right|,\tag{3.1}$$

where $u_{t,real}$ represents the observed weighted $|\Delta \text{CoVaR}|$. The first row of Table 5 shows that the mean of relative accuracy (MRA) for Cluster 1, Cluster 2, and Cluster 3 is 97.85%, 97.56%, and 96.96%, respectively, for the period from 2019 to 2022 using 4-week data for prediction.

Table 5. Comparison of MRA between PDE, BP and RF

	Cluster 1	Cluster 2	Cluster 3
MRA(PDE)	97.65%	97.56%	96.96%
MRA(BP)	96.06%	90.94%	89.28%
MRA(RF)	88.37%	92.54%	88.78%

Figure 6 visually illustrates the value of RA of the prediction for the three clusters. It implies that PDE model (2.1) demonstrates good short-term prediction capability for risk contagion.



Figure 6. The relative accuracy (RA) in Cluster 1, Cluster 2 and Cluster 3 from 2019 to 2022.

Additionally, we compare the PDE model's prediction capability with other machine learning techniques such as BP neural network [37] and Random Forest [38]. To ensure consistency in the prediction methodology, the BP neural network and Random Forest models are trained using data from the first four weeks. The MRA of the BP neural network and Random Forest is presented in the second and third rows of Table 5, respectively. The PDE model outperforms both machine learning approaches, achieving higher MRA in all three clusters. Figure 7 provides a comparison of the RA for the three methods, demonstrating that the PDE model exhibits smaller changes in amplitude compared to the other machine learning approaches. Consequently, the PDE model (2.1) demonstrates superior accuracy compared to the other two machine learning techniques.



Figure 7. Comparison of the relative accuracy (RA) among PDE model, BP neural network model and Random Forest model

3.2. Robustness check

3.2.1. The frequency of the data



Figure 8. Month-Forecast of weighted $|\Delta CoVaR|$ for Cluster 1, Cluster 2 and Cluster 3 from 2019 to 2022 with monthly data.

The results demonstrated that the model remained robust to these variations, indicating its ability to capture the underlying dynamics of the data. Note that some points fit not well with the model. One possible explanation could be the differences in data distribution and dynamics between weekly and monthly frequencies, which could affect the stability of the model. The lower frequency of data could result in a loss of information and potentially miss short-term fluctuations in the data. However, the trend of changing of Month-Forecast weighted $|\Delta CoVaR|$ remains consistent with the actual data. As for daily data, the prediction accuracy is similar to weekly data.

3.2.2. Other periods

Next, we assessed the robustness of the model by using data from a different time period, specifically the years 2009 to 2014, to predict the stock market turbulence in 2015. The Chinese stock market turbulence started on 12 June 2015 and ended in 30 August 2015. During this period, the Shanghai and Shenzhen composite indices have experienced a nearly 50% decline, with a significant number of individual stock prices plummeting. As depicted in Figure 9, the grey area represents the period when stock market turbulence occurred. The predicted values (the blue line) fit well with the actual values (the red line). In the week of 12 July 2015, the values of $|\Delta CoVaR|$ in all three clusters reached their peak, with prediction accuracies of 98.38%, 97.63%, and 99.79%, respectively. The average accuracies for the stock market turbulence from 12 June 2015 to 30 August 2015 are 95.51%, 97.87%, and 98.82%, respectively. It illustrates that the model performed well in fitting the data. The successful prediction of the 2015 stock market crash highlights its ability to generalize across diverse market conditions and time periods. This suggests that the model is applicable to the market under various economic scenarios. However, it is important to note that the accuracy of the predictions may depend on the economic conditions and market dynamics. Therefore, it is recommended to exercise caution when applying the model to new market conditions and adjust the model parameters accordingly.



Figure 9. Week-Forecast of weighted $|\Delta CoVaR|$ for Cluster 1, 2, 3 in 2015.

4. Sensitivity analysis and policy simulation

4.1. Sensitivity analysis on h(x)

During financial crises, monetary authorities and regulators tighten their monitoring of banks. They will take some regulatory measures to control the risk. Sensitivity analysis on the h(x) in the PDE model (2.1) is conducted to explore the impact of policies.

