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HIGHLY DISPERSIVE OPTICAL SOLITONS WITH QUADRATIC-CUBIC NONLINEAR REFRACTIVE INDEX BY LIE SYMMETRY*

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Abstract This paper addresses highly dispersive optical solitons with quadratic–cubic nonlinear form of self–phase modulation. Lie symmetry analysis reduced the governing model to an ordinary differential equation which was further analyzed using two approaches. The series expansion approach and the F–expansion scheme yielded soliton solutions as well as an abundance of additional solutions to the model. The parameter restrictions were also enumerated to provide a formidable structure to the solutions.

Keywords Solitons, Lie symmetry, highly dispersive.

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1. Introduction

The concept of highly dispersive (HD) optical solitons was conceived a few years ago as an extension to cubic–quartic (CQ) optical solitons. These concepts emerged when chromatic dispersion carries a low count in optical fibers whence it had to be supplemented with additional dispersion terms. Thus, CQ solitons were introduced and examined.

The concept of HD solitons conceived from possible catastrophic consequences that would ensue when the chromatic dispersion (CD) count would be low with a possibility of its depletion. Therefore, the concept of CQ solitons emerged to compensate for the low count of CD. Thus, with two dispersive effect being in place the contribution towards the existence of the delicate balance between CD and self–phase modulation (SPM) would still be questionable. Hence, the inclusion of the six dispersive terms would ensure the existence of this delicate balance between dispersive and SPM effects. This would guarantee the stable propagation of solitons for intercontinental distances. The cons of the inclusion of six dispersion terms are nevertheless overlooked and discarded. These include the pronounced soliton radiation and slow–down of solitons. With this reasoning in place, the concept of HD solitons emerged.

The governing model is the nonlinear Schrödinger's equation (NLSE) that was considered with various forms of nonlinear refractive index structure. Moreover, it is well known that the nonlinear Schrödinger's equation belongs to the class of nonlinear partial differential equations. For nonlinear partial differential equations, there are many literatures to study them, such as [9–12, 19, 21–23] and their cited papers.

The current paper will therefore address the governing NLSE with CQ form of self–phase modulation (SPM). The integration methodology is two–fold. First of all, the model which is a partial differential equation (PDE) is reduced to a pair of ordinary differential equations (ODEs) with Lie symmetry reduction. Subsequently, these ODEs, upon integration, provide the velocity with which the HD solitons travel and the soliton solutions to the model along with the parameter constraints that must be maintained for the solitons to sustain. The two distinct approaches to analyze the ODEs are the series solution method and the F–expansion approach. These give way to a number of solutions to the model including optical solitons. The details are sequenced in the rest of the paper after a succinct intro to the governing model.

1.1. Governing model

The HD–NLSE with non–Kerr law of nonlinearity, in general, is written as [1–7]

$$iq_t+ia_1q_x+a_2q_{xx}+ia_3q_{xxx}+a_4q_{xxxx}+ia_5q_{xxxxx}+a_6q_{xxxxxx}+F\left(|q|^2\right)q=0. \quad (1.1)$$

In the dimensionless form of equation (1.1), the dependent variable $q(x,t)$ arises from the soliton profile and denotes a complex–valued function where x and t are the independent variables which depict the spatial and temporal co–ordinates in sequence. The first term represents temporal evolution with $i=\sqrt{-1}$. The coefficients of a_j for $j=1\cdots 6$ are the represent inter–modal dispersion, CD, third–order dispersion, fourth–order dispersion, fifth–order dispersion and sixth–order dispersion in sequence. The functional F stems from the nonlinearity structure of the fiber refractive index that forms the SPM.

2. Quadratic–cubic law

The HD–NLSE with QC nonlinearity is [1–3]

$$iq_t+ia_1q_x+a_2q_{xx}+ia_3q_{xxx}+a_4q_{xxxx}+ia_5q_{xxxxx}+a_6q_{xxxxxx}+\left(b_1\left|q\right|+b_2\left|q\right|^2\right)q=0. \tag{2.1}$$

Equation (2.1) is a special case of (1.1) in which the refractive index structure is of QC form. The SPM effects comes from the coefficients of b_j for $j=1,2$ which give the quadratic and cubic effect in sequence.

Now, $q(x,t)$ being a complex–valued function, splitting into imaginary and real parts in the form $q=u+iv$, which decomposes Eq. (2.1) into the following couple of relations:

$$\begin{aligned}&u_t+a_1u_x+a_2v_{xx}+a_3u_{xxx}+a_4v_{xxxx}+a_5u_{xxxxx}+a_6v_{xxxxxx}\\&+b_1\sqrt{u^2+v^2}v+b_2u^2v+b_2v^3=0, \qquad (2.2)\\&-v_t-a_1v_x+a_2u_{xx}-a_3v_{xxx}+a_4u_{xxxx}-a_5v_{xxxxx}+a_6u_{xxxxxx}\\&+b_1\sqrt{u^2+v^2}u+b_2v^2u+b_2u^3=0. \qquad (2.3)\end{aligned}$$

2.1. Symmetry analysis and symmetry reduction

2.1.1. Symmetry analysis

Consider the Lie group of point transformation [8,13]

$$V=\xi^t(x,t,u,v)\frac{\partial}{\partial t}+\xi^x(x,t,u,v)\frac{\partial}{\partial x}+\eta_u(x,t,u,v)\frac{\partial}{\partial u}+\eta_v(x,t,u,v)\frac{\partial}{\partial v}, \tag{2.4}$$

and

$$\begin{cases}t^*=t+\epsilon\xi^t(x,t,u,v)+O(\epsilon^2),\ \ x^*=x+\epsilon\xi^x(x,t,u,v)+O(\epsilon^2),\\ u^*=u+\epsilon\eta_u(x,t,u,v)+O(\epsilon^2),\ \ v^*=v+\epsilon\eta_v(x,t,u,v)+O(\epsilon^2).\end{cases} \tag{2.5}$$

For equations (2.2) and (2.3), sixth order prolongations formula is given by

$$\begin{aligned}Pr^{(6)}V=&V+\eta_u^t\frac{\partial}{\partial u_t}+\eta_v^t\frac{\partial}{\partial v_t}+\eta_u^x\frac{\partial}{\partial u_x}+\eta_v^x\frac{\partial}{\partial v_x}+\eta_u^{xx}\frac{\partial}{\partial u_{xx}}+\eta_v^{xx}\frac{\partial}{\partial v_{xx}}\\&+\eta_u^{xxx}\frac{\partial}{\partial u_{xxx}}+\eta_v^{xxx}\frac{\partial}{\partial v_{xxxx}}+\eta_u^{xxxx}\frac{\partial}{\partial u_{xxxx}}+\eta_v^{xxxx}\frac{\partial}{\partial v_{xxxx}}\\&+\eta_u^{xxxxx}\frac{\partial}{\partial u_{xxxxx}}+\eta_v^{xxxxx}\frac{\partial}{\partial v_{xxxxx}}+\eta_u^{xxxxxx}\frac{\partial}{\partial u_{xxxxxx}}\\&+\eta_v^{xxxxxx}\frac{\partial}{\partial v_{xxxxxx}},\end{aligned} \tag{2.6}$$

where $\eta_u^t,\eta_v^t,\eta_u^x,\eta_v^x,\eta_u^{xx},\eta_v^{xx},\eta_u^{xxx},\eta_v^{xxx},\eta_u^{xxxx},\eta_v^{xxxx},\eta_u^{xxxxx},\eta_v^{xxxxx},\eta_u^{xxxxxx}$ and η_v^{xxxxxx} are functions to be determined. Consider invariance conditions $Pr^{(6)}V(\Delta)=0$, where $\Delta=0$ in equations (2.2) and (2.3).

