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# IMPROVED CONVERGENCE THEOREM FOR THE GENERAL MODULUS-BASED MATRIX SPLITTING METHOD

Yali Liu[1], Shiliang Wu[2,3] and Cuixia Li[2,†]

**Abstract** In this note, based on the published work by Li [A general modulus-based matrix splitting method for linear complementarity problems of H-matrices, Appl. Math. Lett. 26 (2013) 1159-1164], we further study the convergence property of the general modulus-based matrix splitting (GMMS) method for linear complementarity problems. A new sufficient condition of the GMMS method is obtained, which is weaker than the result in the above work.

**Keywords** Linear complementarity problems, GMMS method, convergence.

**MSC(2010)** 65F10, 90C33.

## 1. Introduction

The linear complementarity problems (denoted by LCP$(q, A)$) is that we need to find that $z \in \mathbb{R}^n$ satisfies

$$w := Az + q \geq 0,\ z \geq 0 \text{ and } z^T w = 0, \tag{1.1}$$

where $A \in \mathbb{R}^{n\times n}$ and $q \in \mathbb{R}^n$ are given in [4, 8]. Li in [7] presented a general modulus-based matrix splitting (GMMS) method for solving the LCP$(q, A)$ with matrix $A$ being an $H$-matrix. Essentially, the idea of the GMMS method is to introduce two positive diagonal matrices for the equivalent absolute value equation of the LCP$(q, A)$. Concretely, using

$$z = \Omega_1(|x| + x) \text{ and } w = \Omega_2(|x| - x), \tag{1.2}$$

where $|\cdot|$ denotes the absolute value, the LCP$(q, A)$ can be equivalently transformed into the following absolute value equation

$$(\Omega_2 + M_{\Omega_1})x = N_{\Omega_1}x + (\Omega_2 - A\Omega_1)|x| - q, \tag{1.3}$$

where $A\Omega_1 = M_{\Omega_1} - N_{\Omega_1}$ with $\det(M_{\Omega_1}) \neq 0$ is a matrix splitting of matrix $A\Omega_1$, $\Omega_1$ and $\Omega_2$ are two positive diagonal matrices.

†The corresponding author.
[1]Faculty of Information Engineering, The College of Arts and Sciences·Kunming, Kunming, Yunnan 650222, China
[2]School of Mathematics, Yunnan Normal University, Kunming, Yunnan 650500, China
[3]Yunnan Key Laboratory of Modern Analytical Mathematics and Applications, Yunnan Normal University, Kunming, Yunnan 650500, China
Email: liuyali80818@126.com(Y. Liu), wushiliang1999@126.com(S. Wu), lixiatkynu@126.com(C. Li)

Based on (1.3), the GMMS method works below.

**Method 1.1.** [7] Let $\Omega_1$ and $\Omega_2$ be given positive diagonal matrices. Then for any initial vector $x^{(0)} \in \mathbb{R}^n$, calculate $x^{(k+1)}$ by

$$(\Omega_2 + M_{\Omega_1})x^{(k+1)} = N_{\Omega_1}x^{(k)} + (\Omega_2 - A\Omega_1)|x^{(k)}| - q, \text{ for } k = 0, 1, 2, \dots, \tag{1.4}$$

where $A\Omega_1 = M_{\Omega_1} - N_{\Omega_1}$ is a matrix splitting of matrix $A\Omega_1$. Then set

$$z^{(k+1)} = \Omega_1(|x^{(k+1)}| + x^{(k+1)})$$

until the iteration sequence $\{z^{(k)}\}_{k=1}^{+\infty} \subset \mathbb{R}^n$ is convergent.

Let $A = M - N$. For $\gamma > 0$, if we take

$$\Omega_1 = \frac{1}{\gamma}I, \Omega_2 = \frac{1}{\gamma}\Omega, M_{\Omega_1} = \frac{1}{\gamma}M, N_{\Omega_1} = \frac{1}{\gamma}N,$$

then the GMMS method reduces to the modulus-based matrix splitting (MMS) method, see Method 3.1 in [1].