Proposition 4.1. Sensitivity analysis on h(x) based on the PDE model (2.1) shows that a significant reduction in systemic risk as the decreasing of h(x). In particular, the effect of joint intervention is better than that of individual intervention.

Method: We present two types of interventions for reducing systemic risk: joint intervention for all bank clusters, and single intervention targeting a specific bank cluster. Each time, we fit the model (2.1) initially using actual weighted $|\Delta \text{CoVaR}|$ data for four weeks. Then, without changing the estimated parameters, we conduct forecasts by decreasing the value of $h(x_i)$, i = 1, 2, 3. The benchmark is the weighted $|\Delta \text{CoVaR}|$ without any additional interventions.

In Case 1, as shown in Figure 10, with increasingly intensive joint interventions for all bank clusters $(90\%h(x_i), 70\%h(x_i), 50\%h(x_i), i = 1, 2, 3)$, the weighted $|\Delta \text{CoVaR}|$ in each cluster gradually decrease. These results demonstrate that stricter control measures can lead to lower $|\Delta \text{CoVaR}|$ values.



Figure 10. The weighted $|\Delta CoVaR|$ versus the benchmark under different levels of external joint-interventions for all clusters

Cluster 1 represents a subsystem of banking industry, encompassing four distinct categories of banks. It not only has the highest number of banks among the three clusters but also exhibits the most complex structure among them. It plays a significant role in stabilizing financial risk in the banking system. Therefore, it is beneficial for financial regulators to implement targeted supervision in Cluster 1.

In Case 2, we focus on decreasing the values of $h(x_1)$, which means implementing interventions only on Cluster 1 at levels of 50%. As depicted in Figure 11, with stricter interventions on Cluster 1, not only does the $|\Delta \text{CoVaR}|$ of Cluster 1 decrease, but the values of Cluster 2 and Cluster 3 also decrease. Comparing Figure 10 with Figure 11, we can observe that joint intervention for all bank clusters can significantly reduce the level of systemic risk. These findings demonstrate that our model can reflect the actual phenomenon of risk contagion among bank clusters.

4.2. Policy simulation

In view of the importance of joint governance, we add common intervention strategy control factor α to PDE model (2.1). Consider the model



Figure 11. The weighted $|\Delta CoVaR|$ versus the benchmark under different levels of external joint-interventions for only Cluster 1

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = \frac{\partial}{\partial x} \left[d(x) \frac{\partial u(x,t)}{\partial x} \right] + r(t) \alpha u(x,t) \left[h(x) - \frac{u(x,t)}{K} \right], \\ u(x,1) = \varphi(x), 1 < x < 3, \\ \frac{\partial u}{\partial x}(1,t) = \frac{\partial u}{\partial x}(3,t) = 0, t > 1. \end{cases}$$

$$(4.1)$$

Motivated by [36, 43], we define control factor α as follows:

$$\alpha = f(\kappa, \xi, \phi) = e^{-2\kappa\xi} (1 - e^{-\frac{1}{\phi}}).$$
(4.2)

Here,

- The parameter $\kappa(0 < \kappa < 1)$ reflects the timeliness of central bank intervention in promoting financial stability. When κ is closer to 1, the central bank's intervention in the asset markets becomes increasingly timely, aiding in stabilizing asset prices and preventing a spiral decline.
- The parameter $\xi(0 < \xi < 1)$ reflects the intensity of financial regulators. As ξ approaches 1, the level of supervision becomes stronger, allowing regulator to utilize robust measures to adjust bank expectations and mitigate risk during periods of heightened risk.
- The parameter $\phi(0 < \phi < 1)$ represents the variations in bank responses. When ϕ approaches 1, the differences in bank responses become more significant. This can help avoid mutual locking caused by consistency and adjusting asset prices expectations in the market. Banks actively adapting to these changes can mitigate systemic risk contagion.

