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ANALYSIS OF NEW TRANSFER FUNCTIONS WITH SUM INTEGRAL TRANSFORMATION

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Abstract We explore the novel SUM integral transform method for solving ordinary and partial differential equations, offering an effective approach beyond conventional Laplace and Sumudu transforms. Using this method, we address various differential equations, deriving transfer functions for classical and fractional derivatives. The resultant transfer functions provide valuable insights into diverse mathematical models.

Keywords SUM integral transform, classical derivatives, transfer functions, pole analysis, fractional derivatives.

MSC(2010) 35A22, 26A33.

1. Introduction

Integral transformations are strong mathematical tools for converting functions or problems from one domain to others [3, 16, 18, 22, 24, 28, 29]. They enable us to study functions in new ways by changing them into new representations, frequently facilitating problem-solving in domains like as science, technology, data processing, and computational mathematics. Integral transforms work by describing a function in terms of another set of functions, often via integral operations. The transformation is a process of transferring functions from their original domain (time, space, frequency, etc.) to a new domain, allowing a problem to be investigated from a different angle. The Fourier transform, Laplace transform, and Mellin transform are examples of notable integral transforms, each with their own set of features and uses. These transformations are frequently used to simplify the use of differential equations, convolutions, and complex calculations, enabling more easy analysis and solution of issues that would otherwise be difficult to solve. Studying integral transforms entails knowing their underlying concepts, characteristics, and applica-

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tions to diverse issues. Mastery of these transformations may considerably improve problem-solving abilities in a variety of fields, making them vital tools in the field of mathematics and its various applications. Transfer functions are critical instruments in control systems and signal processing. They serve as mathematical models of a system's input and output, providing a succinct manner to comprehend and study the behavior of complex systems. Some recent studies on integral transformations are as follows [1, 2, 17, 20, 21]. A transfer function, in essence, shows how a system responds to multiple inputs, whether it's an electrical circuit, a mechanical system, or a chemical process. It's a strong notion since it incorporates the dynamics of the system in a concise mathematical form, usually in the frequency domain. Researchers and engineers can acquire insights about a system's behavior by evaluating its transfer function without having to dive into the system's underlying workings. This abstraction helps engineers forecast and regulate the efficiency of diverse systems by facilitating their design, analysis, and optimization. Transfer functions are important in many domains, including electrical engineering, theory of control, physics, and others. They serve as a link between a system's input and output, making them essential for understanding and influencing the actions of many systems. Some studies on the SUM integral transformation: Abubakar U.M. et al [4] the SUM integral transform finds application in various domains including nuclear physics, population growth modeling, analysis of electric circuits, pharmacokinetics, beam deflection studies, investigations related to Newton's law of cooling, and within other realms of mechanics. In this article, analysis and simulations of transfer functions of classical and fractional derivatives were obtained by applying the SUM integral transform (introduced by Hasan, Sameer Q. at el. [15]). Our study goes further than traditional integral transforms by covering a wide range of fractional derivatives, such as the Caputo, Modified Caputo-Fabrizio, Modified Atangana-Baleanu, and Constant Proportional Caputo derivatives. The fractional derivatives have garnered significant attention due to their ability to describe complex physical phenomena with memory and non-local effects. These derivatives find applications across various disciplines including mathematical modeling of biological systems, analysis of complex networks, and characterization of anomalous diffusion processes [9, 14, 25–27]. By calculating transfer functions for these fractional derivatives, we show how the SUM integral transform can effectively analyze various mathematical models. Our research offers new paths for solving differential equations and understanding complex systems, which may not be easily attainable using only traditional integral transforms.

The rest of our article is planned as follows: Section 2 presents the basic definitions and theorems relevant to our work. In section 3, some basic definitions and theorems of the SUM integral transform are given. Additionally, the SUM integral transform was applied to classical and fractional derivatives. In section 4, transfer functions were obtained using classical and fractional derivatives. In this section we also show examples of numerical simulations. In section 5, information about pole analysis was given and some figures were interpreted. In the last part, section 6, we evaluated the results obtained in the previous chapters.

2. SUM integral transform

Before exploring the SUM integral transform method, we introduce two preliminary definitions:

Definition 2.1 ([19]). A function $f(t)$ is denoted to have exponential growth with a positive rate $\partial(>0)$ as t approaches 1 if there is a value $L>0$ for which

$$|f(t)| < Le^{\partial t}, \quad L, \partial > 0, \quad \forall \geqslant 0. \tag{2.1}$$

Definition 2.2 ([23]). A function $f(t)$ is termed as piecewise continuous over the closed interval $[a,b]$ if it is established and exhibits continuity within the interval $[a,b]$, except for a finite set of points $p_1, p_2, p_3, \cdots, p_n$. At each of these points, both the left and right limits of $f(t)$ are existent.

In accordance with the sequence of the previous integral transforms, the subsequent integral transform is presented as follows [15]:

$$S_a\{f(t)\}_{(s)} = \frac{1}{s^r}\int_{t=0}^{\infty} f(t)a^{-st}dt = \frac{1}{s^r}\int_{t=0}^{\infty} f(t)e^{-st\log(a)}dt = G_a(s), \tag{2.2}$$

if $t \geq 0, r \in \mathbb{Z}, a \in (0,\infty)\backslash\{1\}, n_1 \leq s \leq n_2, n_1, n_2 > 0$ and $f(t)$ represent sectionally continuous with exponential order, respectively. The subsequent theorem gives the sufficient condition for an existence of the SUM integral transform.

Theorem 2.1 ([15]). *If $f(t)$ demonstrates piecewise continuity within every finite interval from 0 to t_0, and it displays exponential order $(\partial > 0)$ as t approaches infinity, then the existence of $S_a f(t)_{(s)}$ holds true for all s with real part greater than $\frac{\partial}{\log(a)}$.*

3. The SUM integral transform of some elementary functions

In this section, we assess the integral transformation of certain fundamental functions through the SUM integral transform [15].

- Supposing $f(t) = k$, where k represents a constant, then

$$S_a\{k\}_{(s)} = \frac{k}{s^r[s\log(a)]}. \tag{3.1}$$

- Supposing $f(t) = t^m, m \in \mathbb{N}_0$, then

$$S_a\{t^m\}_{(s)} = \frac{\Gamma(m+1)}{s^r[s\log(a)]^{m+1}}, Re(m+1) > 0. \tag{3.2}$$

- Supposing $f(t) = e^{\rho t}$ where ρ remains constant, then

$$S_a\{e^{\rho t}\}_{(s)} = \frac{1}{s^r\{[s\log(a)] - \rho\}}. \tag{3.3}$$

Definition 3.1 ([15]). If a function $f(t)$ is n-times continuously differentiable on $[0,\infty)$ and of exponential order $\partial(>0)$, then $S_a\{f'(t)\}_{(s)}, S_a\{f''(t)\}_{(s)}, \cdots,$ $S_a\{f^{(n)}(t)\}_{(s)}$ exist for $Re(s) > \frac{\partial}{\log(a)}$ and

$$S_a\{f^{(n)}(t)\}_{(s)} = [s\log(a)]^n S_a\{f(t)\}_{(s)} - \frac{1}{s^r}\sum_{w=0}^{n-1}[s\log(a)]^{n-w-1}f^{(n-1)-w}(0). \tag{3.4}$$

When we apply the definition of the SUM integral transform as shown in equation (3.1) for $n = 1$ we obtain

$$S_a\{f'(t)\}_{(s)} = [s\log(a)]S_a\{f(t)\}_{(s)} - \frac{f(0)}{s^r}. \tag{3.5}$$

Theorem 3.1. *If $F_1(s)$ and $H_1(s)$ represent the SUM integral transforms of $f(t)$ and $h(t)$, respectively, then*

$$S_a\{f * h\}_{(s)} = s^r F_1(s)H_1(s). \tag{3.6}$$

Proof. We have

$$f * h = \int_0^t f(\tau)h(t-\tau)d\tau. \tag{3.7}$$

By employing the SUM integral transform in conjunction with Leibniz's theorem, we obtain

$$\begin{aligned} S_a\{f * h\}_{(s)} =& S_a\left\{\int_0^t f(\tau)h(t-\tau)d\tau\right\} \\ =& \frac{1}{s^r}\int_0^\infty \left[\int_0^t f(\tau)h(t-\tau)d\tau\right] \\ =& \frac{1}{s^r}\int_0^\infty f(\tau)d\tau \int_\tau^\infty h(t-\tau)e^{-st\log(a)}dt, \end{aligned} \tag{3.8}$$

by setting $u = t - \tau$, we get

$$\begin{aligned} S_a\{f * h\}_{(s)} &= \frac{1}{s^r}\int_0^\infty f(\tau)d\tau \int_0^\infty h(u)e^{-s(u+\tau)\log(a)}du \\ &= \frac{1}{s^r}\int_0^\infty e^{-s\tau\log(a)}f(\tau)d\tau \int_0^\tau h(u)e^{-s\tau\log(a)}du \\ &= s^r \frac{s^r}{\int}{}^\infty_0 e^{-s\tau\log(a)}f(\tau)d\tau \frac{1}{s^r}\int_0^\infty h(u)e^{-su\log(a)}du \\ &= s^r F(s)H(s). \end{aligned} \tag{3.9}$$