For later discussion, some necessary concepts, notations and lemmas are reminded. Let $A = (a_{ij}) \in \mathbb{R}^{n\times n}$. It is called as a $Z$-matrix if $a_{ij} \le 0$ for $i \ne j$; a nonsingular $M$-matrix if $A$ is a $Z$-matrix and $A^{-1} \ge 0$; an $H$-matrix if its comparison matrix $\langle A\rangle = (\langle a\rangle_{ij}) \in \mathbb{R}^{n\times n}$ ($\langle a\rangle_{ii} = |a_{ii}|$ and $\langle a\rangle_{ij} = -|a_{ij}|$ for $i \ne j$) is a nonsingular $M$-matrix; a strictly diagonally dominant (SDD) (by rows) matrix if

$$|a_{ii}| > \sum_{j\ne i} |a_{ij}|, i = 1, 2, \dots, n,$$

see [3]. In addition, an $H$-matrix with positive diagonal is called an $H_+$-matrix in [1]. If $A \le B$ with $A$ being an $M$-matrix and $B$ being a $Z$-matrix, then $B$ is an $M$-matrix [3]. The matrix splitting, $A = M - N$, of $A \in \mathbb{R}^{n\times n}$ is called as an $H$-splitting if $\langle M\rangle - |N|$ is a nonsingular $M$-matrix with $|N| = (|n_{ij}|)$. $\rho(A)$, $\|A\|_\infty$ and $D_A$ denote the spectral radius, the infinite norm and the diagonal part of the matrix $A$, respectively. It is well known that $\mathrm{LCP}(q, A)$ has a unique solution if $A$ is an $H_+$-matrix.

**Lemma 1.1.** [2] *Let $A \in \mathbb{R}^{n\times n}$ with $A \ge 0$. If there exists $u \in \mathbb{R}^n$ with $u > 0$ such that $Au < u$, then $\rho(A) < 1$.*

**Lemma 1.2.** [5] *Let $A \in \mathbb{R}^{n\times n}$ be an $H$-matrix, and $B = D_A - A$. Then $|A^{-1}| \le \langle A\rangle^{-1}$ and $\rho(|D_A|^{-1}|B|) < 1$.*

For the convergence of the GMMS method, Li in [7] gave the following main result.

**Theorem 1.1.** [7] *Let $A\Omega_1 = M_{\Omega_1} - N_{\Omega_1}$ be an $H$-splitting of the $H_+$-matrix $A$, and $\Omega_1$ and $\Omega_2$ be two positive diagonal matrices. If*

$$\Omega_2 e > D_A\Omega_1 e - V^{-1}(\langle M_{\Omega_1}\rangle - |N_{\Omega_1}|)Ve \tag{1.5}$$

*for any positive diagonal matrix $V$ such that $(\langle M_{\Omega_1}\rangle - |N_{\Omega_1}|)V$ is an SDD matrix, where $e = (1, 1, \dots, 1)^T$, then Method 1.1 is convergent for any initial guess $x^{(0)} \in \mathbb{R}^n$.*

The purpose of this paper is to establish a new sufficient condition for convergence of the GMMS method, which is superior to those previously published works in [1, 7, 9].

## 2. Main result

To give our main result, we first present Lemma 2.1.

**Lemma 2.1.** *Let $A \in \mathbb{R}^{n\times n}$ be an $H_+$-matrix, and $A = M - N$ be its an $H$-splitting. Then $\langle M\rangle - |N| \le \langle A\rangle$.*

**Proof.** By the simple computations, we have

$$a_{ii} = m_{ii} - n_{ii} = |m_{ii} - n_{ii}| \ge |m_{ii}| - |n_{ii}|$$

and

$$-|a_{ij}| = -|m_{ij} - n_{ij}| \ge -|m_{ij}| - |n_{ij}|,$$

where $A = (a_{ij}), M = (m_{ij})$ and $N = (n_{ij})$. Hence, the result of Lemma 2.1 is valid. □

Next, for the GMMS method, we give the following main result, see Theorem 2.1.