As a result, the following results are derived based on Ref. [15]

$$\eta_u = c_3 v,\ \eta_v = -c_3 u,\ \xi^x = c_2,\ \xi^t = c_1. \tag{2.7}$$

Thus, the Lie algebra is spanned by the infinitesimal generators

$$V_1 = \frac{\partial}{\partial x},\ V_2 = \frac{\partial}{\partial t},\ V_3 = -u\frac{\partial}{\partial v} + v\frac{\partial}{\partial u}. \tag{2.8}$$

In this case, the one-parameter Lie symmetry group comes out as

$$\begin{aligned}
&G_1 : (x,t,u,v) \to (x-\varepsilon_1, t, u, v)\,,\\
&G_2 : (x,t,u,v) \to (x, t-\varepsilon_2, u, v)\,,\\
&G_3 : (x,t,u,v) \to (x, t, u\cos\varepsilon_3 - v\sin\varepsilon_3, v\cos\varepsilon_3 + u\sin\varepsilon_3)\,.
\end{aligned} \tag{2.9}$$

Also, Lie-Backlund symmetry for high order are as follows:

$$\begin{aligned}
&(u_x a_6 + v a_5)\frac{\partial}{\partial u} + (v_x a_6 - u a_5)\frac{\partial}{\partial v},\\
&u_x\frac{\partial}{\partial u} + v_x\frac{\partial}{\partial v},
\end{aligned} \tag{2.10}$$

where a_5 and a_6 are constants.

2.1.2. Symmetry reductions

Based on paper [16], we also consider the following cases:

Case–1. V_1

Invariant and invariant function are given by

$$\xi = t,\ u = f(\xi),\ v = g(\xi). \tag{2.11}$$

Inserting (2.11) into (2.2) and (2.3), the responding ordinary differential equations are

$$\begin{aligned}
&f_\xi + b_1\sqrt{f^2+g^2}g + b_2 f^2 g + b_2 g^3 = 0,\\
&-g_\xi + b_1\sqrt{f^2+g^2}f + b_2 g^2 f + b_2 f^3 = 0.
\end{aligned} \tag{2.12}$$

For this systems, some results are given by

$$f = C1,\ g == \pm i f, \tag{2.13}$$

$$f(\xi) = C1,\ g(\xi) = \pm\frac{\sqrt{-{b_2}^2\left(f(\xi)\right)^2 + {b_1}^2}}{b_2}. \tag{2.14}$$

Substituting them into $q = u + iv$, one can get the solutions of original equation (2.1).

Case–2. V_2

For this case, invariant and invariant function are displayed by

$$\xi = x,\ u = f(\xi),\ v = g(\xi), \tag{2.15}$$

also, inserting (2.15) into (2.2) and (2.3), we arrive

$$\begin{aligned}
&a_1 f_\xi + a_2 g_{\xi\xi} + a_3 f_{\xi\xi\xi} + a_4 g_{\xi\xi\xi\xi} + a_5 f_{\xi\xi\xi\xi\xi} + a_6 g_{\xi\xi\xi\xi\xi\xi} \\
&+ b_1\sqrt{f^2+g^2}g + bf^2 g + bg^3 = 0, \\
&- a_1 g_\xi + a_2 f_{\xi\xi} - a_3 g_{\xi\xi\xi} + a_4 f_{\xi\xi\xi\xi} - a_5 g_{\xi\xi\xi\xi\xi} + a_6 f_{\xi\xi\xi\xi\xi\xi} \\
&+ b_1\sqrt{f^2+g^2}f + bg^2 f + bf^3 = 0.
\end{aligned} \tag{2.16}$$

Case–1. $\lambda V_1 + V_2$

For traveling wave transformation, one can derive invariant and invariant function

$$\xi = x - \lambda t,\ \ u = f(\xi),\ \ v = g(\xi). \tag{2.17}$$

Plugging (2.17) into (2.2) and (2.3) paves way to

$$\begin{aligned}
&- \lambda f_\xi + a_1 f_\xi + a_2 g_{\xi\xi} + a_3 f_{\xi\xi\xi} + a_4 g_{\xi\xi\xi\xi} + a_5 f_{\xi\xi\xi\xi\xi} + a_6 g_{\xi\xi\xi\xi\xi\xi} \\
&+ b_1\sqrt{f^2+g^2}g + bf^2 g + bg^3 = 0, \\
&\lambda g_\xi - a_1 g_\xi + a_2 f_{\xi\xi} - a_3 g_{\xi\xi\xi} + a_4 f_{\xi\xi\xi\xi} - a_5 g_{\xi\xi\xi\xi\xi} + a_6 f_{\xi\xi\xi\xi\xi\xi} \\
&+ b_1\sqrt{f^2+g^2}f + bg^2 f + bf^3 = 0.
\end{aligned} \tag{2.18}$$

3. Explicit solutions using series expansion method

Consider the transformation [1–3]

$$q(x,t) = g(s)e^{i\phi(x,t)}. \tag{3.1}$$

Here ϕ and $g(s)$ are given by Refs. [1–3].

Substituting (3.1) into (2.2) and (2.3), one can get imaginary part is the same result to Refs. [3, 16]

$$\begin{aligned}
&\left(-v + a_1 - 2a_2\kappa - 3a_3\kappa^2 + 4a_4\kappa^3 + 5a_5\kappa^4 - 6a_6\kappa^5\right) g' \\
&+ \left(a_3 - 4\kappa a_4 - 10\kappa^2 a_5 + 20\kappa^3 a_6\right) g''' + \left(a_5 - 6\kappa a_6\right) g^{(5)} = 0,
\end{aligned} \tag{3.2}$$

the velocity is given by $v = a_1 - 2a_2\kappa - 3a_3\kappa^2 + 4a_4\kappa^3 + 5a_5\kappa^4 - 6a_6\kappa^5$, the constraint conditions are

$$\begin{aligned}
&a_3 - 4\kappa a_4 - 10\kappa^2 a_5 + 20\kappa^3 a_6 = 0, \\
&a_5 - 6\kappa a_6 = 0.
\end{aligned} \tag{3.3}$$

However, one more item $b_1 g^2$ in the real part

$$\begin{aligned}
&\left(-\omega + \kappa a_1 - \kappa^2 a_2 - \kappa^3 a_3 + \kappa^4 a_4 + a_5\kappa^5 + a_6\kappa^6\right) g + b_1 g^2 + b_2 g^3 \\
&+ \left(a_2 + 3a_3\kappa - 6a_4\kappa^2 - 10a_5\kappa^3 + 15a_6\kappa^4\right) g'' \\
&+ \left(a_4 + 5a_5\kappa - 15a_6\kappa^2\right) g^{(4)} + a_6 g^{(6)} = 0.
\end{aligned} \tag{3.4}$$

