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STUDY ON THE ESTIMATION OF THE NUMBER OF POTENTIAL HIV-INFECTED INDIVIDUALS AND PREVENTION AND CONTROL STRATEGIES BASED ON DYNAMIC MODEL*

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Abstract **Motivation:** AIDS is a chronic and fatal infectious disease caused by the human immunodeficiency virus (HIV). According to the Chinese Center for Disease Control and Prevention (CDC), there are currently 689,000 cases of HIV-infected patients, with 103,350 new HIV infections in 2019 and the number of HIV cases continues to rise. Therefore, a reasonable estimation of the number of potentially HIV-infected individuals in the population is essential for the prevention and control of AIDS. **Method:** We develop a dynamic model of a compartment containing potential HIV-infected individuals and fitted the parameters of the model using nonlinear least squares based on the data from 2004 to 2019. **Results:** The basic reproduction number of the model is calculated to be R_0=1.514 and the mean number of potential HIV-infected individuals in the population is estimated by the model to be 16847 (95%CI [15047,18846]). In 2004, there are 1.7835 times more potential HIV-infected individuals K than HIV-infected individuals, this then decreases each year. The average ratio of potentially HIV-infected individuals to HIV-infected individuals from 2004 to 2019 is 38.41%. **Conclusions:** The number of potential HIV-infected individuals has an important impact on the transmission and control of HIV virus and the proportion of changing sexual behavior habits can greatly reduce the number of new potential HIV-infected individuals. The existence of potential HIV-infected individuals should be taken into account in prevention and control strategies and publicity efforts should be intensified so that more people can understand the transmission pathways, pathogenesis and precautions against AIDS, in order to better reduce the number of new HIV infections and the spread of HIV virus in the population.

Keywords AIDS/HIV, stability analysis, basic reproduction number, ratio of potentially infected.

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1. Introduction

Acquired Immunodeficiency Syndrome (AIDS) is a chronic and fatal infectious disease caused by Human Immunodeficiency Virus (HIV) [27]. Once a person becomes infected with the HIV virus, it invades T-lymphocytes leading to a disruption of the immune system, primarily characterized by a decline in cellular immune function, resulting in susceptibility to infections or tumors, ultimately progressing to AIDS and causing the death of the individual. According to data from the World Health Organization [26], in 2022, 630,000 [480,000-880,000] people worldwide will die of HIV-related illnesses and 1.3 million [1.0 million-1.7 million] people will be infected with the human immunodeficiency virus (HIV). HIV/AIDS has been ranked as one of the top 10 diseases that seriously threaten human health and social development [11]. In China, AIDS is classified as a class B notifiable infectious disease and is one of the infectious diseases under border health surveillance. From 2004 to 2020, the annual data on new HIV cases and deaths in China [4], are shown in Figure 1.

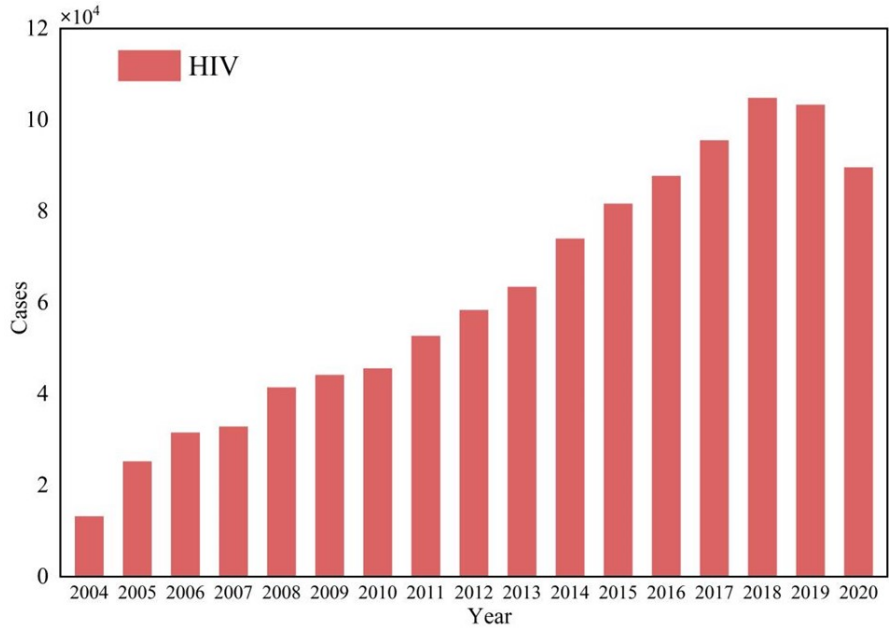

Figure 1. Number of annual HIV incidence from 2004 to 2020.

With an average of 61,519 new cases per year, with the lowest number in 2004 at 13,258 and the highest in 2018 at 104,838. The average annual number of deaths of HIV is 9,333, with the lowest number 2 in 2004 and the highest number 21,407 in 2019.

The average annual death rate from HIV is 0.6813(1/100000), with the lowest death rate 0.0002 in 2004 and the highest death rate 1.5329(1/100000) in 2019, as shown in Figure 2.