**Theorem 2.1.** *Let $A$, $\Omega_1$ and $\Omega_2$ be defined in Theorem 1.1, and $A\Omega_1 = M_{\Omega_1} - N_{\Omega_1}$ be an $H$-splitting of $A\Omega_1$. If*

$$\Omega_2 e > D_A\Omega_1 e - \frac{1}{2}V^{-1}(\langle A\Omega_1\rangle + \langle M_{\Omega_1}\rangle - |N_{\Omega_1}|)Ve \tag{2.1}$$

*for any positive diagonal matrix $V$ such that $(\langle M_{\Omega_1}\rangle - |N_{\Omega_1}|)V$ is an SDD matrix, where $e = (1, 1, \ldots, 1)^T$, then Method 1.1 is convergent for any initial guess $x^{(0)} \in \mathbb{R}^n$.*

**Proof.** Assume that $(z^*, w^*)$ is a solution of the LCP$(q, A)$. Using Eq. (1.2), we get that $x^* = \frac{1}{2}(\Omega_1^{-1}z^* - \Omega_2^{-1}w^*)$ and $|x^*| = \frac{1}{2}(\Omega_1^{-1}z^* + \Omega_2^{-1}w^*)$, which meets

$$(\Omega_2 + M_{\Omega_1})x^* = N_{\Omega_1}x^* + (\Omega_2 - A\Omega_1)|x^*| - q. \tag{2.2}$$

Combining (1.4) with (2.2), we obtain

$$(\Omega_2 + M_{\Omega_1})(x^{(k+1)} - x^*) = N_{\Omega_1}(x^{(k)} - x^*) + (\Omega_2 - A\Omega_1)(|x^{(k)}| - |x^*|). \tag{2.3}$$

Since $A\Omega_1 = M_{\Omega_1} - N_{\Omega_1}$ is an $H$-splitting of $A\Omega_1$, $\langle M_{\Omega_1}\rangle - |N_{\Omega_1}|$ is an $M$-matrix. Clearly, we have

$$\langle M_{\Omega_1}\rangle - |N_{\Omega_1}| \le \langle M_{\Omega_1}\rangle,$$

which implies that matrix $\langle M_{\Omega_1}\rangle$ is an $M$-matrix. Further, we can obtain that $\Omega_2 + M_{\Omega_1}$ is an $H_+$-matrix. Based on Lemma 1.2, we have

$$|(\Omega_2 + M_{\Omega_1})^{-1}| \le \langle \Omega_2 + M_{\Omega_1}\rangle^{-1} = (\Omega_2 + \langle M_{\Omega_1}\rangle)^{-1}.$$

From (2.3), we have

$$\begin{aligned} |x^{(k+1)} - x^*| &= |(\Omega_2 + M_{\Omega_1})^{-1}||N_{\Omega_1}(x^{(k)} - x^*) + (\Omega_2 - A\Omega_1)(|x^{(k)}| - |x^*|)| \\ &\le T|x^{(k)} - x^*|, \end{aligned} \tag{2.4}$$

where

$$T = (\Omega_2 + \langle M_{\Omega_1}\rangle)^{-1}(|N_{\Omega_1}| + |\Omega_2 - A\Omega_1|).$$

Obviously, the GMMS method is convergent for $\rho(T) < 1$.

Next, we consider two cases: $\Omega_2 e \geq D_A\Omega_1 e$ and

$$D_A\Omega_1 e - \frac{1}{2}V^{-1}(\langle A\Omega_1\rangle + \langle M_{\Omega_1}\rangle - |N_{\Omega_1}|)Ve < \Omega_2 e < D_A\Omega_1 e.$$

**Case (I).** Since $\Omega_2 e \geq D_A\Omega_1 e$, we know that $\Omega_2 \geq D_A\Omega_1$. In this case, we have

$$\Omega_2 - |\Omega_2 - A\Omega_1| = \langle A\Omega_1\rangle$$

and

$$\begin{aligned} T &= (\Omega_2 + \langle M_{\Omega_1}\rangle)^{-1}(\Omega_2 + \langle M_{\Omega_1}\rangle - \Omega_2 - \langle M_{\Omega_1}\rangle + |N_{\Omega_1}| + |\Omega_2 - A\Omega_1|) \\ &= I - (\Omega_2 + \langle M_{\Omega_1}\rangle)^{-1}(\Omega_2 + \langle M_{\Omega_1}\rangle - |N_{\Omega_1}| - |\Omega_2 - A\Omega_1|) \\ &= I - (\Omega_2 + \langle M_{\Omega_1}\rangle)^{-1}(\langle M_{\Omega_1}\rangle - |N_{\Omega_1}| + \langle A\Omega_1\rangle) \\ &\leq I - 2(\Omega_2 + \langle M_{\Omega_1}\rangle)^{-1}(\langle M_{\Omega_1}\rangle - |N_{\Omega_1}|). \end{aligned}$$