Combining them together, one can derive

$$\begin{aligned}
&\left(-\omega + \kappa a_1 - \kappa^2 a_2 - 3\kappa^4 a_4 - 33a_6\kappa^6\right) g + b_1 g^2 + b_2 g^3 \\
&+ \left(a_2 + 6a_4\kappa^2 + 75a_6\kappa^4\right) g'' + \left(a_4 + 15a_6\kappa^2\right) g^{(4)} + a_6 g^{(6)} = 0,
\end{aligned} \tag{3.5}$$

$$Mg + b_1 g^2 + b_2 g^3 + N g'' + A g^{(4)} + a_6 g^{(6)} = 0, \tag{3.6}$$

where

$$M = -\left(\omega + \kappa\left(-a_1 + \kappa\left(a_2 + \kappa\left(a_3 - \kappa\left(a_4 + a_5\kappa\right) + a_6\kappa^3\right)\right)\right)\right),$$
$$A = \left(a_4 + 5a_5\kappa - 15a_6\kappa^2\right),$$

and

$$N = \left(a_2 + 3a_3\kappa - 6a_4\kappa^2 - 10a_5\kappa^3 + 15a_6\kappa^4\right).$$

It is clear that this equation is different from equation (24) in Ref. [16], because this equation has an additional squared term $b_1 g^2$. In order to get explicit solutions we still assume that this equation (3.6) has solutions of the form

$$g(s) = c_0 + c_1 s + c_2 s^2 + \cdots = \sum_{n=0}^{\infty} c_n s^n. \tag{3.7}$$

Putting (3.7) into (3.6), one has

$$\begin{aligned} &Mc_0 + M\sum_{n=1}^{\infty} c_n s^n + b_1 c_0^2 + b_1 \sum_{n=1}^{\infty}\sum_{k=0}^{n} c_k c_{n-k} s^n \\ &b_2 c_0^3 + b_2 \sum_{n=1}^{\infty}\sum_{k=0}^{n}(n+1-k)c_{n+1-k}\sum_{l_1+l_2+l_3=k} c_{l_1}c_{l_2}c_{l_3}s^n \\ &+ 2Nc_2 + N\sum_{n=1}^{\infty}(n+1)(n+2)c_{n+2}s^n \\ &+ 24Ac_4 + A\sum_{n=1}^{\infty}(n+1)(n+2)(n+3)(n+4)c_{n+4}s^n \\ &720a_6c_6 + a_6\sum_{n=1}^{\infty}(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)c_{n+6}s^n = 0. \end{aligned} \tag{3.8}$$

Comparing coefficients for $n = 0$ in Eq. (3.8), we can get

$$c_6 = -\frac{Mc_0 + b_1 c_0^2 + b_2 c_0^3 + 2Nc_2 + 24Ac_4}{720a_6}. \tag{3.9}$$

Therefore, for $n \geq 1$, one arrives

$$\begin{aligned} c_{n+6} = &-\frac{1}{a_6(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)} \\ &\times \Big(b_2 \sum_{k=0}^{n}(n+1-k)c_{n+1-k}\sum_{l_1+l_2+l_3=k} c_{l_1}c_{l_2}c_{l_3} \\ &+ A(n+1)(n+2)(n+3)(n+4)c_{n+4} \\ &+ N(n+1)(n+2)c_{n+2} + Mc_n + b_1\sum_{k=0}^{n} c_k c_{n-k}\Big). \end{aligned} \tag{3.10}$$

In this case, one can obtain

$$\begin{aligned}g(s) =&c_0 + c_1 s + c_2 s^2 + c_3 s^3 + c_4 s^4 + c_5 s^5 + c_6 s^6 + \sum_{n=1}^{\infty} c_{n+6} s^{n+6}\\=&c_0 + c_1 s + c_2 s^2 + c_3 s^3 + c_4 s^4 + c_5 s^5 - \frac{Mc_0 + b_1 c_0^2 + b_2 c_0^3 + 2Nc_2 + 24Ac_4}{720a_6} s^6\\&+ \sum_{n=1}^{\infty} - \frac{1}{a_6(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)}\\&\times \Big(b \sum_{k=0}^{n} (n+1-k) c_{n+1-k} \sum_{l_1+l_2+l_3=k} c_{l_1} c_{l_2} c_{l_3}\\&\times A(n+1)(n+2)(n+3)(n+4) c_{n+4} + N(n+1)(n+2) c_{n+2}\\&+ Mc_n + b_1 \sum_{k=0}^{n} c_k c_{n-k}\Big) s^{n+6},\end{aligned} \tag{3.11}$$

in the next step, substituting the traveling wave transform $s = x - \lambda t$ into the above equation (3.11), it generates the following results

$$\begin{aligned}g(s) =&c_0 + c_1 (x - \lambda t) + c_2 (x - \lambda t)^2 + c_3 (x - \lambda t)^3 + c_4 (x - \lambda t)^4 + c_5 (x - \lambda t)^5\\&+ c_6 (x - \lambda t)^6 + \sum_{n=1}^{\infty} c_{n+6} (x - \lambda t)^{n+6}\\=&c_0 + c_1 (x - \lambda t) + c_2 (x - \lambda t)^2 + c_3 (x - \lambda t)^3 + c_4 (x - \lambda t)^4 + c_5 (x - \lambda t)^5\\&- \frac{Mc_0 + b_1 c_0^2 + b_2 c_0^3 + 2Nc_2 + 24Ac_4}{720a_6} (x - \lambda t)^6\\&+ \sum_{n=1}^{\infty} - \frac{1}{a_6(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)}\\&\times \Big(b \sum_{k=0}^{n} (n+1-k) c_{n+1-k} \sum_{l_1+l_2+l_3=k} c_{l_1} c_{l_2} c_{l_3}\\&\times A(n+1)(n+2)(n+3)(n+4) c_{n+4} + N(n+1)(n+2) c_{n+2}\\&+ Mc_n + b_1 \sum_{k=0}^{n} c_k c_{n-k}\Big) (x - \lambda t)^{n+6},\end{aligned} \tag{3.12}$$

where $c_i, (i = 1 \cdots 6)$ denote nonzero constants.

Finally, substituting Eq. (3.12) into (3.1), one can get the solutions of original equation (2.1) as follows

$$\begin{aligned}q(x,t) =&g(s) e^{i\phi(x,t)}\\=&\Bigg(c_0 + c_1 (x - \lambda t) + c_2 (x - \lambda t)^2 + c_3 (x - \lambda t)^3 + c_4 (x - \lambda t)^4 + c_5 (x - \lambda t)^5\\&+ c_6 (x - \lambda t)^6 + \sum_{n=1}^{\infty} c_{n+6} (x - \lambda t)^{n+6}\Bigg) e^{i\phi(x,t)}\end{aligned}$$

$$=\Bigg(c_0+c_1(x-\lambda t)+c_2(x-\lambda t)^2+c_3(x-\lambda t)^3+c_4(x-\lambda t)^4+c_5(x-\lambda t)^5$$
$$-\frac{Mc_0+b_1c_0^2+b_2c_0^3+2Nc_2+24Ac_4}{720a_6}(x-\lambda t)^6$$
$$+\sum_{n=1}^{\infty}-\frac{1}{a_6(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)}$$
$$\times\Big(b\sum_{k=0}^{n}(n+1-k)c_{n+1-k}\sum_{l_1+l_2+l_3=k}c_{l_1}c_{l_2}c_{l_3}$$
$$\times A(n+1)(n+2)(n+3)(n+4)c_{n+4}+N(n+1)(n+2)c_{n+2}+Mc_n$$
$$+b_1\sum_{k=0}^{n}c_kc_{n-k}\Big)(x-\lambda t)^{n+6}\Bigg)e^{i\phi(x,t)}. \quad (3.13)$$