Proposition 4.2. Numerical simulation implies that all three interventions can effectively reduce banking systemic risks.

Method: To examine the impact of these interventions on systemic risk contagion, we set the basic values of $\kappa = 0.3$, $\xi = 0.3$, $\phi = 0.4$ in model (4.1). Then, we adjust each parameter to values 0.5, 0.7, 0.9, respectively, while keeping the rest of the forecast process the same as in references [36, 43].

Based on the analysis, we find that, in most periods, increasing the values of κ , ξ , and ϕ results in a decrease in the weighted $|\Delta \text{CoVaR}|$ as shown in Figure 12, Figure 13, and Figure 14. All three interventions show a negative correlation with interbank systemic risk contagion, supporting the conclusions in reference [36]. Meanwhile, we can also find the effect of heterogeneity about different policy intensity and bank clusters.



Figure 12. The $|\Delta CoVaR|$ versus the benchmark under joint-interventions with $\xi = 0.3, \phi = 0.4$



Figure 13. The $|\Delta CoVaR|$ versus the benchmark under joint-interventions with $\kappa = 0.3, \phi = 0.4$



Figure 14. The $|\Delta CoVaR|$ versus the benchmark under joint-interventions with $\kappa = 0.3, \xi = 0.3$

These results suggest that timely intervention measures by the central bank and strengthened control efforts by financial regulatory authorities are crucial in controlling the contagion of systemic risk. Additionally, differential response strategies by banks when risks arise are essential for rapidly curbing risk contagion. By implementing these macro measures and promoting coordinated governance, systemic risks can be effectively controlled and mitigated.

5. Conclusion and suggestion

This work provides a comprehensive analysis of systemic risk in China's banking industry using a combination of network analysis, spectral clustering, and a spatiotemporal PDE model. The numerical results demonstrate the effectiveness of the PDE model in predicting systemic risk. Compared to the BP neural network model and the Random Forest model, the PDE model shows superior prediction ability, especially in short-term predictions using only a few weeks of historical data. Sensitivity analysis illustrate that joint-control of all clusters is more efficient than single-control of one cluster. Through policy simulations, we examine the impact of various interventions on systemic risk contagion. This helps us better understand the dynamic transmission of systemic risk and provide valuable insights for policymakers and regulators in implementing effective risk management strategies.

In fact, an important trend in studying complex problems is to combine deterministic models with machine learning, especially deep learning, social networks, and large-scale real data [4, 47]. The method developed in this paper can also be used to study more financial risk, such as systemic risk on stock market, bond market, the financial market of some region or even even global financial market etc. More complex differential equation or systems can be considered based on different problem.

Conflict of interest

The authors declare that they have no competing interests.

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Author contributions

Haiyan Wang: Conceptualization; Software; Validation; Writing-review and editing;

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Xiaofeng Yan: Data curation; Methodology; Software; Validation; Visualization; Writing–original draft.

Bank Node	Full name of the banks	Abbreviation
1	Ping An Bank	PAB
2	Ningbo Bank	BNB
3	Jiangsu Jiangyin Rural Commercial Bank	JJRCB
4	Shanghai Pudong Development Bank	SPDB
5	Huaxia Bank	HB
6	China Minsheng Bank	CMDB
7	China Merchants Bank	CMB
8	Wuxi Rural Commercial Bank	WRCB
9	Jiangsu Bank	JB
10	Hangzhou Bank	BH
11	Nanjing Bank	BN
12	Jiangsu Changshu Rural Commercial Bank	JCRCB
13	Industrial Bank	IB
14	Bank of Beijing	BB
15	Shanghai Bank	BS
16	Agricultural Bank of China	ABC
17	Bank of Communications	BC
18	Industrial and Commercial Bank of China	ICBC
19	China Everbright Bank	CEB
20	China Construction Bank	CCB
21	Bank of China	BOC
22	Guiyang bank	BG
23	China CITIC Bank	CITICB
24	Jiangsu Suzhou Rural Commercial Bank	JSRCB

Appendix: 24 listed banks in China

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