Noticing that $\langle M_{\Omega_1}\rangle - |N_{\Omega_1}|$ is an $M$-matrix, there exists a positive vector $u$ so that

$$(\langle M_{\Omega_1}\rangle - |N_{\Omega_1}|)u > 0.$$

Therefore,

$$Tu \leq (I - 2(\Omega_2 + \langle M_{\Omega_1}\rangle)^{-1}(\langle M_{\Omega_1}\rangle - |N_{\Omega_1}|))u < u.$$

It follows that $\rho(T) < 1$ from Lemma 1.1.

**Case (II).** Since $\langle M_{\Omega_1}\rangle - |N_{\Omega_1}|$ is an $M$-matrix, there exists a positive diagonal matrix $V$ such that $(\langle M_{\Omega_1}\rangle - |N_{\Omega_1}|)V$ is an SDD matrix. Then from the equivalent statement $M_{35}$ of the nonsingular $M$-matrix in [3, Page 137] we have

$$\langle M_{\Omega_1}\rangle Ve \geq (\langle M_{\Omega_1}\rangle - |N_{\Omega_1}|)Ve > 0.$$

So,

$$\langle M_{\Omega_1}\rangle Ve + \Omega_2 Ve > (|N_{\Omega_1}| + \Omega_2)Ve > 0.$$

Moreover, we can get that the interval $(D_A\Omega_1 e - \frac{1}{2}V^{-1}(\langle A\Omega_1\rangle + \langle M_{\Omega_1}\rangle - |N_{\Omega_1}|)Ve$, $D_A\Omega_1 e)$ is nonempty from Lemma 2.1.

Since

$$D_A\Omega_1 e - \frac{1}{2}V^{-1}(\langle A\Omega_1\rangle + \langle M_{\Omega_1}\rangle - |N_{\Omega_1}|)Ve < \Omega_2 e,$$

we obtain

$$\begin{aligned} &2VD_A\Omega_1 e - (\langle A\Omega_1\rangle + \langle M_{\Omega_1}\rangle - |N_{\Omega_1}|)Ve < 2V\Omega_2 e \\ \Leftrightarrow &[2D_A\Omega_1 - (\langle A\Omega_1\rangle + \langle M_{\Omega_1}\rangle - |N_{\Omega_1}|)]Ve < 2\Omega_2 Ve \\ \Leftrightarrow &(2D_A\Omega_1 - \langle A\Omega_1\rangle - \langle M_{\Omega_1}\rangle + |N_{\Omega_1}|)Ve < 2\Omega_2 Ve \\ \Leftrightarrow &(|A\Omega_1| - \langle M_{\Omega_1}\rangle + |N_{\Omega_1}|)Ve < 2\Omega_2 Ve \\ \Leftrightarrow &(2\Omega_2 + \langle M_{\Omega_1}\rangle - |N_{\Omega_1}| - |A\Omega_1|)Ve > 0. \end{aligned}$$

In addition, when $\Omega_2 e < D_A\Omega_1 e$, we have

$$|\Omega_2 - A\Omega_1| = |A\Omega_1| - \Omega_2 \geq 0.$$

Let

$$\bar{M} = \Omega_2 + \langle M_{\Omega_1} \rangle, \bar{N} = |N_{\Omega_1}| + |\Omega_2 - A\Omega_1|.$$

Then $T = \bar{M}^{-1}\bar{N}$ and

$$\begin{aligned}(\bar{M} - \bar{N})Ve &= (\Omega_2 + \langle M_{\Omega_1} \rangle - |N_{\Omega_1}| - |\Omega_2 - A\Omega_1|)Ve \\ &= (2\Omega_2 + \langle M_{\Omega_1} \rangle - |N_{\Omega_1}| - |A\Omega_1|)Ve \\ &> 0.\end{aligned}$$

By using Theorem 3 in [6], we have

$$\begin{aligned}\rho(T) =& \rho(V^{-1}TV) \\ \leq& \|V^{-1}TV\|_\infty \\ =& \|((\Omega_2 + \langle M_{\Omega_1} \rangle)V)^{-1}(|N_{\Omega_1}| + |\Omega_2 - A\Omega_1|)V\|_\infty \\ \leq& \max_{1 \leq i \leq n} \frac{((|N_{\Omega_1}| + |\Omega_2 - A\Omega_1|)Ve)_i}{((\Omega_2 + \langle M_{\Omega_1} \rangle)Ve)_i} \\ <& 1.\end{aligned}$$