4. Explicit solutions using F–expansion approach

In this part, we use the F–expansion method [3,16] to deal with equation (3.6). As pointed out in [3,16], from the terms g^3 and $g^{(6)}$, let

$$g=m_0+m_1\phi+m_2\phi^2+m_3\phi^3, \quad (4.1)$$

in this paper, however, we assuming that ϕ satisfies the following equation

$$\phi'=n_0+n_1\phi+n_2\phi^2. \quad (4.2)$$

Putting them into equation (3.6), one has

$$\phi^9: 20160a_6m_3n_2^6+b_2m_3^3=0,$$
$$\phi^8: 5040a_6m_2n_2^6+83160a_6m_3n_1n_2^5+3b_2m_2m_3^2=0,$$
$$\phi^7: 720a_6m_1n_2^6+19440a_6m_2n_1n_2^5+59760a_6m_3n_0n_2^5+136800a_6m_3n_1^2n_2^4$$
$$+360Am_3n_2^4+3b_2m_1m_3^2+3b_2m_2^2m_3=0,$$
$$\phi^6: 2520a_6m_1n_1n_2^5+13440a_6m_2n_0n_2^5+29400a_6m_2n_1^2n_2^4+189000a_6m_3n_0n_1n_2^4$$
$$+113400a_6m_3n_1^3n_2^3+120m_2n_2^4+1080m_3n_1n_2^3+3b_2m_0m_3^2+6b_2m_1m_2m_3$$
$$+b_2m_2^3+b_1$$
$$m_3^2=0,$$
$$\phi^5: 1680a_6m_1n_0n_2^5+3360a_6m_1n_1^2n_2^4+38640a_6m_2n_0n_1n_2^4+21840a_6m_2n_1^3n_2^3$$
$$+62832a_6m_3n_0^2n_2^4+223104m_0n_1^2n_2^3+48972a_6m_3n_1^4n_2^2+24Am_1n_2^4$$
$$+336Am_2n_1n_2^3+816Am_3n_0n_2^3+1164m_3n_1^2n_2^2+12NM_3n_2^2+6b_2m_0m_2m_3$$
$$+3b_2m_1^2m_3+3b_2m_1m_2^2+2b_1m_2m_3=0, \quad (4.3)$$
$$\phi^4: 4200a_6m_1n_0n_1n_2^4+2100a_6m_1n_1^3n_2^3+12096a_6m_2n_0^2n_2^4+40152a_6m_2n_0n_1^2n_2^3$$
$$+8106a_6m_2n_1^4n_2^2+138936a_6m_3n_0^2n_1n_2^3+119532a_6m_3n_0n_1^3n_2^2$$
$$+10101a_6m_3n_1^5n_2+60\ \mathrm{A}m_1n_1n_2^3+240\ \mathrm{A}m_2n_0n_2^3+330\ \mathrm{A}m_2n_1^2n_2^2$$
$$+1680\ \mathrm{A}m_3n_0n_1n_2^2+525\ \mathrm{A}m_3n_1^3n_2+6Nm_2n_2^2+21\mathrm{Nm}_3n_1n_2$$

$$+ 6b_2m_0m_1m_3 + 3b_2m_0m_2^2 + 3b_2m_1^2m_2 + 2b_1m_1m_3 + b_1m_2^2 = 0,$$

$$\phi^3 : 1232a_6m_1n_0^2n_2^4 + 3584a_6m_1n_0n_1^2n_2^3 + 602a_6m_1n_1^4n_2^2 + 22960a_6m_2n_0^2n_1n_2^3 + 17920a_6m_2n_0n_1^3n_2^2 + 1330a_6m_2n_1^5n_2 + 26928a_6m_3n_0^3n_2^3 + 101520a_6m_3n_0^2n_1^2n_2^2 + 27666a_6m_3n_0n_1^4n_2 + 729a_6m_3n_1^6 + 40\ \mathrm{A}m_1n_0n_2^3 + 50\ \mathrm{A\ m_1}n_1^2n_2^2 + 440\ \mathrm{A}m_2n_0n_1n_2^2 + 130\ \mathrm{A_2}n_1^3n_2 + 576\ \mathrm{A}m_3n_0^2n_2^2 + 1062\ \mathrm{A}m_3n_0n_1^2n_2 + 81\ \mathrm{A}m_3n_1^4 + 2\mathrm{Nm_1}n_2^2 + 10\mathrm{Nm_2}n_1n_2 + 18\ \mathrm{N_3}n_0n_2 + 9\mathrm{Nm_3}n_1^2 + 3b_2m_0^2m_3 + 6b_2m_0m_1m_2 + b_2m_1^3 + 2b_1m_0m_3 + 2b_1m_1m_2 + M_3m_3 = 0,$$

$$\phi^2 : 1848a_6m_1n_0^2n_1n_2^3 + 1176a_6m_1n_0n_1^3n_2^2 + 63a_6m_1n_1^5n_2 + 3968a_6m_2n_0^3n_2^3 + 13320a_6m_2n_0^2n_1^2n_2^2 + 3096a_6m_2n_0n_1^4n_2 + 64a_6m_2n_1^6 + 34440a_6m_3n_0^3n_1n_2^2 + 26880a_6m_3n_0^2n_1^3n_2 + 1995a_6m_3n_0n_1^5 + 60\ \mathrm{A}m_1n_0n_1n_2^2 + 15\ \mathrm{A}m_1n_1^3n_2 + 136\ \mathrm{A}m_2n_0^2n_2^2 + 232\ \mathrm{A}m_2n_0n_1^2n_2 + 16\ \mathrm{A}m_2n_1^4 + 660\ \mathrm{A}m_3n_0^2n_1n_2 + 195\ \mathrm{A}m_3n_0n_1^3 + 3\mathrm{Nm_1}n_1n_2 + 8m_2n_0n_2 + 4m_2n_1^2 + 15\mathrm{Nm_3}n_0n_1 + 3b_2m_0^2m_2 + 3b_2m_0m_1^2 + 2b_1m_0m_2 + b_1m_1^2 + Mm_2 = 0,$$

$$\phi^1 : 272a_6m_1n_0^3n_2^3 + 720a_6m_1n_0^2n_1^2n_2^2 + 114a_6m_1n_0n_1^4n_2 + a_6m_1n_1^6 + 3696a_6m_2n_0^3n_1n_2^2 + 2352a_6m_2n_0^2n_1^3n_2 + 126a_6m_2n_0n_1^5 + 3696a_6m_3n_0^4n_2^2 + 10752a_6m_3n_0^3n_1^2n_2 + 1806a_6m_3n_0^2n_1^4 + 16Am_1n_0^2n_2^2 + 22Am_1n_0n_1^2n_2 + Am_1n_1^4 + 120Am_2n_0^2n_1n_2 + 30Am_2n_0n_1^3 + 120Am_3n_0^3n_2 + 150\mathrm{Am_3}n_0^2n_1^2 + 2N_1n_0n_2 + m_1n_1^2 + 6m_2n_0n_1 + 6N_3n_0^2 + 3b_2m_0^2m_1 + 2b_1m_0m_1 + M_1 = 0,$$

$$\phi^0 : a_6m_1n_0n_1^5 + 272a_6m_2n_0^4n_2^2 + 62a_6m_2n_0^2n_1^4 + 540a_6m_3n_0^3n_1^3 + Am_1n_0n_1^3 + 16Am_2n_0^3n_2 + 14Am_2n_0^2n_1^2 + 36Am_3n_0^3n_1 + Nm_1n_0n_1 + 2Nm_2n_0^2 + Mm_0 + b_1m_0^2 + b_2m_0^3 + 136a_6m_1n_0^3n_1n_2^2 + 52a_6m_1n_0^2n_1^3n_2 + 584a_6m_2n_0^3n_1^2n_2 + 1440a_6m_3n_0^4n_1n_2 + 8\ \mathrm{A}m_1n_0^2n_1n_2 = 0. \tag{4.4}$$