□

The relations for the Caputo [10, 11], Caputo-Fabrizio [12], Modified Caputo-Fabrizio [13], Atangana-Baleanu [7], Modified Atangana-Baleanu [5], and Constant Proportional Caputo [8] derivatives are expressed as follows:

$$\begin{aligned} &{}^{C}_{0}\mathrm{D}^{\phi}_{t} f(t) = \frac{df(t)}{dt} * \frac{t^{-\phi}}{\Gamma(1-\phi)}, \\ &{}^{CF}_{0}\mathrm{D}^{\phi}_{t} f(t) = \frac{df(t)}{dt} * \frac{M(\phi)}{1-\phi}\exp(\frac{-\phi}{1-\phi}t), \\ &{}^{MCF}_{0}\mathrm{D}^{\phi}_{t} f(t) = \frac{d}{dt}\left\{(f(t) - f_0) * \frac{1}{1-\phi}\exp\left(-\frac{\phi}{1-\phi}t\right)\right\}, \\ &{}^{ABC}_{0}\mathrm{D}^{\phi}_{t} f(t) = \frac{df(t)}{dt} * \frac{AB(\phi)}{1-\phi}E_\phi\left(\frac{-\phi}{1-\phi}t^\phi\right), \\ &{}^{MABC}_{0}\mathrm{D}^{\phi}_{t} f(t) = \frac{AB(\phi)}{1-\phi}\left(f(t) - E_\phi\left(-\mu_\phi t^\phi\right)f(0) - \mu_\phi t^{\phi-1}E_{\phi,\phi}\left(-\mu_\phi t^\phi\right) * f(t)\right), \end{aligned}$$

$$ {}^{CPC}_{\ 0}\mathrm{D}_t^{\phi} f(t) = k_1(\phi)\left(\frac{t^{-\phi}}{\Gamma(1-\phi)} * f(t)\right) + k_0(\phi)\left(\frac{t^{-\phi}}{\Gamma(1-\phi)} * f'(t)\right), $$

where $\mu_\phi = \frac{\phi}{1-\phi}$. The SUM integral transforms corresponding to these derivatives are given by:

$$ \begin{aligned} & S_a\{{}^{C}_{0}\mathrm{D}_t^{\phi} f(t)\}_{(s)} \\ =& S_a\left\{\frac{t^{-\phi}}{1-\phi} * f'(t)\right\}_{(s)} \\ =& S^r S_a\left\{\frac{t^{-\phi}}{1-\phi}\right\}_{(s)} S_a\left\{f'(t)\right\}_{(s)} \\ =& (s\log(a))^{\phi} S_a\left\{f(t)\right\}_{(s)} - (s\log(a))^{\phi-1}\frac{f(0)}{s^r}, \end{aligned} \tag{3.10} $$

$$ \begin{aligned} & S_a\{{}^{MCF}_{\ \ 0}\mathrm{D}_t^{\phi} f(t)\}_{(s)} \\ =& S_a\left\{\frac{d}{dt}\left\{(f(t)-f_0) * \frac{1}{1-\phi}\exp\left(-\frac{\phi}{1-\phi}t\right)\right\}\right\}_{(s)} \\ =& s\log(a) S_a\left\{(f(t)-f(0)) * \frac{1}{1-\phi}\exp\left(-\frac{\phi}{1-\phi}t\right)\right\}_{(s)} \\ & - \frac{1}{s^r}\left[(f(t)-f(0)) * \frac{1}{1-\phi}\exp\left(-\frac{\phi}{1-\phi}t\right)\right]_{t=0} \\ =& s\log(a) s^r S_a\left\{f(t)-f(0)\right\}_{(s)} S_a\left\{\frac{1}{1-\phi}\exp\left(-\frac{\phi}{1-\phi}t\right)\right\}_{(s)} \\ =& s\log(a) s^r \left(S_a\left\{f(t)\right\}_{(s)} - S_a\left\{f(0)\right\}_{(s)}\right)\frac{1}{1-\phi} S_a\left\{\exp\left(-\frac{\phi}{1-\phi}t\right)\right\}_{(s)} \\ =& s\log(a) s^r\left(S_a\left\{f(t)\right\}_{(s)} - \frac{f(0)}{s^r\left(s\log(a)\right)}\right)\frac{1}{1-\phi}\frac{1}{s^r\left(s\log(a)+\frac{\phi}{1-\phi}\right)} \\ =& \frac{s\log(a)}{s\log(a)(1-\phi)+\phi} S_a\{f\}_{(s)} - \frac{f(0)}{s^r\left(s\log(a)(1-\phi)+\phi\right)}, \end{aligned} \tag{3.11} $$

$$ \begin{aligned} & S_a\{{}^{MABC}_{\ \ \ \ 0}\mathrm{D}_t^{\phi} f(t)\}_{(s)} \\ =& S_a\left\{\frac{AB(\phi)}{1-\phi}\left(f(t) - E_\phi\left(-\mu_\phi t^\phi\right)f(0) - \mu_\phi t^{\phi-1}E_{\phi,\phi}\left(-\mu_\phi t^\phi\right) * f(t)\right)\right\}_{(s)} \\ =& \frac{AB(\phi)}{1-\phi}\Big(S_a\left\{f(t)\right\}_{(s)} - S_a\left\{E_\phi\left(-\mu_\phi t^\phi\right)\right\}_{(s)} f(0) - \mu_\phi S_a\big\{t^{\phi-1} \\ & \times E_{\phi,\phi}\left(-\mu_\phi t^\phi\right) * f(t)\big\}_{(s)}\Big) \\ =& \frac{AB(\phi)}{1-\phi}\left(S_a\left\{f(t)\right\}_{(s)} - \frac{\left(s\log(a)\right)^{\phi-1}}{s^r\left(\left(s\log(a)\right)^{\phi}+\mu_\phi\right)} f(0) - \mu_\phi s^r\right. \\ & \left.\times\frac{1}{s^r\left(\left(s\log(a)\right)^{\phi}+\mu_\phi\right)} S_a\left\{f(t)\right\}_{(s)}\right) \end{aligned} $$

$$
\begin{aligned}
&= \frac{AB(\phi)}{1-\phi}\left(S_a\left\{f(t)\right\}_{(s)}\left(1-\frac{\mu_\phi}{(s\log(a))^\phi+\mu_\phi}\right)-\frac{(s\log(a))^{\phi-1}}{s^r\left((s\log(a))^\phi+\mu_\phi\right)}f(0)\right)\\
&= \frac{AB(\phi)}{1-\phi}\left(S_a\left\{f(t)\right\}_{(s)}\frac{(s\log(a))^\phi}{(s\log(a))^\phi+\mu_\phi}-\frac{(s\log(a))^{\phi-1}}{s^r\left((s\log(a))^\phi+\mu_\phi\right)}f(0)\right)\\
&= S_a\left\{f(t)\right\}_{(s)}\frac{AB(\phi)\,(s\log(a))^\phi}{(s\log(a))^\phi(1-\phi)+\phi}-\frac{AB(\phi)\,(s\log(a))^{\phi-1}}{s^r\left((s\log(a))^\phi(1-\phi)+\phi\right)}f(0), \qquad (3.12)
\end{aligned}
$$

$$
\begin{aligned}
&S_a\{{}^{CPC}_{\ 0}\mathrm{D}_t^\phi f(t)\}_{(s)}\\
&= S_a\left\{k_1(\phi)\left(\frac{t^{-\phi}}{\Gamma(1-\phi)}*f(t)\right)+k_0(\phi)\left(\frac{t^{-\phi}}{\Gamma(1-\phi)}*f'(t)\right)\right\}_{(s)}\\
&= \left\{k_1(\phi)(s\log(a))^{\phi-1}+k_0(\phi)(s\log(a))^\phi\right\}S_a\{f(t)\}_{(s)}-\frac{1}{s^r}(s\log(a))^{\phi-1}\\
&\quad\times k_0(\phi)f(0). \qquad (3.13)
\end{aligned}
$$