Data published by the Chinese Center for Disease Control and Prevention (CDC) can be used to obtain the HIV incidence data by region from 2004 to 2019 [4]. The HIV incidence in each region increased significantly in 2005, 2008, 2011, 2017, 2019

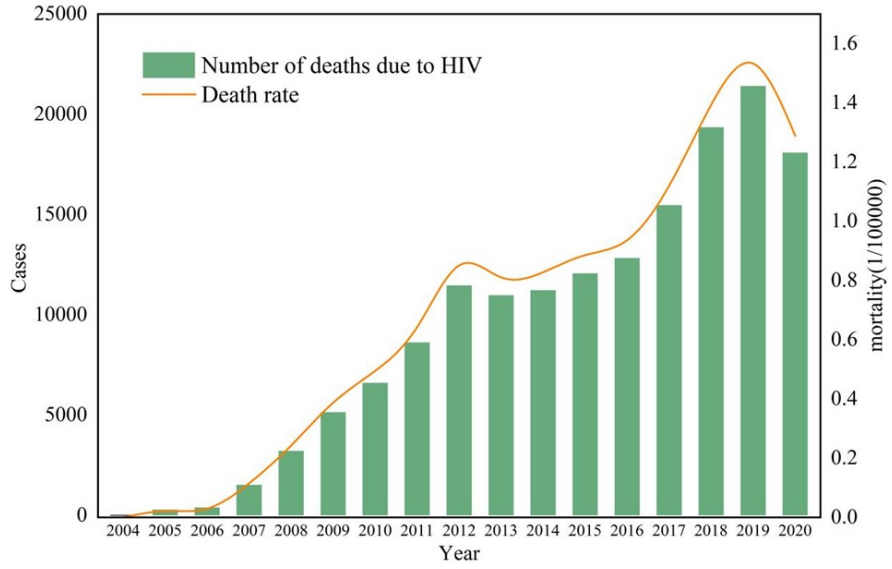

Figure 2. The HIV mortality rate and deaths from 2004 to 2020.

and the spatial distribution of the number of HIV incidence is plotted in Figure 3. In 2005, the annual number of HIV cases in Xinjiang Uygur Autonomous Region and parts of southwestern China exceed 1,000. Over time, by 2008, the annual incidence in parts of the Southwest exceed 5,000 cases. In 2011, the annual number of cases begins to increase in some parts of the eastern seaboard, with more than 1,000 cases per year. In 2017, there is a significant increase in the number of annual cases in Sichuan Province compared to 2005, with more than 10,000 cases per year and the number of annual cases in Xinjiang Uygur Autonomous Region increases from 1,000 in 2005 to 5,000, and the number of cases in the rest of China's provinces is also increasing. In 2019, there is a significant increase in the number of HIV incidence across China, making the study of HIV of great importance.

Both HIV virus carriers and AIDS patients are sources of infection. HIV is a virus that can only be transmitted under specific conditions, typically through the transmission routes of blood, sexual and mother-to-child vertical transmission [16]. After an incubation period of several years or even up to 10 years or more, HIV-infected persons can develop into AIDS patients. During this period, their immune system becomes severely compromised, leading to various infections. In the later stages, it often develops into malignancies and causes long-term physical depletion, eventually leading to systemic failure and death. Once infected with the HIV virus, infected individuals can transmit the virus to others within a few days and they will then become infectious for life [12].

There is no effective treatment for AIDS, nor is there an effective vaccine to prevent it. The active effective measures to prevent and control AIDS are health education and behavioral intervention strategies, calling on everyone to do it: Insisting on cleanliness, refraining from prostitution and soliciting prostitutes and avoiding high-risk sexual behavior. It is strictly forbidden to use drugs and not to share syringes with others; do not have unauthorized blood transfusions or use blood products, but use them under the guidance of a doctor; do not borrow or

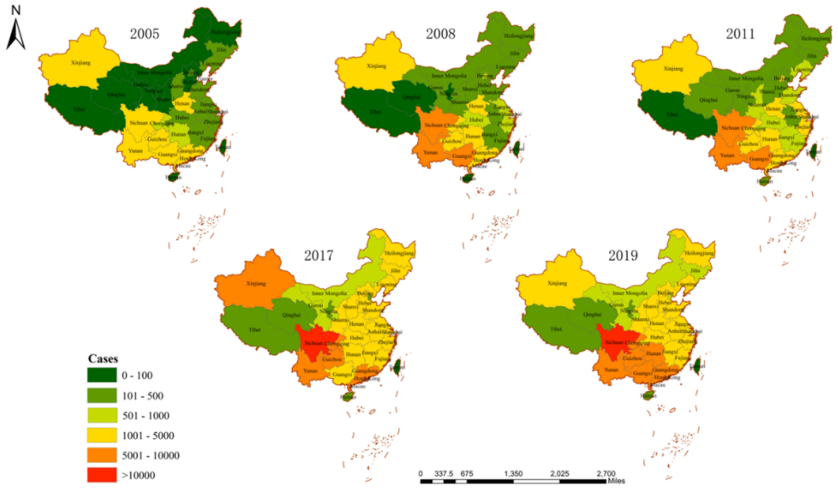

Figure 3. Spatial distribution of new HIV cases in China, 2005-2019.

share personal items such as toothbrushes, razors, shavers, etc. The use of condoms is one of the most effective preventive measures against STDs and AIDS during sex; avoiding direct contact with the blood, semen and breast milk of AIDS patients in order to cut off their transmission channels [4].