Thus, we arrive a set of special solutions:

$$A = 83\,a_6\left(4\,n_0n_2 - {n_1}^2\right),$$
$$M = 2520\,a_6\left(64\,{n_0}^3{n_2}^3 - 48\,{n_1}^2{n_0}^2{n_2}^2 + 12\,{n_1}^4n_0n_2 - {n_1}^6\right),$$
$$N = 946\,a_6\left(16\,{n_0}^2{n_2}^2 - 8\,n_0{n_1}^2n_2 + {n_1}^4\right),\ a_6 = a_6,$$
$$b_1 = 15120\,\frac{{n_2}^3a_6\left(4\,RootOf\left(4\,{n_2}^2\_Z^2+16\,{n_0}^3n_2-3\,{n_0}^2{n_1}^2+\left(-12\,n_0n_1n_2+2\,{n_1}^3\right)\_Z\right){n_2}^2\right)}{m_3} - 15120\,\frac{{n_2}^3a_6\left(6\,n_0n_1n_2+{n_1}^3\right)}{m_3},$$
$$b_2 = -20160\,\frac{a_6{n_2}^6}{{m_3}^2},$$
$$m_0 = \frac{RootOf\left(4\,{n_2}^2\_Z^2+16\,{n_0}^3n_2-3\,{n_0}^2{n_1}^2+\left(-12\,n_0n_1n_2+2\,{n_1}^3\right)\_Z\right)m_3}{n_2},$$
$$m_1 = 3\,\frac{m_3n_0}{n_2}, m_2 = 3/2\,\frac{m_3n_1}{n_2}, m_3 = \sqrt{\frac{-20160a_6n_2^6}{b_2}}, n_0 = n_0, n_1 = n_1, n_2 = n_2. \tag{4.5}$$

Equation (4.2) [17] has a great of many solutions, therefore, one can get solutions of (2.1):

Family–1: When $n_1n_2 \neq 0$ (or $n_0n_2 \neq 0$) and $n_1^2 - 4n_0n_2 > 0$,

$$\begin{aligned}&g(x,t)\\ =&m_0 + \frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(-\frac{1}{2n_2}\left[n_1 + \sqrt{n_1^2-4n_0n_2}\tanh\left(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s\right)\right]\right)\\ &+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(-\frac{1}{2n_2}\left[n_1 + \sqrt{n_1^2-4n_0n_2}\tanh\left(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s\right)\right]\right)^2\\ &+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(-\frac{1}{2n_2}\left[n_1 + \sqrt{n_1^2-4n_0n_2}\tanh\left(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s\right)\right]\right)^3,\end{aligned} \tag{4.6}$$

$$\begin{aligned}&g(x,t)\\ =&m_0 - \frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\frac{1}{2n_2}\left[n_1 + \sqrt{n_1^2-4n_0n_2}\coth\left(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s\right)\right]\\ &+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(-\frac{1}{2n_2}\left[n_1 + \sqrt{n_1^2-4n_0n_2}\coth\left(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s\right)\right]\right)^2\\ &+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(-\frac{1}{2n_2}\left[n_1 + \sqrt{n_1^2-4n_0n_2}\coth\left(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s\right)\right]\right)^3,\end{aligned} \tag{4.7}$$

$$\begin{aligned}&g(x,t)\\ =&m_0 - \frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\frac{1}{2n_2}\left[n_1 + \sqrt{n_1^2-4n_0n_2}\left(\tanh\left(\sqrt{n_1^2-4n_0n_2}s\right)\right.\right.\\ &\left.\left.\pm i\,\mathrm{sech}\left(\sqrt{n_1^2-4n_0n_2}s\right)\right)\right]\\ &+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(-\frac{1}{2n_2}\left[n_1 + \sqrt{n_1^2-4n_0n_2}\left(\tanh\left(\sqrt{n_1^2-4n_0n_2}s\right)\right.\right.\right.\\ &\left.\left.\left.\pm i\,\mathrm{sech}\left(\sqrt{n_1^2-4n_0n_2}s\right)\right)\right]\right)^2\\ &+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(-\frac{1}{2n_2}\left[n_1 + \sqrt{n_1^2-4n_0n_2}\left(\tanh\left(\sqrt{n_1^2-4n_0n_2}s\right)\right.\right.\right.\\ &\left.\left.\left.\pm i\,\mathrm{sech}\left(\sqrt{n_1^2-4n_0n_2}s\right)\right)\right]\right)^3,\end{aligned} \tag{4.8}$$

$$
\begin{aligned}
&g(x,t)\\
&=m_0-\frac{1}{2n_2}\Bigg[n_1+\sqrt{n_1^2-4n_0n_2}\Bigg(\coth\left(\sqrt{n_1^2-4n_0n_2}s\right)\\
&\quad\pm i\mathrm{csch}\left(\sqrt{n_1^2-4n_0n_2}s\right)\Bigg)\Bigg]\\
&\quad+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\Bigg(-\frac{1}{2n_2}\Bigg[n_1+\sqrt{n_1^2-4n_0n_2}\Bigg(\coth\left(\sqrt{n_1^2-4n_0n_2}s\right)\\
&\quad\pm i\mathrm{csch}\left(\sqrt{n_1^2-4n_0n_2}s\right)\Bigg)\Bigg]\Bigg)^2\\
&\quad+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\Bigg(-\frac{1}{2n_2}\Bigg[n_1+\sqrt{n_1^2-4n_0n_2}\Bigg(\coth\left(\sqrt{n_1^2-4n_0n_2}s\right)\\
&\quad\pm i\mathrm{csch}\left(\sqrt{n_1^2-4n_0n_2}s\right)\Bigg)\Bigg]\Bigg)^3,
\end{aligned}\tag{4.9}
$$

$$
\begin{aligned}
&g(x,t)\\
&=m_0-\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\frac{1}{4n_2}\Bigg[2n_1+\sqrt{n_1^2-4n_0n_2}\Bigg(\tanh\left(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s\right)\\
&\quad+\coth\left(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s\right)\Bigg)\Bigg]\\
&\quad+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\Bigg(-\frac{1}{4n_2}\Bigg[2n_1+\sqrt{n_1^2-4n_0n_2}\Bigg(\tanh\left(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s\right)\\
&\quad+\coth\left(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s\right)\Bigg)\Bigg]\Bigg)^2\\
&\quad+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\Bigg(-\frac{1}{4n_2}\Bigg[2n_1+\sqrt{n_1^2-4n_0n_2}\Bigg(\tanh\left(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s\right)\\
&\quad+\coth\left(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s\right)\Bigg)\Bigg]\Bigg)^3,
\end{aligned}\tag{4.10}
$$