We examine the subsequent set of problems:

$$
\begin{aligned}
&Df=\lambda f,\\
&{}^{C}_{0}\mathrm{D}_t^\phi f=\lambda f,\\
&{}^{MCF}_{\ \ \ 0}\mathrm{D}_t^\phi f=\lambda f,\\
&{}^{MABC}_{\ \ \ \ 0}\mathrm{D}_t^\phi f=\lambda f,\\
&{}^{CPC}_{\ \ 0}\mathrm{D}_t^\phi f=\lambda f.
\end{aligned}
$$

We determine the SUM integral transformation of these equations as follows:

$$
\begin{aligned}
&Df=\lambda f,\\
&S_a\{Df\}=S_a\{\lambda f\}_{(s)},\\
&(s\log(a))S_a\{f\}_{(s)}-\frac{f(0)}{s^r}=\lambda S_a\{f\}_{(s)},\\
&S_a\{f\}_{(s)}(s\log(a)-\lambda)=\frac{f(0)}{s^r},\\
&S_a\{f\}_{(s)}=f(0)\frac{1}{s^r}\frac{1}{s\log(a)-\lambda},
\end{aligned}
$$

let $r=0, a=e, f(0)=1, \lambda=1$, then we get:

$$
S_a\{f\}_{(s)}=\frac{1}{s-1}. \qquad (3.14)
$$

We can then plot the SUM integral transform with the classical derivative in Figure 1.

$$
\begin{aligned}
&{}^{C}_{0}\mathrm{D}_t^\phi f=\lambda f,\\
&S_a\{{}^{C}_{0}\mathrm{D}_t^\phi f\}_{(s)}=S_a\{\lambda f\}_{(s)},\\
&(s\log(a))^\phi S_a\{f\}_{(s)}-(s\log(a))^{\phi-1}\frac{f(0)}{s^r}=\lambda S_a\{f\}_{(s)},
\end{aligned}
$$

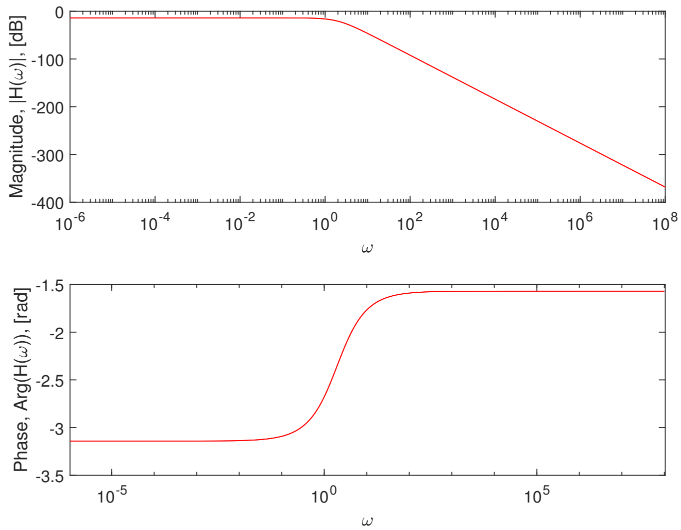

Figure 1. SUM integral transform with the classical derivative

$$S_a\{f\}_{(s)}\left((s\log(a))^{\phi}-\lambda\right)=\frac{1}{s^r}\left(S\log(a)^{\phi-1}f(0)\right),$$
$$S_a\{f\}_{(s)}=\frac{1}{s^r}\frac{(s\log(a))^{\phi-1}}{(s\log(a))^{\phi}-\lambda}f(0),$$

let $r=1, a=e, f(0)=1, \lambda=1, \phi=1$, then we get

$$S_a\{f\}_{(s)}=\frac{1}{s(s-1)}. \tag{3.15}$$

We can then plot the SUM integral transform with the Caputo derivative in Figure 2.

$$\begin{aligned}
&{}^{MCF}_{\ \ 0}\mathrm{D}^{\phi}_{t}f=\lambda f,\\
&S_a\{{}^{MCF}_{\ \ 0}\mathrm{D}^{\phi}_{t}f\}_{(s)}=S_a\{\lambda f\}_{(s)},\\
&\frac{s\log(a)}{s\log(a)(1-\phi)+\phi}S_a\{f\}_{(s)}-\frac{f(0)}{s^r\left(s\log(a)(1-\phi)+\phi\right)}=\lambda S_a\{f\}_{(s)},\\
&S_a\{f\}_{(s)}\left(\frac{s\log(a)}{s\log(a)(1-\phi)+\phi}-\lambda\right)=\frac{f(0)}{s^r\left(s\log(a)(1-\phi)+\phi\right)},\\
&S_a\{f\}_{(s)}\frac{s\log(a)-\lambda\left(s\log(a)(1-\phi)+\phi\right)}{s\log(a)(1-\phi)+\phi}=\frac{f(0)}{s^r\left(s\log(a)(1-\phi)+\phi\right)},
\end{aligned}$$

then,

$$S_a\{f\}_{(s)}=\frac{1}{s^r\left(s\log(a)-\lambda\left(s\log(a)(1-\phi)-\lambda\phi\right)\right)}f(0),$$

let $f(0)=1, r=2, a=e, \lambda=2, \phi=1$, then we get

$$S_a\{f\}_{(s)}=\frac{1}{s^2(s-2)}. \tag{3.16}$$

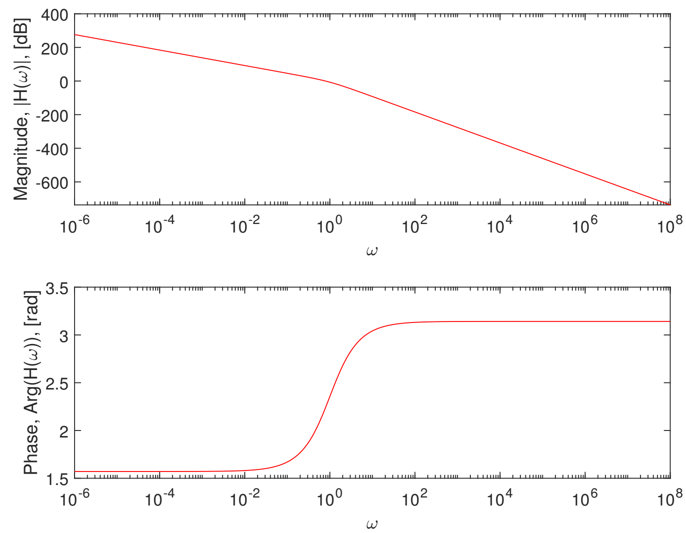

Figure 2. SUM integral transform with the Caputo derivative

We can then plot the SUM integral transform with the Modified Caputo-Fabrizio derivative in Figure 3.

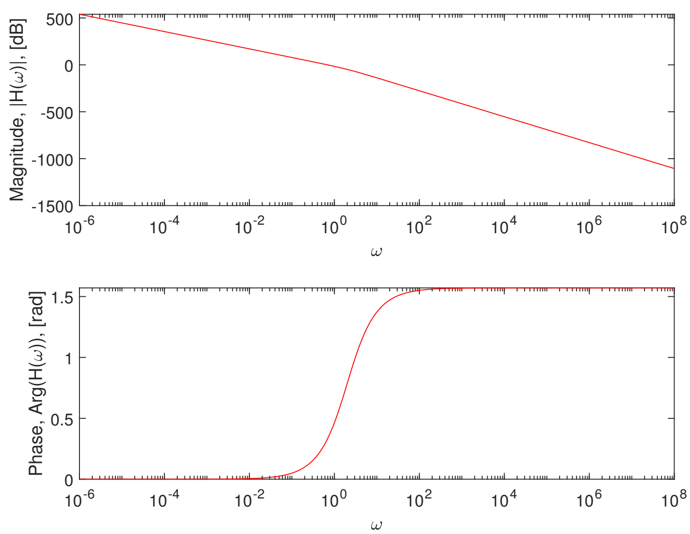

Figure 3. SUM integral transform with the Modified Caputo-Fabrizio derivative

$$
\begin{aligned}
&{}^{MABC}_{\quad 0}\mathrm{D}_t^{\phi} f = \lambda f,\\
&S_a\{{}^{MABC}_{\quad 0}\mathrm{D}_t^{\phi} f\}_{(s)} = S_a\{\lambda f\}_{(s)},
\end{aligned}
$$

$$S_a\left\{f(t)\right\}_{(s)} \frac{AB(\phi)\left(s\log(a)\right)^{\phi}}{\left(s\log(a)\right)^{\phi}(1-\phi)+\phi} - \frac{AB(\phi)\left(s\log(a)\right)^{\phi-1}}{s^r\left(\left(s\log(a)\right)^{\phi}(1-\phi)+\phi\right)} f(0)$$