Many researchers at home and abroad have studied the transmission process of the HIV virus and the impact of preventive and control measures on its transmission by building mathematical models. Cristiana J. Silva et al. [23] proposed a fractional-order model of HIV/AIDS transmission and investigated the local and uniform stability of the fractional-order model. Zhao et al. [33] developed a new concept of segmented fractional order differential equations to study the dynamics of HIV/AIDS infections. Huo et al. [10] developed a new model of the HIV/AIDS epidemic with treatment to study the impact of treatment on the dynamics of the spread of the AIDS epidemic. Shiv Mangal et al. [17] introduced a deterministic Fractional Order Epidemiologic Model (FOEM) for studying the transmission dynamics of HIV and AIDS. Li et al. [14] established a new control model and gave the global asymptotic stability of the disease-free equilibrium and endemic equilibrium by constructing the Lyapunov function, obtaining that prevention and treatment are the main factors in preventing and controlling the HIV/AIDS epidemic.Li et al. [13] established a broader class of viral kinetic models for HIV infection of $CD4^+$ T-cells and obtained the existence and stability conditions of the uninfected equilibrium and the infected equilibrium of the model. Wang et al. [25] developed a model of syphilis and HIV coinfection, analyzed the model according to the different situation and evaluated the transmission hazards of syphilis and HIV using data from the United States as an example. Attaullah et al. [2] developed a novel HIV/AIDS model through a higher-order Galerkin time discretization scheme and analyzed the transmission dynamics of the model. Sohaib et al. [1] established an HIV infection model and performed numerical simulations to assess the feasibility of different scenarios. Qi et al. [22] explored the effect of the stochastic environmental fluctuations on the dynamics of an HIV system with both virus-to-cell and cell-to-cell transmissions.Olaniyi et al. [21] established a new mathematical model for human immunodeficiency virus and acquired immune deficiency syndrome fea-

turing vertical transmission and nonlinear treatment is presented. The model is analyzed through the implementation of some dynamical system tools with a view to assessing the behavior of trajectories of the system governing the dynamics of HIV/AIDS. Xu et al. [29] developed a model of hepatitis B transmission dynamics that included potentially infected individuals and vaccination and assessed the number of potentially infected individuals with hepatitis B in China to be approximately 450,000. However, there are fewer studies of potential HIV infections based on the transmission process. China has taken strict measures to control the spread of HIV. Diagnosed HIV patients are required to strictly control their behavior and are not allowed to re-infect others. However, the number of new HIV infections in China continues to grow, and we believe that there are still undetected HIV infections (potential HIV infections) in the population. Therefore, it is very important to assess the number of potentially HIV-infected people. Based on the existing case data, this paper establishes a dynamic model that includes potential HIV-infected individuals based on the transmission mechanism of HIV, assesses the number of potential HIV-infected individuals in the population and explores effective strategies to control the transmission of HIV.

In Section 1, this paper introduces the situation of the HIV virus transmission in China. In Section 2, we build the model, calculate the basic reproduction number and analyze the equilibrium points of the model. In Section 2, we conduct a numerical simulation to give the effects of the parameters on the changes of the number of potential HIV-infected individuals and HIV-infected individuals as well as on the basic reproduction number R_0. In Section 4, it is the discussion section.

2. Model building and analysis

2.1. Model building

Sexual transmission is the most common mode of HIV and it is first spread among male homosexuals. Health education, intervention measures and drug interventions targeted at women of childbearing age can effectively prevent mother-to-child vertical transmission of HIV [9], and that the impact of mother-to-child vertical transmission on potential HIV-infected individuals is minimal, with an infection rate of only 0.000335% for the age group 0 to 14 years and a similarly low rate of 0.0024047% for individuals aged 75 and older. Therefore, this study focuses exclusively on the population aged 15 to 74. The aim of this study is to establish a dynamic model of the inclusion of potentially HIV-infected compartments based on the mechanism of HIV virus transmission and to estimate the number of potential HIV-infected individuals. Modeling assumptions include the following: HIV transmission primarily occurs through sexual contact, and mother-to-child vertical transmission can be effectively blocked through various means. Given the very low infection rate of HIV among individuals aged 0 to 14, the study exclusively focuses on the population aged 15 to 74. HIV carriers are considered infectious during the latent period, with potential HIV-infected individuals being highly infectious. Additionally, control measures are in place for people living with HIV and AIDS who have been tested and documented by the relevant agencies and such groups are not contagious.

According to the transmission process of the HIV virus, the population is divided into five compartments: susceptible individuals (S), latent infected (E), HIV-

infected individuals (H, those who have already been diagnosed with HIV), potential HIV-infected individuals (K, those who are infected with the HIV virus but remain undiagnosed or undiscovered) and AIDS patients (A). Based on the HIV virus transmission process, the following flowchart is established, shown Figure 4.