$$
\begin{aligned}
&g(x,t)\\
&=m_0+\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\frac{1}{2n_2}\Bigg[-n_1\\
&\quad+\frac{\sqrt{(E^2+F^2)(n_1^2-4n_0n_2)}-E\sqrt{n_1^2-4n_0n_2}\cosh(\sqrt{n_1^2-4n_0n_2}s)}{E\sinh(\sqrt{n_1^2-4n_0n_2}s)+F}\Bigg]\\
&\quad+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\Bigg(\frac{1}{2n_2}\Bigg[-n_1
\end{aligned}
$$

$$+\frac{\sqrt{(E^2+F^2)(n_1^2-4n_0n_2)}-E\sqrt{n_1^2-4n_0n_2}\cosh(\sqrt{n_1^2-4n_0n_2}s)}{E\sinh(\sqrt{n_1^2-4n_0n_2}s)+F}\Bigg]\Bigg)^2$$

$$+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\Bigg(\frac{1}{2n_2}\Bigg[-n_1$$

$$+\frac{\sqrt{(E^2+F^2)(n_1^2-4n_0n_2)}-E\sqrt{n_1^2-4n_0n_2}\cosh(\sqrt{n_1^2-4n_0n_2}s)}{E\sinh(\sqrt{n_1^2-4n_0n_2}s)+F}\Bigg]\Bigg)^3, \quad (4.11)$$

$$g(x,t)$$

$$=m_0+\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\frac{1}{2n_2}\Bigg[-n_1$$

$$-\frac{\sqrt{(F^2-E^2)(n_1^2-4n_0n_2)}+E\sqrt{n_1^2-4n_0n_2}\sinh(\sqrt{n_1^2-4n_0n_2}s)}{E\cosh(\sqrt{n_1^2-4n_0n_2}s)+F}\Bigg]$$

$$+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\Bigg(\frac{1}{2n_2}\Bigg[-n_1$$

$$-\frac{\sqrt{(F^2-E^2)(n_1^2-4n_0n_2)}+E\sqrt{n_1^2-4n_0n_2}\sinh(\sqrt{n_1^2-4n_0n_2}s)}{E\cosh(\sqrt{n_1^2-4n_0n_2}s)+F}\Bigg]\Bigg)^2$$

$$+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\Bigg(\frac{1}{2n_2}\Bigg[-n_1$$

$$-\frac{\sqrt{(F^2-E^2)(n_1^2-4n_0n_2)}+E\sqrt{n_1^2-4n_0n_2}\sinh(\sqrt{n_1^2-4n_0n_2}s)}{E\cosh(\sqrt{n_1^2-4n_0n_2}s)+F}\Bigg]\Bigg)^3. \quad (4.12)$$

Here $F^2-E^2>0$. Also, F and E are non-zero constants. Eq. (4.6) stands for dark soliton, Eq. (4.7) depicts singular soliton, Eq. (4.10) denotes combo dark–singular soliton, Eq. (4.11) signifies combo singular soliton and Eq. (4.12) purports combo bright–dark soliton. Eqs. (4.8) and (4.9) also depict complexion solutions.

$$g(x,t)$$

$$=m_0+\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left[\frac{2n_0\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)}{\sqrt{n_1^2-4n_0n_2}\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)-n_1\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)}\right]$$

$$+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\left[\frac{2n_0\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)}{\sqrt{n_1^2-4n_0n_2}\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)-n_1\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)}\right]\right)^2$$

$$+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\left[\frac{2n_0\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)}{\sqrt{n_1^2-4n_0n_2}\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)-n_1\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)}\right]\right)^3, \quad (4.13)$$

$$g(x,t)$$
$$=m_0+\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left[\frac{-2n_0\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)}{-\sqrt{n_1^2-4n_0n_2}\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)+n_1\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)}\right]$$
$$+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\left[\frac{-2n_0\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)}{-\sqrt{n_1^2-4n_0n_2}\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)+n_1\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)}\right]\right)^2$$
$$+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\left[\frac{-2n_0\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)}{-\sqrt{n_1^2-4n_0n_2}\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)+n_1\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{2}s)}\right]\right)^3, \tag{4.14}$$

$$g(x,t)$$
$$=m_0+\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$
$$\times\left[\frac{2n_0\cosh(\sqrt{n_1^2-4n_0n_2}s)}{\sqrt{n_1^2-4n_0n_2}\sinh(\sqrt{n_1^2-4n_0n_2}s)-n_1\cosh(\sqrt{n_1^2-4n_0n_2}s)\pm i\sqrt{n_1^2-4n_0n_2s}}\right]$$
$$+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$
$$\times\left(\left[\frac{2n_0\cosh(\sqrt{n_1^2-4n_0n_2}s)}{\sqrt{n_1^2-4n_0n_2}\sinh(\sqrt{n_1^2-4n_0n_2}s)-n_1\cosh(\sqrt{n_1^2-4n_0n_2}s)\pm i\sqrt{n_1^2-4n_0n_2s}}\right]\right)^2$$
$$+\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$
$$\times\left(\left[\frac{2n_0\cosh(\sqrt{n_1^2-4n_0n_2}s)}{\sqrt{n_1^2-4n_0n_2}\sinh(\sqrt{n_1^2-4n_0n_2}s)-n_1\cosh(\sqrt{n_1^2-4n_0n_2}s)\pm i\sqrt{n_1^2-4n_0n_2s}}\right]\right)^3, \tag{4.15}$$

$$g(x,t)$$
$$=m_0+\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$
$$\times\left[\frac{2n_0\sinh(\sqrt{n_1^2-4n_0n_2}s)}{\sqrt{n_1^2-4n_0n_2}\cosh(\sqrt{n_1^2-4n_0n_2}s)-n_1\sinh(\sqrt{n_1^2-4n_0n_2}s)\pm\sqrt{n_1^2-4n_0n_2s}}\right]$$
$$+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$
$$\times\left(\left[\frac{2n_0\sinh(\sqrt{n_1^2-4n_0n_2}s)}{\sqrt{n_1^2-4n_0n_2}\cosh(\sqrt{n_1^2-4n_0n_2}s)-n_1\sinh(\sqrt{n_1^2-4n_0n_2}\xi)\pm\sqrt{n_1^2-4n_0n_2s}}\right]\right)^2$$
$$+\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$

$$\times\left(\left[\frac{2n_0\sinh(\sqrt{n_1^2-4n_0n_2}s)}{\sqrt{n_1^2-4n_0n_2}\cosh(\sqrt{n_1^2-4n_0n_2}s)-n_1\sinh(\sqrt{n_1^2-4n_0n_2}\xi)\pm\sqrt{n_1^2-4n_0n_2}s}\right]\right)^3, \tag{4.16}$$

$$\begin{aligned}g(x,t)\\ =&m_0+\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\\ &\times\left[\frac{4n_0\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)}{-2n_1\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)+2\sqrt{n_1^2-4n_0n_2}\cosh^2(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)-\sqrt{n_1^2-4n_0n_2}}\right]\\ &+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\\ &\times\left(\left[\frac{4n_0\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)}{-2n_1\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)+2\sqrt{n_1^2-4n_0n_2}\cosh^2(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)-\sqrt{n_1^2-4n_0n_2}}\right]\right)^2\\ &+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\\ &\times\left(\left[\frac{4n_0\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)}{-2n_1\sinh(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)\cosh(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)+2\sqrt{n_1^2-4n_0n_2}\cosh^2(\frac{\sqrt{n_1^2-4n_0n_2}}{4}s)-\sqrt{n_1^2-4n_0n_2}}\right]\right)^3.\end{aligned} \tag{4.17}$$

Eqs. (4.13), (4.14), (4.16) and (4.17) stands for combo singular soliton and Eq. (4.15) depicts complexion solutions.