$$= \lambda S_a\{f\}_{(s)},$$

$$S_a\{f\}_{(s)}\left(\frac{AB(\phi)\left(s\log(a)\right)^{\phi}}{\left(s\log(a)\right)^{\phi}(1-\phi)+\phi} - \lambda\right) = \frac{AB(\phi)\left(s\log(a)\right)^{\phi-1}}{s^r\left(\left(s\log(a)\right)^{\phi}(1-\phi)+\phi\right)} f(0),$$

$$S_a\{f\}_{(s)} \frac{AB(\phi)\left(s\log(a)\right)^{\phi} - \lambda\left(\left(s\log(a)\right)^{\phi}(1-\phi)+\phi\right)}{\left(s\log(a)\right)^{\phi}(1-\phi)+\phi}$$

$$= \frac{AB(\phi)\left(s\log(a)\right)^{\phi-1}}{s^r\left(\left(s\log(a)\right)^{\phi}(1-\phi)+\phi\right)} f(0),$$

$$S_a\{f\}_{(s)} = \frac{AB(\phi)\left(s\log(a)\right)^{\phi-1} f(0)}{s^r\left(AB(\phi)\left(s\log(a)\right)^{\phi} - \lambda\left(\left(s\log(a)\right)^{\phi}(1-\phi)+\phi\right)\right)},$$

let $r = 0, a = e, f(0) = 1, \lambda = 1$, and $\phi = \frac{1}{2}$, we get

$$S_a\{f\}_{(s)} = \frac{1+\sqrt{\pi}}{s-\sqrt{\pi}\sqrt{s}}. \tag{3.17}$$

We can then plot the SUM integral transform with the Modified Atangana-Baleanu derivative in Figure 4.

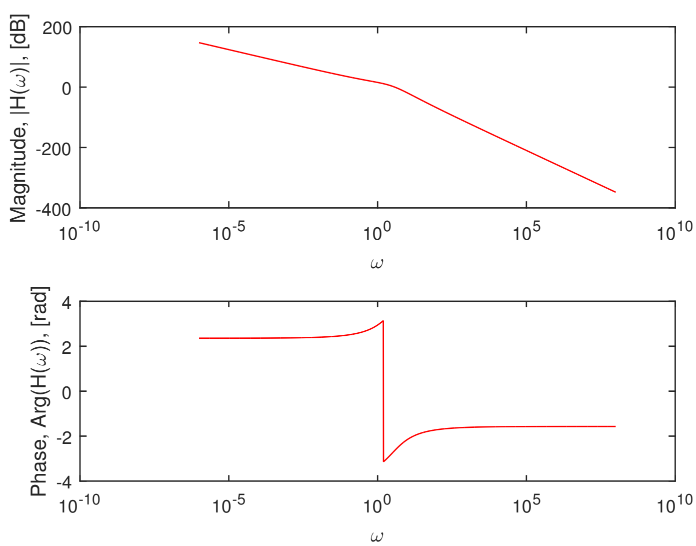

Figure 4. SUM integral transform with the Modified Atangana-Baleanu derivative

$${}^{CPC}_{\ \ 0}\mathrm{D}_t^{\phi} f = \lambda f,$$

$$S_a\{{}^{CPC}_{\ \ 0}\mathrm{D}_t^{\phi} f\}_{(s)} = S_a\{\lambda f\}_{(s)},$$

$$S_a\{f\}_{(s)}\{k_1(\phi)(s\log(a))^{\phi-1}+k_0(\phi)(s\log(a))^{\phi}\}-\frac{1}{s^r}((s\log(a))^{\phi-1})k_0(\phi)f(0)$$

$$=\lambda S_a\{f\}_{(s)},$$

$$S_a\{f\}_{(s)}\left(k_1(\phi)(s\log(a))^{\phi-1}+k_0(\phi)(s\log(a))^{\phi}-\lambda\right)$$

$$=\frac{1}{s^r}((s\log(a))^{\phi-1})k_0(\phi)f(0),$$

$$S_a\{f\}_{(s)}=\frac{1}{s^2}\frac{((s\log(a))^{\phi-1})k_0(\phi)f(0)}{k_1(\phi)(s\log(a))^{\phi-1}+k_0(\phi)(s\log(a))^{\phi}-\lambda},$$

let $a=e, r=0, \lambda=1, f(0)=1, \phi=\frac{1}{2}, k_1(\phi)=1, k_0(\phi)=1$, then we have

$$\begin{aligned} S_a\{f\}_{(s)} =&\frac{s^{\phi-1}k_0(\phi)}{k_1(\phi)s^{\phi-1}+k_0(\phi)s^{\phi}-1},\\ =&\frac{s^{\phi-1}}{s^{\phi-1}+s^{\phi}-1},\\ =&\frac{s^{-1/2}}{s^{-1/2}+s^{1/2}-1},\\ =&\frac{1}{s-\sqrt{s}+1}. \end{aligned} \tag{3.18}$$

We can then plot the SUM integral transform with the Constant Proportional Caputo derivative in Figure 5.

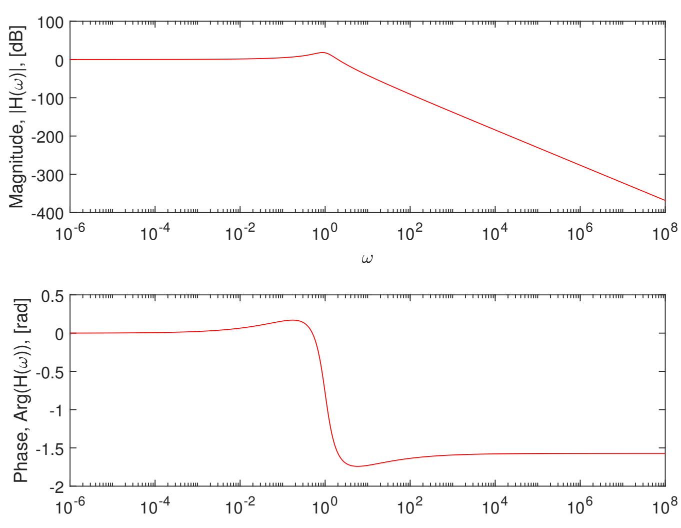

Figure 5. SUM integral transform with the Constant Proportional Caputo derivative

4. Transfer function

Definition 4.1. The transfer function $H(s)$ is defined for a continuous-time input signal $x(t)$ and output $y(t)$ as [6]:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{L(y(t))}{L(x(t))}. \tag{4.1}$$

In signal processing, communication theory, and control theory, transfer functions are essential to the understanding of systems like single-input single-output filters. These operations are basic instruments that come from classical control engineering. We provide the decay differential equation's transfer function in this context, which includes four different differential operators: the classical derivative (shown in Figure 6), Caputo fractional differentiation (shown in Figure 7), Modified Caputo-Fabrizio (shown in Figure 8), Modified Atangana-Baleanu fractional operators (shown in Figure 9), and conventional differentiation (shown in Figure 10). For the following equations, we choose $f(0) = 0$ as the starting condition. As such, we extract the transfer functions using the SUM integral transformations.

$$\begin{aligned}
&S_a\{Df\}_{(s)} = S_a\{g\}_{(s)},\\
&(s\log(a))S_a\{f\}_{(s)} - \frac{f(0)}{s^r} = S_a\{g\}_{(s)},\\
&(s\log(a))S_a\{f\}_{(s)} = S_a\{g\}_{(s)},\\
&\frac{S_a\{g\}_{(s)}}{S_a\{f\}_{(s)}} = (s\log(a)),
\end{aligned}$$