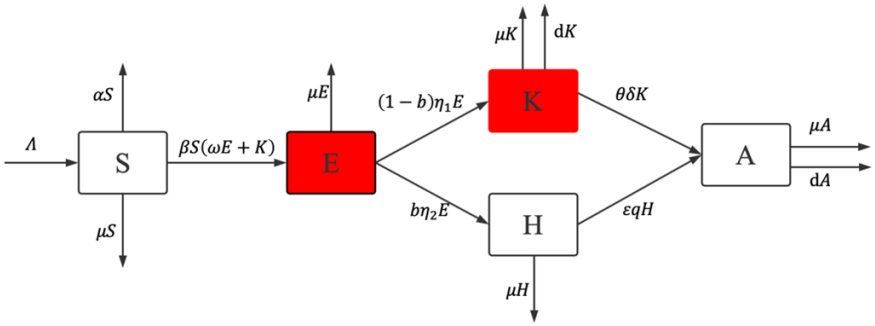

Figure 4. Flow chart of HIV virus transmission.

The meaning of each parameter symbol: Λ is the new susceptible individuals each year. The parameter β represents the basic transmission rate, b is the conversion rate from E to H. μ is the natural mortality rate, d is the mortality rate due to disease. η_1 is the rate of transfer from E to K, η_2 is the rate of transfer from E to H. q is the rate of conversion from H to A, ε is the rate of transfer from H to A. θ is the transfer rate from K to A, δ is the rate of conversion from K to A. α is the proportion of susceptible individuals who change their sexual habits per unit of time, ω is the relative transmission rate. All the parameters are nonnegative.

Based on Figure 4, the following dynamic model of HIV virus transmission is developed:

$$\begin{cases} \dfrac{dS}{dt} = \Lambda - \beta S(\omega E + K) - \alpha S - \mu S, \\ \dfrac{dE}{dt} = \beta S(\omega E + K) - (1-b)\eta_1 E - b\eta_2 E - \mu E, \\ \dfrac{dK}{dt} = (1-b)\eta_1 E - \theta\delta K - dK - \mu K, \\ \dfrac{dH}{dt} = b\eta_2 E - \varepsilon q H - \mu H, \\ \dfrac{dA}{dt} = \varepsilon q H + \theta\delta K - dA - \mu A. \end{cases} \tag{2.1}$$

2.2. Calculation of disease-free equilibrium point and basic reproduction number

There is always a disease-free equilibrium point $Z_0 = (S_0, 0, 0, 0, 0)$ in system (2.1), where $S_0 = \Lambda/(\alpha + \mu)$.

The basic reproduction number (R_0) refers to the number of secondary cases after an infected individual enters the susceptible population [6]. The next generation matrix method [5] is used to calculate R_0.

To calculate the basic reproduction number of system (2.1), let:

$$\mathcal{F} = \begin{bmatrix} \beta S(\omega E + K) \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathcal{V} = \begin{bmatrix} (1-b)\eta_1 E + b\eta_2 E + \mu E \\ -(1-b)\eta_1 E + \theta\delta K + dK + \mu K \\ -b\eta_2 E + \varepsilon q H + \mu H \\ -\theta\delta K - \varepsilon q H + dA + \mu A \end{bmatrix}.$$

The Jacobian matrices of $\mathcal{F}$, $\mathcal{V}$ at the disease-free equilibrium point Z_0 are:

$$F = \begin{bmatrix} \omega\beta S_0 & \beta S_0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} (1-b)\eta_1 + b\eta_2 + \mu & 0 & 0 & 0 \\ -(1-b)\eta_1 & \theta\delta + d + \mu & 0 & 0 \\ -b\eta_2 & 0 & \varepsilon q + \mu & 0 \\ 0 & -\theta\delta & -\varepsilon q & d + \mu \end{bmatrix}.$$

At the disease-free equilibrium point, we can derive

$$FV^{-1} = \begin{bmatrix} \dfrac{\omega\beta S_0}{(1-b)\eta_1 + b\eta_2 + \mu} + \dfrac{\beta S_0 (1-b)\eta_1}{(\theta\delta + d + \mu)[(1-b)\eta_1 + b\eta_2 + \mu]} & \dfrac{\beta S_0}{\theta\delta + d + \mu} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The basic reproduction number is the spectral radius of the FV^{-1}, therefore we get

$$R_0 = \frac{\omega\beta S_0}{(1-b)\eta_1 + b\eta_2 + \mu} + \frac{\beta S_0 (1-b)\eta_1}{(\theta\delta + d + \mu)[(1-b)\eta_1 + b\eta_2 + \mu]}. \tag{2.2}$$

2.3. Existence of the endemic equilibrium point

Theorem 2.1. *When $R_0 > 1$ the system (2.1) has a unique endemic equilibrium point $Z^* = (S^*, E^*, K^*, H^*, A^*)$.*

Proof. Using the equilibrium equation of system (2.1) at the endemic equilibrium point, we can get

$$\begin{aligned} H^* &= \frac{b\eta_2}{\varepsilon q + \mu} E^*, \\ K^* &= \frac{(1-b)\eta_1}{\theta\delta + d + \mu} E^*, \\ S^* &= \frac{\Lambda}{\beta E^* \left(\omega + \frac{(1-b)\eta_1}{\theta\delta + d + \mu}\right) + \alpha + \mu}, \\ A^* &= \left[\frac{\varepsilon q b\eta_2}{(d+\mu)(\varepsilon q + \mu)} + \frac{\theta\delta(1-b)\eta_1}{(\theta\delta + d + \mu)(d+\mu)}\right] E^*. \end{aligned}$$

According to the equilibrium equation of the system (2.1) we can obtain

$$\beta S^*(\omega E^* + K^*) - (1-b)\eta_1 E^* - b\eta_2 E^* - \mu E^* = 0$$

From this, we can get

$$E^* = 0 \quad \text{or} \quad E^* = \frac{\Lambda(R_0 - 1)}{R_0[(1-b)\eta_1 + b\eta_2 + \mu]}.$$

When $R_0 > 1$,we obtain $E^* > 0$ and $S^* > 0$.