Family–2: When $n_1^2-4n_0n_2<0$ and $n_1n_2\neq 0$ (or $n_0n_2\neq 0$),

$$\begin{aligned}g(x,t)\\ =&m_0+\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\frac{1}{2n_2}\left[-n_1+\sqrt{4n_0n_2-n_1^2}\tan\left(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s\right)\right]\\ &+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\frac{1}{2n_2}\left[-n_1+\sqrt{4n_0n_2-n_1^2}\tan\left(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s\right)\right]\right)^2\\ &+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\frac{1}{2n_2}\left[-n_1+\sqrt{4n_0n_2-n_1^2}\tan\left(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s\right)\right]\right)^3,\end{aligned} \tag{4.18}$$

$$\begin{aligned}g(x,t)\\ =&m_0-\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\frac{1}{2n_2}\left[n_1+\sqrt{4n_0n_2-n_1^2}\cot\left(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s\right)\right]\\ &+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(-\frac{1}{2n_2}\left[n_1+\sqrt{4n_0n_2-n_1^2}\cot\left(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s\right)\right]\right)^2\end{aligned}$$

$$+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(-\frac{1}{2n_2}\left[n_1+\sqrt{4n_0n_2-n_1^2}\cot\left(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s\right)\right]\right)^3, \tag{4.19}$$

$$\begin{aligned}
&g(x,t)\\
=&m_0+\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\frac{1}{2n_2}\left[-n_1+\sqrt{4n_0n_2-n_1^2}\left(\tan\left(\sqrt{4n_0n_2-n_1^2}s\right)\right.\right.\\
&\left.\left.\pm\sec\left(\sqrt{4n_0n_2-n_1^2}s\right)\right)\right]\\
&+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\frac{1}{2n_2}\left[-n_1+\sqrt{4n_0n_2-n_1^2}\left(\tan\left(\sqrt{4n_0n_2-n_1^2}s\right)\right.\right.\right.\\
&\left.\left.\left.\pm\sec\left(\sqrt{4n_0n_2-n_1^2}s\right)\right)\right]\right)^2\\
&+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\frac{1}{2n_2}\left[-n_1+\sqrt{4n_0n_2-n_1^2}\left(\tan\left(\sqrt{4n_0n_2-n_1^2}s\right)\right.\right.\right.\\
&\left.\left.\left.\pm\sec\left(\sqrt{4n_0n_2-n_1^2}s\right)\right)\right]\right)^3,
\end{aligned} \tag{4.20}$$

$$\begin{aligned}
&g(x,t)\\
=&Lm_0-\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\frac{1}{2n_2}\left[n_1+\sqrt{4n_0n_2-n_1^2}\left(\cot\left(\sqrt{4n_0n_2-n_1^2}s\right)\right.\right.\\
&\left.\left.\pm\csc\left(\sqrt{4n_0n_2-n_1^2}s\right)\right)\right]\\
&+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(-\frac{1}{2n_2}\left[n_1+\sqrt{4n_0n_2-n_1^2}\left(\cot\left(\sqrt{4n_0n_2-n_1^2}s\right)\right.\right.\right.\\
&\left.\left.\left.\pm\csc\left(\sqrt{4n_0n_2-n_1^2}s\right)\right)\right]\right)^2\\
&+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(-\frac{1}{2n_2}\left[n_1+\sqrt{4n_0n_2-n_1^2}\left(\cot\left(\sqrt{4n_0n_2-n_1^2}s\right)\right.\right.\right.\\
&\left.\left.\left.\pm\csc\left(\sqrt{4n_0n_2-n_1^2}s\right)\right)\right]\right)^3,
\end{aligned} \tag{4.21}$$

$$
\begin{aligned}
g(x,t)
&= m_0 - \frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\frac{1}{4n_2}\Bigg[-2n_1 \\
&+ \sqrt{4n_0n_2-n_1^2}\left(\tan\left(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s\right)-\cot\left(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s\right)\right)\Bigg] \\
&+ \frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\Bigg(-\frac{1}{4n_2}\Bigg[-2n_1 \\
&+ \sqrt{4n_0n_2-n_1^2}\left(\tan\left(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s\right)-\cot\left(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s\right)\right)\Bigg]\Bigg)^2 \\
&+ \sqrt{\frac{-2016a_6n_2^6}{b_2}}\Bigg(-\frac{1}{4n_2}\Bigg[-2n_1 \\
&+ \sqrt{4n_0n_2-n_1^2}\left(\tan\left(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s\right)-\cot\left(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s\right)\right)\Bigg]\Bigg)^3,
\end{aligned}
\tag{4.22}
$$

$$
\begin{aligned}
g(x,t)
&= m_0 + \frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\frac{1}{2n_2}\Bigg[-n_1 \\
&+ \frac{\pm\sqrt{(F^2-E^2)(4n_0n_2-n_1^2)}-E\sqrt{4n_0n_2-n_1^2}\cos(\sqrt{4n_0n_2-n_1^2}s)}{E\sin(\sqrt{4n_0n_2-n_1^2}s)+F}\Bigg] \\
&+ \frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\Bigg(\frac{1}{2n_2}\Bigg[-n_1 \\
&+ \frac{\pm\sqrt{(F^2-E^2)(4n_0n_2-n_1^2)}-E\sqrt{4n_0n_2-n_1^2}\cos(\sqrt{4n_0n_2-n_1^2}s)}{E\sin(\sqrt{4n_0n_2-n_1^2}s)+F}\Bigg]\Bigg)^2 \\
&+ \sqrt{\frac{-2016a_6n_2^6}{b_2}}\Bigg(\frac{1}{2n_2}\Bigg[-n_1 \\
&+ \frac{\pm\sqrt{(F^2-E^2)(4n_0n_2-n_1^2)}-E\sqrt{4n_0n_2-n_1^2}\cos(\sqrt{4n_0n_2-n_1^2}s)}{E\sin(\sqrt{4n_0n_2-n_1^2}s)+F}\Bigg]\Bigg)^3,
\end{aligned}
\tag{4.23}
$$

$$
\begin{aligned}
g(x,t)
&= m_0 + \frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\frac{1}{2n_2}\Bigg[-n_1 \\
&+ \frac{\pm\sqrt{(F^2-E^2)(4n_0n_2-n_1^2)}+E\sqrt{4n_0n_2-n_1^2}\sinh(\sqrt{4n_0n_2-n_1^2}s)}{E\cos(\sqrt{4n_0n_2-n_1^2}s)+F}\Bigg]
\end{aligned}
$$

$$+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\frac{1}{2n_2}\left[-n_1 + \frac{\pm\sqrt{(F^2-E^2)(4n_0n_2-n_1^2)}+E\sqrt{4n_0n_2-n_1^2}\sinh(\sqrt{4n_0n_2-n_1^2}s)}{E\cos(\sqrt{4n_0n_2-n_1^2}s)+F}\right]\right)^2$$
$$+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\frac{1}{2n_2}\left[-n_1 + \frac{\pm\sqrt{(F^2-E^2)(4n_0n_2-n_1^2)}+E\sqrt{4n_0n_2-n_1^2}\sinh(\sqrt{4n_0n_2-n_1^2}s)}{E\cos(\sqrt{4n_0n_2-n_1^2}s)+F}\right]\right)^3. \tag{4.24}$$