$a = e$, then we get

$$\frac{S_a\{g\}_{(s)}}{S_a\{f\}_{(s)}} = s, \tag{4.2}$$

$$\begin{aligned}
&S_a\{{}^{C}_{0}\mathrm{D}^{\phi}_{t} f\}_{(s)} = S_a\{g\}_{(s)},\\
&S_a\{f\}_{(s)}(s\log(a))^{\phi}(s\log(a))^{\phi-1}\frac{f(0)}{s^r} = S_a\{g\}_{(s)},\\
&\frac{S_a\{g\}_{(s)}}{S_a\{f\}_{(s)}} = (s\log(a))^{\phi},
\end{aligned}$$

let $\phi = 0.5, a = e$ we get

$$\frac{S_a\{g\}_{(s)}}{S_a\{f\}_{(s)}} = \sqrt{s}, \tag{4.3}$$

$$\begin{aligned}
&S_a\{{}^{MCF}_{0}\mathrm{D}^{\phi}_{t} f\}_{(s)} = S_a\{g\}_{(s)},\\
&\frac{s\log(a)}{s\log(a)(1-\phi)+\phi}S_a\{f\}_{(s)} - \frac{f(0)}{s^r\,(s\log(a)(1-\phi)+\phi)} = S_a\{g\}_{(s)},\\
&\frac{S_a\{g\}_{(s)}}{S_a\{f\}_{(s)}} = \frac{s\log(a)}{s\log(a)(1-\phi)+\phi},
\end{aligned}$$

let $\phi = 1$ and $a = e$, we get

$$\frac{S_a\{g\}_{(s)}}{S_a\{f\}_{(s)}} = s, \tag{4.4}$$

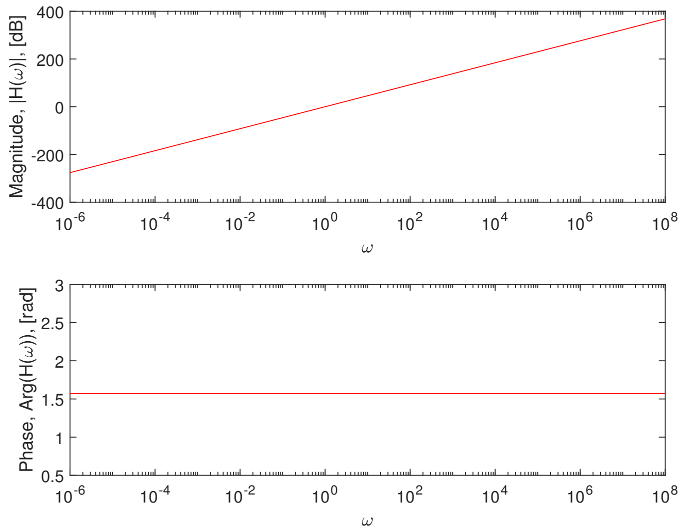

Figure 6. Transfer function for $a = e$

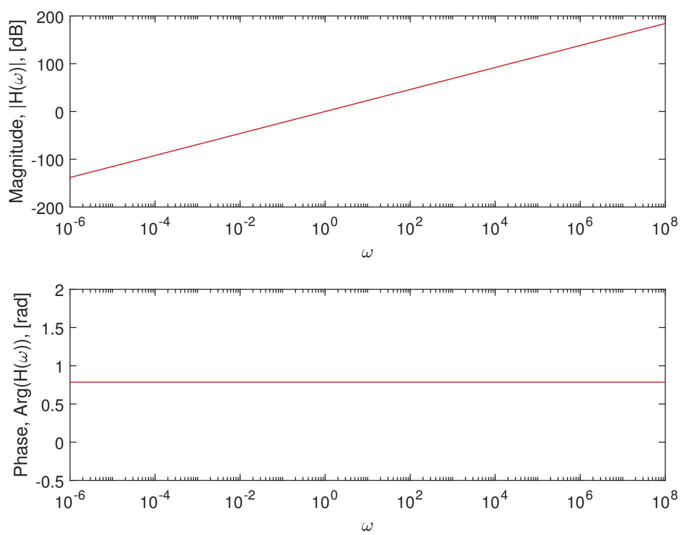

Figure 7. Transfer function for $\phi = 0.5, a = e$

$$S_a\{{}^{MABC}{}_0\mathrm{D}_t^{\phi} f\}_{(s)} = S_a\{g\}_{(s)},$$

$$S_a\left\{f(t)\right\}_{(s)} \frac{AB(\phi)\left(s\log(a)\right)^{\phi}}{\left(s\log(a)\right)^{\phi}(1-\phi)+\phi} - \frac{AB(\phi)\left(s\log(a)\right)^{\phi-1}}{s^r\left(\left(s\log(a)\right)^{\phi}(1-\phi)+\phi\right)} f(0)$$

$$= S_a\{g\}_{(s)},$$

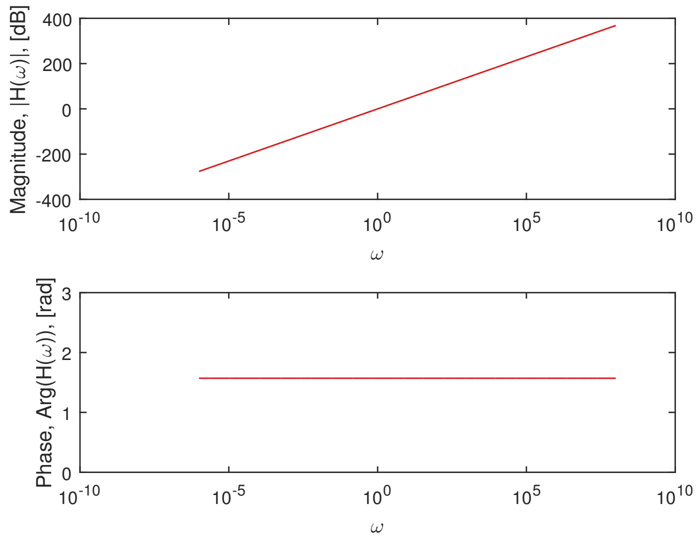

Figure 8. Transfer function (4.4) for $\phi = 1$ and $a = e$

$$\frac{S_a\{g\}_{(s)}}{S_a\{f\}_{(s)}} = \frac{AB(\phi)(s\log(a))^{\phi}}{(s\log(a))^{\phi}(1-\phi)+\phi},$$

let $a = e, f(0) = 1$, and $\phi = \frac{1}{2}$, we get

$$\frac{S_a\{g\}_{(s)}}{S_a\{f\}_{(s)}} = \frac{\sqrt{s}\,(1+\sqrt{\pi})}{\sqrt{\pi}\,(1+\sqrt{s})}, \tag{4.5}$$

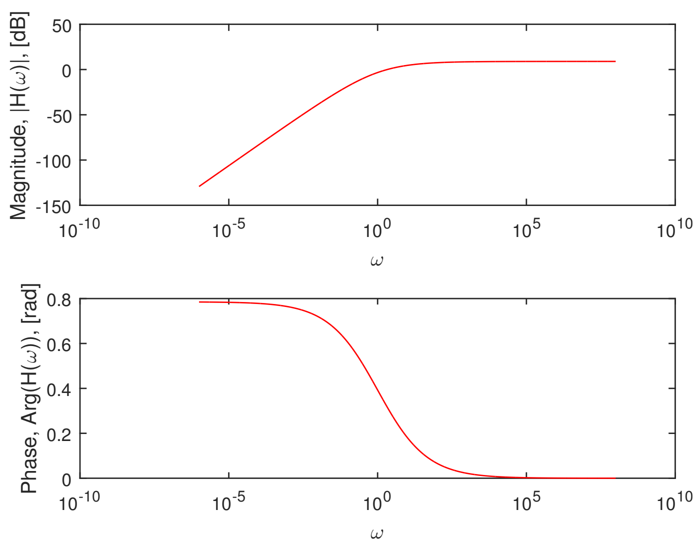

Figure 9. Transfer function (4.5) for $a = e, f(0) = 1$, and $\phi = \frac{1}{2}$

$$S_a\{{}^{CPC}_{\ \ \ 0}\mathrm{D}^{\phi}_{t} f\}_{(s)} = S_a\{g\}_{(s)},$$

$$S_a\{f\}_{(s)}\{k_1(\phi)(s\log(a))^{\phi-1}+k_0(\phi)(s\log(a))^{\phi}\}-\frac{1}{s^r}((s\log(a))^{\phi-1})k_0(\phi)f(0)$$
$$=S_a\{g\}_{(s)},$$
$$S_a\{f\}_{(s)}\left(k_1(\phi)(s\log(a))^{\phi-1}+k_0(\phi)(s\log(a))^{\phi}\right)=S_a\{f\}_{(s)},$$
$$\frac{S_a\{g\}_{(s)}}{S_a\{f\}_{(s)}}=(k_1(\phi)(s\log(a))^{\phi-1}+k_0(\phi)(s\log(a))^{\phi}, \tag{4.6}$$

let $a=e, f(0)=1, \phi=\frac{1}{2}, k_1(\phi)=1, k_0(\phi)=1$, then we have

$$\frac{S_a\{g\}_{(s)}}{S_a\{f\}_{(s)}}=\frac{s+1}{\sqrt{s}}. \tag{4.7}$$

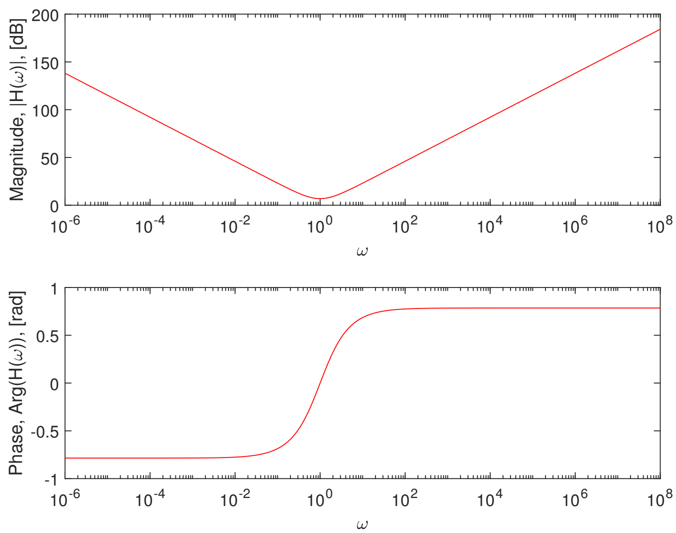

Figure 10. Transfer function for $a=e, f(0)=1, \phi=\frac{1}{2}, k_1(\phi)=1, k_0(\phi)=1$