Therefore, when $R_0 > 1$, E^* has a unique positive root. There is a unique positive equilibrium point Z^* of the system (2.1). □

2.4. Stability at equilibrium point

It is easy to find that the set

$$\Gamma = \left\{ \phi \in \mathbb{R}_+^5 : \sum_{i=1}^{5} \phi_i \leq \frac{\Lambda}{\mu} \right\}$$

is positively invariant for model (2.1). Next, we will discuss the dynamics of model (2.1) in Γ.Then we have the following result.

Theorem 2.2. *The disease-free equilibrium point Z_0 of the system (2.1) is locally asymptotically stable when $R_0 < 1$. When $R_0 > 1$, the disease-free equilibrium point Z_0 of the system (2.1) is unstable.*

Proof. The Jacobian matrix of the system (2.1) at the disease-free equilibrium point Z_0 is J.

$$J = \begin{bmatrix} -\alpha - \mu & -\omega\beta S_0 & -\beta S_0 & 0 & 0 \\ 0 & \omega\beta S_0 - (1-b)\eta_1 - b\eta_2 - \mu & \beta S_0 & 0 & 0 \\ 0 & (1-b)\eta_1 & -\theta\delta - d - \mu & 0 & 0 \\ 0 & b\eta_2 & 0 & -\varepsilon q - \mu & 0 \\ 0 & 0 & \theta\delta & \varepsilon q & -d - \mu \end{bmatrix}.$$

The characteristic equation of this matrix is:

$$\begin{aligned} g(\varphi) = (\varphi + d + \mu)(\varphi + \varepsilon q + \mu)(\varphi + \alpha + \mu)[(\varphi + \theta\delta + d + \mu)(\varphi - \omega\beta S_0 \\ + (1-b)\eta_1 + b\eta_2 + \mu) - \beta S_0(1-b)\eta_1]. \end{aligned}$$

Clearly

$$\varphi_1 = -d - \mu < 0, \quad \varphi_2 = -\varepsilon q - \mu < 0, \quad \varphi_3 = -\alpha - \mu < 0.$$

The other two eigenvalues φ_4 , φ_5 satisfy the equation

$$h(\varphi) = \varphi^2 + A_3\varphi + A_4.$$

Where

$$A_3 = \theta\delta + d + \mu + (1-b)\eta_1 + b\eta_2 + \mu - \omega\beta S_0,$$
$$A_4 = (\theta\delta + d + \mu)[(1-b)\eta_1 + b\eta_2 + \mu](1 - R_0).$$

According to equation (2.2), When $R_0 < 1$, according to equation (2.2), we get $\frac{\omega\beta S_0}{(1-b)\eta_1 + b\eta_2 + \mu} < 1$, $(1-b)\eta_1 + b\eta_2 + \mu - \omega\beta S_0 > 0$.

Therefore, when $R_0 < 1$, $A_3 > 0$, $A_4 > 0$. According to the Routh-Hurwitz discriminant [15], the characteristic roots φ_4 , φ_5 have negative real parts. When $R_0 > 1$, $A_4 < 0$, $g(\varphi)$ has at least one characteristic root with positive real part.

In summary, when $R_0 < 1$ the disease-free equilibrium point Z_0 of the system (2.1) is locally asymptotically stable; when $R_0 > 1$, the disease-free equilibrium point Z_0 of the system (2.1) is unstable [7]. □

Theorem 2.3. *When $R_0 < 1$, the disease-free equilibrium Z_0 of system (2.1) is globally asymptotically stable.*

Proof. Define function

$$h(x) = x - 1 - \ln x, x > 0.$$

For any $x > 0$, there is $h(x) \geq 0$ and $h(x) = 0$ if and only if $x = 1$.

Let $U(t) = (S(t), E(t), K(t), H(t), A(t))$ be the solution of system (2.1) through any $\phi := (\phi_1, \phi_2, \phi_3, \phi_4, \phi_5) \in \Gamma$. Clearly, $U(t)$ is bounded and $S(t) > 0$ holds for any $t > 0$. We define a function V on $\Omega = \{\phi \in \mathbb{R}^5_+ : \phi_1 > 0\} \subseteq \Gamma$ as follows [30].

$$V(\phi) = S_0 \cdot h(\frac{\phi_1}{S_0}) + R_0 \cdot \phi_2 + \frac{\beta S_0}{\theta\delta + d + \mu}\phi_3.$$