Combining Case (I) with Case (II), the result of Theorem 2.1 is valid. □

Comparing the condition (2.1) with the condition (1.5), the former is weaker than the latter. In fact, by simple computation,

$$\begin{aligned}& D_A\Omega_1 e - V^{-1}(\langle M_{\Omega_1} \rangle - |N_{\Omega_1}|)Ve \\ & - [D_A\Omega_1 e - \frac{1}{2}V^{-1}(\langle A\Omega_1 \rangle + \langle M_{\Omega_1} \rangle - |N_{\Omega_1}|)Ve] \\ =& - V^{-1}(\langle M_{\Omega_1} \rangle - |N_{\Omega_1}|)Ve + \frac{1}{2}V^{-1}(\langle A\Omega_1 \rangle + \langle M_{\Omega_1} \rangle - |N_{\Omega_1}|)Ve \\ =& \frac{1}{2}V^{-1}(\langle A\Omega_1 \rangle - (\langle M_{\Omega_1} \rangle - |N_{\Omega_1}|))Ve \\ \geq& 0.\end{aligned}$$

This implies that the convergence condition (1.4) of Theorem 1.1 in [7] is improved.

When $\Omega_2 = \Omega$ and $\Omega_1 = I$, Corollary 2.1 is obtained.

**Corollary 2.1.** *Let $A = M - N$ be an $H$-splitting of the $H_+$-matrix $A$. If*

$$\Omega e > D_A e - \frac{1}{2}V^{-1}(\langle A \rangle + \langle M \rangle - |N|)Ve \tag{2.5}$$

*for any positive diagonal matrix $V$ such that $(\langle M \rangle - |N|)V$ is an SDD matrix, where $e = (1, 1, \ldots, 1)^T$, then the MMS method (see Method 3.1 in [1]) is convergent for any initial guess $x^{(0)} \in \mathbb{R}^n$.*

Further, it is easy to find that the condition (2.5) in Corollary 2.1 is also weaker than those in Theorem 4.3 in [1] and Theorems 3.1 in [9].

It's important to note that the matrix $V$ involved in Theorem 2.1 and Corollary 2.1 may be not easily available, including Theorem 1.1 as well. Whereas, when the $H_+$-matrix $A$ is an SDD matrix, the matrix $V$ can be easily chosen. That is to say, for this situation, we can choose $V = I$, and then make use of the condition (2.1) to judge the convergence of the GMMS method and the condition (2.5) to judge the convergence of the MMS method.

Finally, we use two simple examples to compare Theorem 1.1 [7] with Theorem 2.1.

**Example 2.1.** To compare Theorem 1.1 [7] with Theorem 2.1, we set $V = \Omega_1 = I$ and

$$A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}.$$

Clearly, $A\Omega_1 = A$. Taking

$$M_{\Omega_1} = \begin{bmatrix} 4 & 0 \\ 2 & 4 \end{bmatrix}, N_{\Omega_1} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Then

$$\langle A\Omega_1 \rangle = \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}, \langle M_{\Omega_1} \rangle = \begin{bmatrix} 4 & 0 \\ -2 & 4 \end{bmatrix}, |N_{\Omega_1}| = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

So,

$$\langle M_{\Omega_1} \rangle - |N_{\Omega_1}| = \begin{bmatrix} 4 & -1 \\ -3 & 4 \end{bmatrix} \text{ and } \frac{1}{2}(\langle A\Omega_1 \rangle + \langle M_{\Omega_1} \rangle - |N_{\Omega_1}|) = \begin{bmatrix} 4 & -1 \\ -2 & 4 \end{bmatrix}.$$

By Theorem 1.1 [7], the choice of matrix $\Omega_2$ satisfies $\Omega_2 > 3I$ to ensure the convergence of the GMMS method. Whereas, using Theorem 2.1, the choice of matrix $\Omega_2$ only need to satisfy $\Omega_2 > 2I$ to ensure the convergence of the GMMS method. This shows that Theorem 2.1 is weaker than Theorem 1.1 [7]. Further, for the interval $(2I, 3I]$, if we take $\Omega_2 = 3I$, then $\rho(T) = 0.4918 < 1$. This shows that the GMMS method is convergent. But, in this case, Theorem 1.1 [7] does not satisfy to judge the convergence of the GMMS method.