Here $F^2-E^2>0$. Also, F and E are non-zero constants. Eqs. (4.18)–(4.24) stands for singular periodic solutions.

$$g(x,t) = m_0 + \frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left[\frac{-2n_0\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)}{\sqrt{4n_0n_2-n_1^2}\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)+n_1\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)}\right]$$
$$+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\left[\frac{-2n_0\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)}{\sqrt{4n_0n_2-n_1^2}\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)+n_1\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)}\right]\right)^2$$
$$+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\left[\frac{-2n_0\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)}{\sqrt{4n_0n_2-n_1^2}\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)+n_1\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)}\right]\right)^3, \tag{4.25}$$

$$g(x,t) = m_0 + \frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left[\frac{2n_0\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)}{\sqrt{4n_0n_2-n_1^2}\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)-n_1\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)}\right]$$
$$+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\left[\frac{2n_0\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)}{\sqrt{4n_0n_2-n_1^2}\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)-n_1\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)}\right]\right)^2$$
$$+\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\left[\frac{2n_0\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)}{\sqrt{4n_0n_2-n_1^2}\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)-n_1\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{2}s)}\right]\right)^3, \tag{4.26}$$

$$g(x,t) = m_0 + \frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$
$$\times\left[\frac{-2n_0\cos(\sqrt{4n_0n_2-n_1^2}\theta)}{\sqrt{4n_0n_2-n_1^2}\sin(\sqrt{4n_0n_2-n_1^2}s)+n_1\cos(\sqrt{4n_0n_2-n_1^2}s)\pm i\sqrt{4n_0n_2-n_1^2}s}\right]$$

$$+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$
$$\times\left(\left[\frac{-2n_0\cos(\sqrt{4n_0n_2-n_1^2}s)}{\sqrt{4n_0n_2-n_1^2}\sin(\sqrt{4n_0n_2-n_1^2}s)+n_1\cos(\sqrt{4n_0n_2-n_1^2}s)\pm i\sqrt{4n_0n_2-n_1^2}s}\right]\right)^2$$
$$+\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$
$$\times\left(\left[\frac{-2n_0\cos(\sqrt{4n_0n_2-n_1^2}s)}{\sqrt{4n_0n_2-n_1^2}\sin(\sqrt{4n_0n_2-n_1^2}s)+n_1\cos(\sqrt{4n_0n_2-n_1^2}s)\pm i\sqrt{4n_0n_2-n_1^2}s}\right]\right)^3, \tag{4.27}$$

$$g(x,t)$$
$$=m_0+\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$
$$\times\left[\frac{2n_0\sin(\sqrt{4n_0n_2-n_1^2}s)}{\sqrt{4n_0n_2-n_1^2}\cos(\sqrt{4n_0n_2-n_1^2}s)-n_1\sin(\sqrt{4n_0n_2-n_1^2}s)\pm\sqrt{4n_0n_2-n_1^2}s}\right]$$
$$+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$
$$\times\left(\left[\frac{2n_0\sin(\sqrt{4n_0n_2-n_1^2}s)}{\sqrt{4n_0n_2-n_1^2}\cos(\sqrt{4n_0n_2-n_1^2}s)-n_1\sin(\sqrt{4n_0n_2-n_1^2}s)\pm\sqrt{4n_0n_2-n_1^2}s}\right]\right)^2$$
$$+\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$
$$\times\left(\left[\frac{2n_0\sin(\sqrt{4n_0n_2-n_1^2}s)}{\sqrt{4n_0n_2-n_1^2}\cos(\sqrt{4n_0n_2-n_1^2}s)-n_1\sin(\sqrt{4n_0n_2-n_1^2}s)\pm\sqrt{4n_0n_2-n_1^2}s}\right]\right)^3, \tag{4.28}$$

$$g(x,t)$$
$$=m_0+\frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$
$$\times\left[\frac{4n_0\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)}{-2n_1\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)+2\sqrt{4n_0n_2-B^2}\cos^2(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)-\sqrt{4n_0n_2-n_1^2}}\right]$$
$$+\frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$
$$\times\left(\left[\frac{4n_0\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)}{-2n_1\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)+2\sqrt{4n_0n_2-n_1^2}\cos^2(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)-\sqrt{4n_0n_2-n_1^2}}\right]\right)^2$$
$$+\sqrt{\frac{-2016a_6n_2^6}{b_2}}$$

$$\times\left(\left[\frac{4n_0\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)}{-2n_1\sin(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)\cos(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)+2\sqrt{4n_0n_2-n_1^2}\cos^2(\frac{\sqrt{4n_0n_2-n_1^2}}{4}s)-\sqrt{4n_0n_2-n_1^2}}\right]\right)^3, \tag{4.29}$$

where Eqs. (4.25), (4.26), (4.28) and (4.29) stand for singular periodic solutions and (4.27) depict complexion solutions.

Family–3: When $n_0 = 0$ and $n_1 n_2 \neq 0$,

$$\begin{aligned} g(x,t) =& m_0 + \frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\frac{-n_1 d}{n_1(d+\cosh(n_1 s)-\sinh(n_1 s))}\right) \\ &+ \frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\left(\frac{-n_1 d}{n_1(d+\cosh(n_1 s)-\sinh(n_1 s))}\right)\right)^2 \\ &+ \sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\left(\frac{-n_1 d}{n_1(d+\cosh(n_1 s)-\sinh(n_1 s))}\right)\right)^3. \end{aligned} \tag{4.30}$$

Here d is constant. Eq. (4.30) denotes combo singular soliton.

Family–4: When $n_2 \neq 0$ and $n_0 = n_1 = 0$,

$$\begin{aligned} g(x,t) =& m_0 + \frac{3n_0}{n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\frac{-1}{n_1(s)+k}\right) + \frac{3n_1}{2n_2}\sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\frac{-1}{n_1(s)+k}\right)^2 \\ &+ \sqrt{\frac{-2016a_6n_2^6}{b_2}}\left(\frac{-1}{n_1(s)+k}\right)^3, \end{aligned} \tag{4.31}$$

where k is an arbitrary constant. Eq. (4.31) denotes plane wave solution.

5. Conclusions

In this paper, HD–NLSE with QC law of nonlinearity was first analyzed using Lie symmetry analysis. This Lie symmetry reduction made it possible to reduce the governing PDE to an ODE that was further analyzed with the usage of two integration algorithms. These revealed a plethora of solutions to the present model that included soliton solutions as well as other periodic solutions to the current model. Thus, Lie symmetry analysis together with the series solution technique and F–expansion scheme collectively proved to be a powerful combination of algorithms that gave way to an abundance of solutions. The soliton solutions will be of great importance in optics while other solutions will be applicable to several other physical systems.

These results together with the applied algorithms are encouraging to look at the concept of HD–NLSE with other forms of SPM. Additionally, this concept of HD solitons will be applied to other optoelectronic devices such as Bragg gratings, magneto–optic waveguides, optical couplers, optical metamaterials and several others. Later this would be studied with differential group delay, dispersion–flattened fibers and several additional situations. Additionally HD solitons will be addressed with fractional temporal evolution that is one of the key approaches to mitigate

Internet bottleneck effect. The quiescent optical solitons would also be looked for whenever one or more of the dispersion terms are rendered to be inadvertantly nonlinear. These are on the bucket list. The results will be aligned with the pre–existing reports and made visible sometime down the road [5–7]!

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