The derivative of V with respect to $t\,(t > 0)$ along $U(t)$ is

$$\begin{aligned}
&\frac{dV(U(t))}{dt} \\
&= \left(1 - \frac{S_0}{S(t)}\right) \cdot \dot{S}(t) + R_0 \cdot \dot{E}(t) + \frac{\beta S_0}{\theta\delta + d + \mu} \cdot \dot{K}(t) \\
&= \left(1 - \frac{S_0}{S(t)}\right) \left[\Lambda - \beta S(t)(\omega E(t) + K(t)) - \alpha S(t) - \mu S(t)\right] \\
&\quad + R_0 \cdot \left[\beta S(t)(\omega E(t) + K(t)) - (1-b)\eta_1 E(t) - b\eta_2 E(t) - \mu E(t)\right] \\
&\quad + \frac{\beta S_0}{\theta\delta + d + \mu} \cdot \left[(1-b)\eta_1 E(t) - \theta\delta K(t) - dK(t) - \mu K(t)\right] \\
&= -\frac{\alpha + \mu}{S(t)}[S(t) - S_0]^2 + (R_0 - 1)\beta S(t)(\omega E(t) + K(t)).
\end{aligned}$$

Hence when $R_0 < 1$, there is $dV(U(t))/dt \leq 0$, which means that V is a Lyapunov function on $\{U(t) : t \geq 1\} \subseteq \Omega$. And $dV(U(t))/dt = 0$ if and only if $U(t) = Z_0$. Therefore, the largest invariant set of $\{U(t) | dV(U(t))/dt = 0, U(t) \subseteq \Omega\}$ is $\{Z_0\}$. According to the LaSalle's Invariance Principle [15], when $R_0 < 1$, the disease-free equilibrium Z_0 of system (2.1) is globally asymptotically stable. □

Theorem 2.4. *When $R_0 > 1$, the endemic equilibrium point Z^* of system (2.1) is locally asymptotically stable.*

Proof. Let the Jacobian matrix of the system (2.1) at the endemic equilibrium point Z^* be M.

$$M = \begin{bmatrix} -\beta(\omega E^* + K^*) - \alpha - \mu & -\omega\beta S^* & -\beta S^* & 0 & 0 \\ \beta(\omega E^* + K^*) & \omega\beta S^* - (1-b)\eta_1 - b\eta_2 - \mu & \beta S^* & 0 & 0 \\ 0 & (1-b)\eta_1 & -\theta\delta - d - \mu & 0 & 0 \\ 0 & b\eta_2 & 0 & -\varepsilon q - \mu & 0 \\ 0 & 0 & \theta\delta & \varepsilon q & -d - \mu \end{bmatrix}.$$

The characteristic equation is obtained by calculating

$$\begin{aligned} & f(\lambda) \\ &= (\lambda + d + \mu)(\lambda + \varepsilon q + \mu)(\lambda + \theta\delta + d + \mu)[(\lambda + \alpha + \mu)(\lambda - \omega\beta S^* + (1-b)\eta_1 \\ &\quad + b\eta_2 + \mu) + \beta(\omega E^* + K^*)(\lambda + (1-b)\eta_1 + b\eta_2 + \mu)] - \beta S^*(1-b)\eta_1(\lambda + \alpha + \mu) \\ &= g_1(\lambda) \cdot g_2(\lambda). \end{aligned}$$

Where

$$\begin{aligned} g_1(\lambda) &= (\lambda + d + \mu)(\lambda + \varepsilon q + \mu)(\lambda + \theta\delta + d + \mu), \\ g_2(\lambda) &= (\lambda + \alpha + \mu)\left[\lambda + ((1-b)\eta_1 + b\eta_2 + \mu)\left(1 - \frac{S^*}{S_0}G(\lambda)\right)\right] \\ &\quad + \beta(\omega E^* + K^*)(\lambda + (1-b)\eta_1 + b\eta_2 + \mu), \\ G(\lambda) &= \frac{\omega\beta S_0}{(1-b)\eta_1 + b\eta_2 + \mu} + \frac{\beta S_0(1-b)\eta_1}{(\lambda + \theta\delta + d + \mu)[(1-b)\eta_1 + b\eta_2 + \mu]}. \end{aligned}$$

Let $g_2(\lambda) = 0$, the characteristic equation $f(\lambda)$ can be simplified to

$$\begin{aligned} & (\lambda + \alpha + \mu)(\lambda + (1-b)\eta_1 + b\eta_2 + \mu) \\ & + \beta(\omega E^* + K^*)(\lambda + (1-b)\eta_1 + b\eta_2 + \mu) \\ =& \frac{S^*}{S_0}G(\lambda)(\lambda + \alpha + \mu)((1-b)\eta_1 + b\eta_2 + \mu). \end{aligned} \tag{2.3}$$

Put K^* into $\beta(\omega E^* + K^*)$, we can get

$$\beta(\omega E^* + K^*) = \frac{(1-b)\eta_1 + b\eta_2 + \mu}{S_0} \cdot R_0 \cdot E^*. \tag{2.4}$$

Equation (2.3) can be obtained by simplification as

$$\begin{aligned} & (\lambda + \alpha + \mu)(\lambda + (1-b)\eta_1 + b\eta_2 + \mu) \\ & + \frac{(1-b)\eta_1 + b\eta_2 + \mu}{S_0}R_0E^*(\lambda + (1-b)\eta_1 + b\eta_2 + \mu) \\ =& \frac{S^*}{S_0}G(\lambda)(\lambda + \alpha + \mu)((1-b)\eta_1 + b\eta_2 + \mu). \end{aligned} \tag{2.5}$$