5. Pole analysis

Pole analysis is a method used in various fields such as engineering, control systems, and mathematics to understand the behavior of complex systems, especially systems described by equations or functions. It involves analyzing the polarities of a system to understand its stability, dynamics and performance characteristics. In engineering, especially in control systems, poles represent the roots of the denominator polynomial of a system's transfer function. The transfer function describes the relationship between the input and output of a system. Poles are very important as they determine the stability and response of the system [6].

Let's start by addressing the following problem as our initial experiment [6]:

$$\frac{dV}{dt}=R\frac{di}{dt}+\frac{1}{C}i. \tag{5.1}$$

The transfer function for this specific problem is obtained through SUM integral transforms in the following manner, as depicted in Figure 11:

$$S_a\left\{\frac{dV}{dt}\right\} = S_a\left\{R\frac{di}{dt}\right\} + S_a\left\{\frac{1}{C}i\right\}, \tag{5.2}$$

$$\begin{aligned}
(s\log(a))S_a\{V\}_{(s)} &= R(s\log(a))S_a\{i\}_{(s)} + \frac{1}{C}S_a\{i\}_{(s)},\\
(s\log(a))S_a\{V\}_{(s)} &= S_a\{i\}_{(s)}\left(R\,(s\log(a)) + \frac{1}{C}\right),\\
\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}} &= \frac{s\log(a)}{R(s\log(a)) + \frac{1}{C}}\\
&= \frac{s\log(a)C}{RC(s\log(a)) + 1},
\end{aligned}$$

let $a = e, c = 3, R = 2$, we get

$$\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}} = \frac{3s}{6s+1}. \tag{5.3}$$

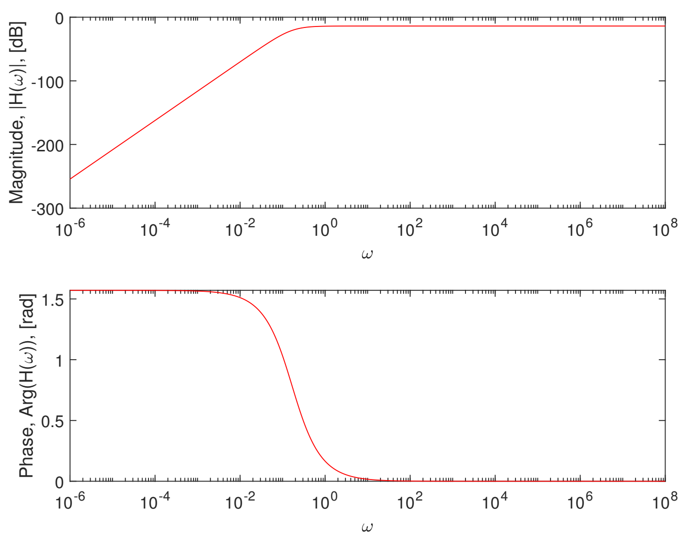

Figure 11. Transfer function for $a = e, c = 3, R = 2$

Let's explore the following problem as our second experiment [6]:

$$\frac{dE}{dt} = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i. \tag{5.4}$$

Upon performing the SUM integral transformation on both sides of the aforementioned equation, the result is, as illustrated in Figure 12:

$$S_a\left\{\frac{dE}{dt}\right\}_{(s)} = S_a\left\{R\frac{di}{dt}\right\}_{(s)} + S_a\left\{L\frac{d^2i}{dt^2}\right\}_{(s)} + S_a\left\{i\frac{1}{C}\right\}_{(s)}, \tag{5.5}$$

$$s\log(a)S_a\{E\}_{(s)} = Rs\log(a)S_a\{i\}_{(s)} + L(s\log(a))^2 S_a\{i\}_{(s)} + \frac{1}{C}S_a\{i\}_{(s)},$$

$$s\log(a)S_a\{E\}_s = S_a\{i\}_{(s)}(Rs\log(a) + L(s\log(a))^2 + \frac{1}{C}),$$

$$\frac{S_a\{i\}_{(s)}}{S_a\{E\}_{(s)}} = \frac{s\log(a)}{Rs\log(a) + L(s\log(a))^2 + \frac{1}{C}},$$

$$\frac{S_a\{i\}_{(s)}}{S_a\{E\}_{(s)}} = \frac{s\log(a)C}{RCs\log(a) + LC(s\log(a))^2 + 1},$$

let $a = e, C = 2, R = 1, L = 1$, we get

$$\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}} = \frac{3s}{6s + s^2 + 1}. \tag{5.6}$$

Let's look at the following problem as our third experiment [6]:

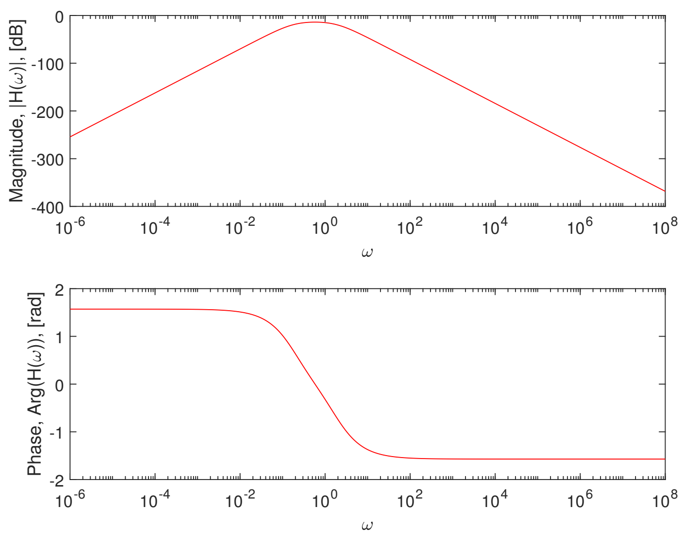

Figure 12. Transfer function for $a = e, C = 2, R = 1, L = 1$

$${}_0^{MABC}D_t^{\phi}V = R\frac{di}{dt} + \frac{1}{C}i, \tag{5.7}$$

$$S_a\left\{{}_0^{MABC}D_t^{\phi}V\right\}_{(s)} = S_a\left\{R\frac{di}{dt}\right\}_{(s)} + S_a\left\{\frac{1}{C}i\right\}_{(s)},$$

$$S_a\left\{V\right\}_{(s)}\frac{AB(\phi)\left(s\log(a)\right)^{\phi}}{\left(s\log(a)\right)^{\phi}(1-\phi)+\phi} - \frac{AB(\phi)\left(s\log(a)\right)^{\phi-1}}{s^r\left(\left(s\log(a)\right)^{\phi}(1-\phi)+\phi\right)}V(0)$$

$$= R\left((s\log(a))S_a\{i\}_{(s)} - \frac{i(0)}{s^r}\right) + \frac{1}{C}S_a\{i\}_{(s)},$$

$$S_a\left\{V\right\}_{(s)}\frac{AB(\phi)\left(s\log(a)\right)^{\phi}}{\left(s\log(a)\right)^{\phi}\left(1-\phi\right)+\phi}=\left(R(s\log(a))+\frac{1}{C}\right)S_a\{i\}_{(s)},$$

$$\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}}=\frac{AB(\phi)(s\log(a))^{\phi}C}{\left((s\log(a))^{\phi}(1-\phi)+\phi\right)\left(RC(s\log(a))+1\right)},$$