**Example 2.2.** Let $V = \Omega_1 = I$ and $A = \text{tridiag}(1, 4, 1) \in \mathbb{R}^{n\times n}$ with $n \geq 3$. Taking $M_{\Omega_1} = \text{tridiag}(2, 4, 0)$, $N_{\Omega_1} = \text{tridiag}(1, 0, -1)$. Then, by the simple calculations, we have

$$\langle M_{\Omega_1} \rangle - |N_{\Omega_1}| = \text{tridiag}(-3, 4, -1), \frac{1}{2}(\langle A\Omega_1 \rangle + \langle M_{\Omega_1} \rangle - |N_{\Omega_1}|) = \text{tridiag}(-2, 4, -1).$$

For $n \geq 3$, by Theorem 1.1 [7], the choice of matrix $\Omega_2$ satisfies $\Omega_2 > 4I$ to ensure the convergence of the GMMS method. Whereas, using Theorem 2.1, the choice of matrix $\Omega_2$ only need to satisfy $\Omega_2 > 3I$ to ensure the convergence of the GMMS method.

Next, we consider $\Omega_2 = 4I$ for $A = M_{\Omega_1} - N_{\Omega_1}$ with $M_{\Omega_1} = \text{tridiag}(2, 4, 0)$ and $N_{\Omega_1} = \text{tridiag}(1, 0, -1)$. In our computations, the starting vector is zero, the relative residual error (denoted by 'RES'), which is defined by

$$\text{RES}(x^{(k)}) = \| \min(Az^{(k)} + p, z^{(k)})\|_2.$$

All the test results are run on an Intel@ Celeron@ G4900, where the CPU 3.10GHz and the memory is 8.00 GB, and the language is MATLAB 7.0.

Table 1 lists some numerical results of the GMMS method with $q = -Az^*$ and $z^* = (1, 2, \ldots, 1, 2)^T$, where 'IT', 'CPU', 'RES', respectively, denote elapsed CPU time in seconds, the iteration steps and the relative residual error. From these numerical results confirm that GMMS is convergent for $\Omega_2 = 4I$.

**Table 1.** Numerical results of GMMS with $\Omega_2 = 4I$ and $\Omega_1 = I$.

| $n$ | 300 | 600 | 900 |
|---|---|---|---|
| $\rho(T)$ | 0.6475 | 0.6557 | 0.6591 |
| IT | 16 | 16 | 16 |
| CPU | 0.0156 | 0.0781 | 0.1719 |
| RES | 3.9499e-7 | 5.6384e-7 | 6.9269e-7 |

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## References

[1] Z. Bai, *Modulus-based matrix splitting iteration methods for linear complementarity problems*, Numerical Linear Algebra with Applications, 2010, 17, 917–933.

[2] Z. Bai and L. Zhang, *Modulus-based synchronous two-stage multisplitting iteration methods for linear complementarity problems*, Numerical Algorithms, 2013, 62, 59–77.

[3] A. Berman and R. J. Plemmons, *Nonnegative Matrices in the Mathematical Sciences*, Academic, New York, 1979.

[4] R. W. Cottle, J. Pang and R. E. Stone, *The Linear Complementarity Problem*, Academic, San Diego, 1992.

[5] A. Frommer and G. Mayer, *Convergence of relaxed parallel multisplitting methods*, Linear Algebra and its Applications, 1989, 119, 141–152.

[6] J. Hu, *Estimates of* $\|B^{-1}A\|_\infty$ *and their applications*, Mathematica Numerica Sinica, 1982, 4, 272–282.

[7] W. Li, *A general modulus-based matrix splitting method for linear complementarity problems of* $H$*-matrices*, Applied Mathematics Letter, 2013, 26, 1159–1164.

[8] K. G. Murty, *Linear Complementarity, Linear and Nonlinear Programming*, Heldermann, Berlin, 1988.

[9] L. Zhang and Z. Ren, *Improved convergence theorems of modulus-based matrix splitting iteration methods for linear complementarity problems*, Applied Mathematics Letter, 2013, 26, 638–642.