The above formula can be obtained by simplification as

$$1+\frac{(1-b)\eta_1+b\eta_2+\mu}{S_0(\lambda+\alpha+\mu)}\cdot R_0\cdot E^*=\frac{S^*((1-b)\eta_1+b\eta_2+\mu)}{S_0(\lambda+(1-b)\eta_1+b\eta_2+\mu)}\cdot G(\lambda). \tag{2.6}$$

Furthermore, from the equilibrium equation we can obtain

$$\beta S^*(\omega E^*+K^*)-(1-b)\eta_1E^*-b\eta_2E^*-\mu E^*=0.$$

Finally, we obtain

$$R_0=\frac{S_0}{S^*}$$

Assumed $\mathrm{Re}(\lambda)\geq 0$, taking the norm [31] on the right side of the equal sign of equation (2.6) can get

$$\begin{aligned}\left|\frac{S^*((1-b)\eta_1+b\eta_2+\mu)}{S_0(\lambda+(1-b)\eta_1+b\eta_2+\mu)}\cdot G(\lambda)\right|&\leq\left|\frac{S^*((1-b)\eta_1+b\eta_2+\mu)}{S_0(\lambda+(1-b)\eta_1+b\eta_2+\mu)}\cdot R_0\right|\\&=\left|\frac{(1-b)\eta_1+b\eta_2+\mu}{\lambda+(1-b)\eta_1+b\eta_2+\mu}\right|\\&\leq 1.\end{aligned} \tag{2.7}$$

Taking the norm on the left side of the equal sign of equation (2.6) can get

$$\left|1+\frac{(1-b)\eta_1+b\eta_2+\mu}{S_0(\lambda+\alpha+\mu)}\cdot R_0\cdot E^*\right|>1. \tag{2.8}$$

At this point we can see that the inequalities (2.7) and (2.8) are contradictory to the equation (2.6), so the initial assumption about λ is invalid [30, 31]. Therefore, all the eigenvalues of the characteristic equation $\det(\lambda E-M)=0$ have negative real parts. When $R_0>1$, the endemic equilibrium of system (2.1) is locally asymptotically stable. □

Theorem 2.5. *When $R_0>1$, the endemic equilibrium point Z^* of system (2.1) is locally asymptotically stable.*

Obviously, where the equations of the endemic equilibrium Z^*

$$\Lambda=\beta S^*(\omega E^*+K^*)+(\alpha+\mu)S^*,(1-b)\eta_1+b\eta_2+\mu=\frac{\beta S^*(\omega E^*+K^*)}{E^*},$$
$$\theta\delta+d+\mu=\frac{(1-b)\eta_1E^*}{K^*}.$$

Define the same function $h(x)$ as above.

Proof. We define a function V_1 as follows [3],

$$V_1(\phi)=S^*h\left(\frac{\phi_1}{S^*}\right)+E^*h\left(\frac{\phi_2}{E^*}\right)+\frac{S^*\beta\mathrm{K}^*}{(1-b)\,\eta_1E^*}K^*h\left(\frac{\phi_3}{K^*}\right).$$

The derivative of V_1 with respect to $t\,(t>0)$ along $U(t)$ is

$$\frac{dV_1(U(t))}{dt}=\left(1-\frac{S^*}{S(t)}\right)\dot{S}(t)+\left(1-\frac{E^*}{E(t)}\right)\dot{E}(t)+\frac{S^*\beta K^*}{(1-b)\eta_1E^*}\left(1-\frac{K^*}{K(t)}\right)\dot{K}(t)$$

$$
\begin{aligned}
&= -\Lambda\left[h\left(\frac{S^*}{S(t)}\right)+h\left(\frac{S(t)}{S^*}\right)\right]+(\beta S^*K^*+\beta\omega S^*E^*)h\left(\frac{S(t)}{S^*}\right)\\
&\quad-\beta S^*K^*\left(\frac{E(t)K^*}{E^*K(t)}+\frac{S(t)E^*K(t)}{S^*E(t)K^*}-\ln\frac{S(t)}{S^*}-2\right)\\
&\quad-\beta\omega S^*E^*\left(\frac{S(t)}{S^*}-1-\ln\frac{S(t)}{S^*}\right)\\
&= -\Lambda h\left(\frac{S^*}{S(t)}\right)-(\alpha S^*+\mu S^*+\beta\omega S^*E^*)h\left(\frac{S(t)}{S^*}\right)\\
&\quad-\beta S^*K^*\left[h\left(\frac{E(t)K^*}{E^*K(t)}\right)+h\left(\frac{S(t)E^*K(t)}{S^*E(t)K^*}\right)\right]. \qquad (2.9)
\end{aligned}
$$

We know $h(x)\geq 0$, so we can get $dV_1(U(t))/dt\leq 0$ and $dV_1(U(t))/dt=0$ if and only if $U(t)=Z^*$. According to the LaSalle's Invariance Principle, when $R_0>1$, the endemic equilibrium Z^* of system (2.1) is globally asymptotically stable [8]. □

3. Results

3.1. Estimation of parameters

Based on the average incubation period for HIV infection being 9 years [24], it is obtained that $\eta_1=\eta_2=0.11$. Calculating from the reported AIDS deaths numbers by the National Health Commission of the People's Republic of China [19], we obtain $d=0.2898$. The average time for an HIV-infected individual to progress to AIDS being 10 years, we can obtain $\varepsilon=\theta=0.1$.