let $a=e, C=2, R=2$, and $\phi=1$, then we get

$$\frac{S_a\{i\}_{(s)}}{Sa\{V\}_{(s)}}=\frac{2s}{4s+1}. \tag{5.8}$$

We can then plot the transfer function in Figure 13. Let's examine the following

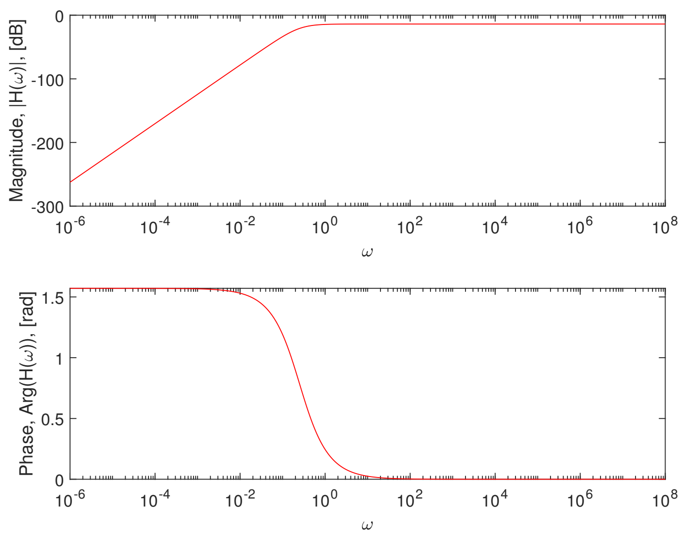

Figure 13. Transfer function (5.8) for $a=e, C=2, R=2$, and $\phi=1$

problem as our fourth experiment [6]:

$$\frac{dV}{dt}=R\left({}_0^{MABC}D_t^{\phi}i\right)+\frac{1}{C}i,$$

$$S_a\left\{\frac{dV}{dt}\right\}_{(s)}=S_a\left\{R\left({}_0^{MABC}D_t^{\phi}i\right)\right\}_{(s)}+S_a\left\{\frac{1}{C}i\right\}_{(s)},$$

$$\left(s\log(a)\right)S_a\left\{V\right\}_{(s)}-\frac{V(0)}{s^r}$$

$$=R\left(S_a\left\{i\right\}_{(s)}\frac{AB(\phi)\left(s\log(a)\right)^{\phi}}{\left(s\log(a)\right)^{\phi}\left(1-\phi\right)+\phi}-\frac{AB(\phi)\left(s\log(a)\right)^{\phi-1}}{s^r\left(\left(s\log(a)\right)^{\phi}\left(1-\phi\right)+\phi\right)}i(0)\right)$$

$$+\frac{1}{C}S_a\left\{i\right\}_{(s)},$$

$$(s\log(a))\, S_a\{V\}_{(s)} = S_a\{i\}_{(s)}\, \frac{AB(\phi)\,(s\log(a))^{\phi}\, RC + (s\log(a))^{\phi}\,(1-\phi)+\phi}{C\left((s\log(a))^{\phi}\,(1-\phi)+\phi\right)},$$

$$\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}} = \frac{(s\log(a))\, C\left((s\log(a))^{\phi}(1-\phi)+\phi\right)}{AB(\phi)(s\log(a))^{\phi}RC + (s\log(a))^{\phi}(1-\phi)+\phi},$$

let $C=1, a=e, R=3$, and $\phi=1$, then we get

$$\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}} = \frac{s}{3s+1}. \tag{5.9}$$

We can then plot the transfer function in Figure 14. Let's take the following problem

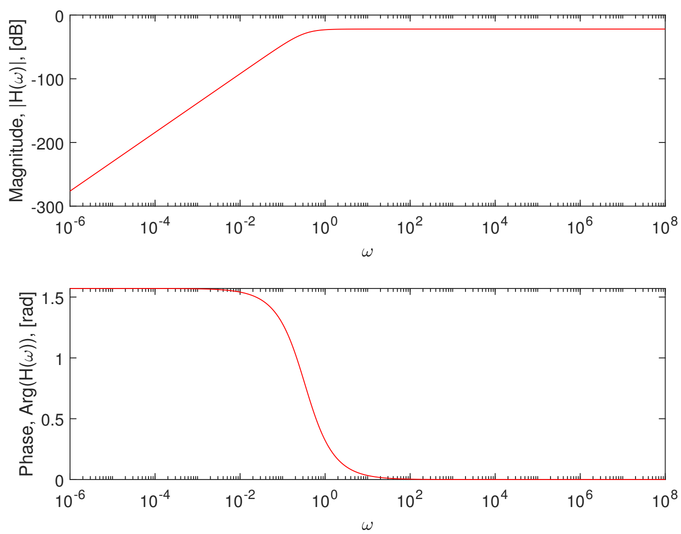

Figure 14. Transfer function (5.9) for $C=1, a=e, R=3$, and $\phi=1$

as our fifth experiment [6]:

$${}_{0}^{MABC}D_t^{\phi}V = R\left({}_{0}^{MABC}D_t^{\phi}i\right) + \frac{1}{C}i,$$

$$S_a\left\{{}_{0}^{MABC}D_t^{\phi}V\right\}_{(s)} = S_a\left\{R\frac{di}{dt}\right\}_{(s)} + S_a\left\{\frac{1}{C}i\right\}_{(s)},$$

$$S_a\{V\}_{(s)}\,\frac{AB(\phi)\,(s\log(a))^{\phi}}{(s\log(a))^{\phi}\,(1-\phi)+\phi} = RS_a\{i\}_{(s)}\,\frac{AB(\phi)\,(s\log(a))^{\phi}}{(s\log(a))^{\phi}\,(1-\phi)+\phi} + \frac{1}{C}S_a\{i\}_{(s)},$$

$$S_a\{V\}_{(s)}\,\frac{AB(\phi)\,(s\log(a))^{\phi}}{(s\log(a))^{\phi}\,(1-\phi)+\phi} = S_a\{i\}_{(s)}\left(\frac{AB(\phi)\,(s\log(a))^{\phi}\,R}{(s\log(a))^{\phi}\,(1-\phi)+\phi} + \frac{1}{C}\right),$$

$$S_a\{V\}_{(s)}\,\frac{AB(\phi)\,(s\log(a))^{\phi}}{(s\log(a))^{\phi}\,(1-\phi)+\phi}$$

$$= S_a\{i\}_{(s)} \frac{AB(\phi)(s\log(a))^{\phi}RC + (s\log(a))^{\phi}(1-\phi)+\phi}{C\left((s\log(a))^{\phi}(1-\phi)+\phi\right)},$$

$$\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}} = \frac{AB(\phi)(s\log(a))^{\phi}C}{AB(\phi)(s\log(a))^{\phi}RC + (s\log(a))^{\phi}(1-\phi)+\phi},$$

let $C = 4, a = e, R = 3$, and $\phi = 1$, then we get

$$\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}} = \frac{4s}{12s+1}. \tag{5.10}$$

We can then plot the transfer function in Figure 15. Let's explore the subsequent

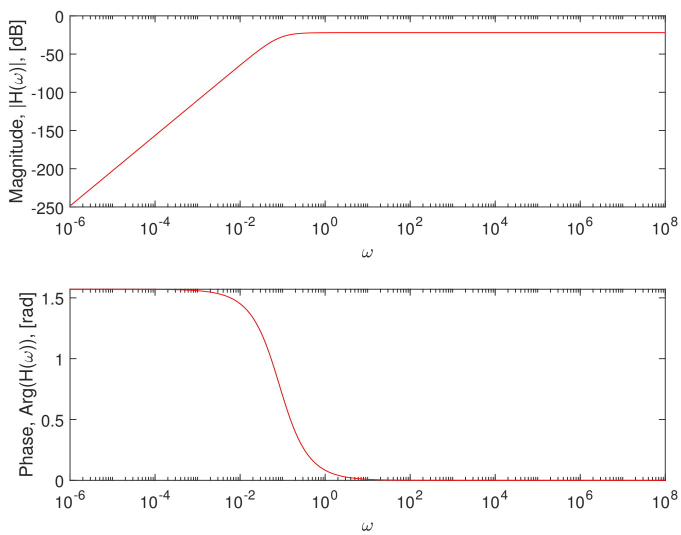

Figure 15. Transfer function (5.10) for $C = 4, a = e, R = 3$, and $\phi = 1$

problem as our sixth experiment [6]:

$${}_{0}^{CPC}D_t^{\phi}V = R\frac{di}{dt} + \frac{1}{C}i,$$

$$S_a\left\{{}_{0}^{CPC}D_t^{\phi}V\right\}_{(s)} = S_a\left\{R\frac{di}{dt}\right\}_{(s)} + S_a\left\{\frac{1}{C}i\right\}_{(s)},$$

$$\left[k_1(\phi)(s\log(a))^{\phi-1} + k_0(\phi)(s\log(a))^{\phi}\right] S_a\{V\}_{(s)}$$

$$= R(s\log(a))S_a\{i\}_{(s)} + \frac{1}{C}S_a\{i\}_{(s)},$$

$$S_a\{V\}_{(s)}\left[k_1(\phi)(s\log(a))^{\phi-1} + k_0(\phi)(s\log(a))^{\phi}\right] = \left(Rs\log(a) + \frac{1}{C}\right)S_a\{i\}_{(s)},$$

$$\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}} = \frac{C\left[k_1(\phi)(s\log(a))^{\phi-1} + k_0(\phi)(s\log(a))^{\phi}\right]}{RCs\log(a)+1},$$

let $C = 4, a = e, R = 3, \phi = 2, k_1(\phi) = 1, k_0(\phi) = 1$, we get

$$\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}} = \frac{4s + 4s^2}{12s + 1}. \tag{5.11}$$

We can then plot the transfer function in Figure 16. Let's approach the following

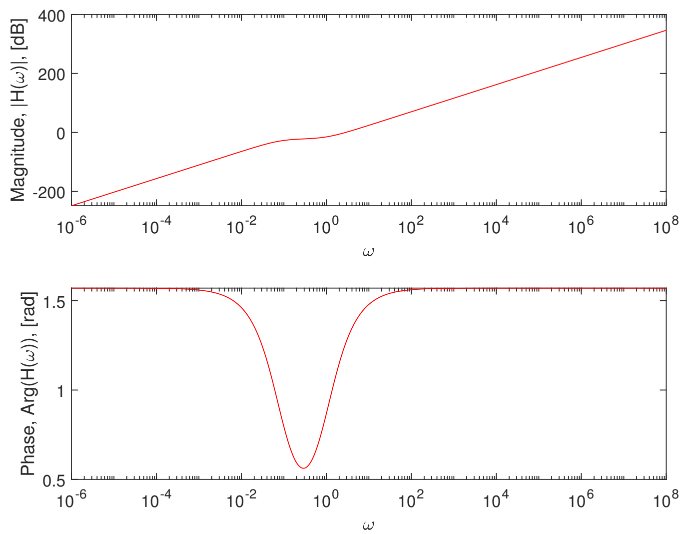

Figure 16. Transfer function for $C = 4, a = e, R = 3, \phi = 2, k_1(\phi) = 1, k_0(\phi) = 1$

problem as our seventh experiment [6]:

$$\frac{dV}{dt} = R\left({}_0^{CPC}D_t^{\phi} i\right) + \frac{1}{C}i,$$

$$S_a\left\{\frac{dV}{dt}\right\}_{(s)} = S_a\left\{R\left({}_0^{CPC}D_t^{\phi} i\right)\right\}_{(s)} + S_a\left\{\frac{1}{C}i\right\}_{(s)},$$

$$s\log(a)S_a\{V\}_{(s)} = RS_a\{i\}_{(s)}\left[k_1(\phi)(s\log(a))^{\phi-1} + k_0(\phi)(s\log(a))^{\phi}\right] + \frac{1}{C}S_a\{i\}_{(s)},$$

$$s\log(a)S_a\{V\}_{(s)} = RS_a\{i\}_{(s)}\left[k_1(\phi)(s\log(a))^{\phi-1} + k_0(\phi)(s\log(a))^{\phi} + \frac{1}{C}\right],$$

$$\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}} = \frac{Cs\log(a)}{RC\left[k_1(\phi)(s\log(a))^{\phi-1} + k_0(\phi)(s\log(a))^{\phi} + 1\right]},$$

let $C = 3, a = e, R = 2, \phi = 3, k_1(\phi) = 1, k_0(\phi) = 1$, we get

$$\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}} = \frac{s}{2s^3 + 2s^2 + 2}. \tag{5.12}$$

We can then plot the transfer function in Figure 17. Let's examine the following

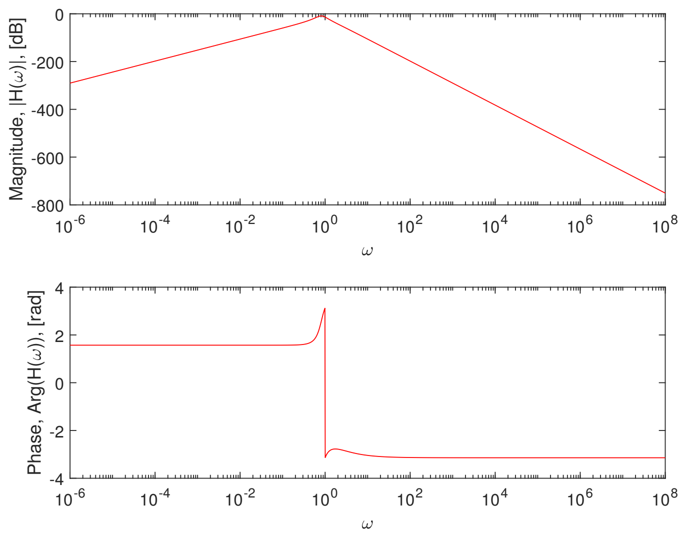

Figure 17. Transfer function for $C = 3, a = e, R = 2, \phi = 3, k_1(\phi) = 1, k_0(\phi) = 1$

problem as our eighth experiment [6]:

$$
\begin{aligned}
&{}_0^{CPC}D_t^{\phi}V = R_0^{CPC}D_t^{\phi}i + \frac{1}{C}i,\\
&S_a\left\{{}_0^{CPC}D_t^{\phi}V\right\}_{(s)} = S_a\left\{R\left({}_0^{CPC}D_t^{\phi}i\right)\right\}_{(s)} + S_a\left\{\frac{1}{C}i_{(s)}\right\},\\
&\quad S_a\{V\}_{(s)}\left[k_1(\phi)(s\log(a))^{\phi-1} + k_0(\phi)(s\log(a))^{\phi}\right]\\
&= R\left[k_1(\phi)(s\log(a))^{\phi-1} + k_0(s\log(a))^{\phi}\right]S_a\{i\}_{(s)} + \frac{1}{C}S_a\{i\}_{(s)},\\
&\quad S_a\{V\}_{(s)}\left[k_1(\phi)(s\log(a))^{\phi-1} + k_0(\phi)(s\log(a))^{\phi}\right]\\
&= R\left[k_1(\phi)(s\log(a))^{\phi-1} + k_0(s\log(a))^{\phi} + \frac{1}{C}\right]S_a\{i\}_{(s)},\\
&\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}} = \frac{C\left[k_1(\phi)(s\log(a))^{\phi-1} + k_0(\phi)(s\log(a))^{\phi}\right]}{RC\left[k_1(\phi)(s\log(a))^{\phi-1} + k_0(\phi)(s\log(a))^{\phi} + 1\right]},
\end{aligned}
$$

let $C = 3, a = e, R = 2, \phi = 2, k_1(\phi) = 1, k_0(\phi) = 1$, we get

$$\frac{S_a\{i\}_{(s)}}{S_a\{V\}_{(s)}} = \frac{s^2+s}{s^2+s+1}. \tag{5.13}$$

We can then plot the transfer function in Figure 18.

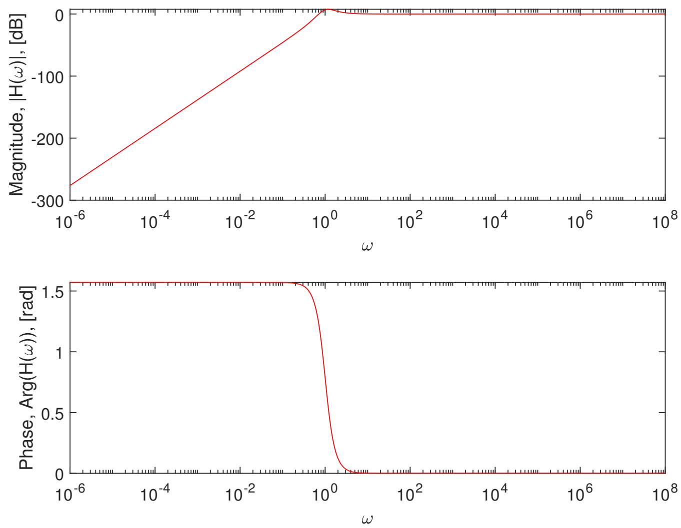

Figure 18. Transfer function for $C = 3, a = e, R = 2, \phi = 2, k_1(\phi) = 1, k_0(\phi) = 1$

6. Conclusion

In this paper, we investigate a new integral transform. We obtained many new transfer functions with this integral transform. We use many different kernels to get effective transfer functions. We demonstrate the simulations of the transfer functions by some figures.

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