The initial value of S is obtained from the total population of individuals aged 15 to 74 in China as reported by the National Bureau of Statistics in 2012 [20]. The initial value of H is derived from the reported number of new HIV cases among individuals aged 15 to 74 in China in 2012 by the National Health Commission of the People's Republic of China [19]. The initial value for A is sourced from the reported number of AIDS cases among individuals aged 15 to 74 in China in 2012, published by National Health Commission of the People's Republic of China [19]. All the initial values for the model are set as follows: $S(0)=1010866460$, $E(0)=46000$, $K(0)=23187$, $H(0)=13001$, $A(0)=2971$.

Table 1. Value and source of parameter values.

Parameter	Value	Source	Parameter	Value	Source
β	1×10^{-9}	Fitted	q	0.4	[18]
b	0.62	Fitted	ε	0.1	Calculated
μ	0.007	[19]	θ	0.1	Calculated
d	0.2898	Calculated	δ	0.85	Fitted
η_1	0.11	Calculated	α	0.03	[17]
η_2	0.11	Calculated	ω	0.1	Fitted
Λ	23960000	[20]			

3.2. Fitting results

A nonlinear least squares fitting method to the annual new case data is used and we obtain the values of the parameters β , b, δ , ω, as shown in Table 1. The conversion rate of latent infected E to HIV-infected individuals H is $b = 0.62$, so the conversion rate of latent infected E to potential HIV-infected individuals K is $(1 - b) = 0.38$. Substituting these parameters into the expression for the basic reproduction number R_0, we obtain R_0=1.514. This means that the basic reproduction number for virus transmission is 1.514, which is greater than 1, without implementing any control measures, the HIV virus will continue to spread in the population.

And performing a goodness-of-fit test, in this paper, we use R^2 to define the goodness-of-fit coefficient, which is expressed as

$$R^2 = 1 - \frac{RSS}{TSS}$$

Where RSS stands for residual sum of squares, which represents the sum of squared deviations between the actual data and the simulated values. TSS stands for total sum of squares, which represents the sum of squares of deviations between actual and expected values, the fitting results are shown in Figure 5. We can get the goodness-of-fit coefficient of 0.95, its value is closer to 1 which means the fit is better and the fitted curves are consistent with the actual trends.

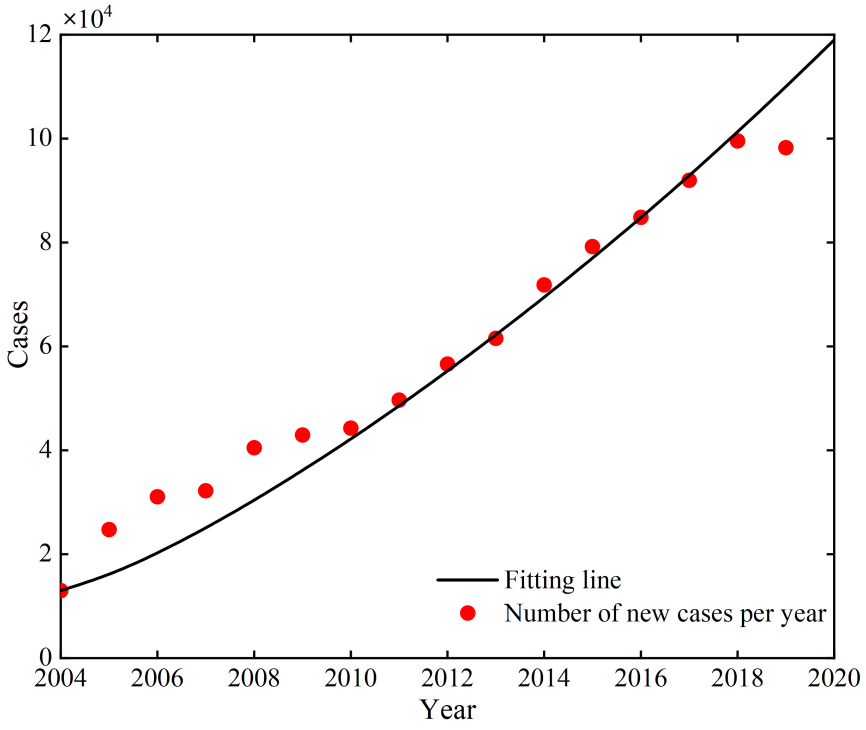

Figure 5. Fitting results for the annual number of new cases of HIV.

4. Simulation analysis

4.1. Proportion of susceptible individuals who changed their sexual habits per unit time α on potential HIV-infected individuals K

The proportion of susceptible individuals who changed their sexual habits per unit time α has an impact on changes of the number of potential HIV-infected individuals, as the proportion of susceptible individuals changed their sexual habits per unit time α increases, the number of potential HIV-infected individuals K reduces, the results are shown in Figure 6. The black curve represents our initial fitted curve, the mean value of the number of potential HIV-infected individuals K in the group of 15-74 years old is estimated to be 16847 and the ratio of potential HIV-infected individuals K to HIV-infected individuals H, is shown in Table 2. It is seen that the percentage of unchecked potential HIV-infected individuals K is the largest in 2004, which is 178.35%, which is 1.7835 times more than that of HIV-infected individuals H. Then it decreases year by year. In 2018, it is 18.74% and in 2019 it increases, with a K/H ratio of 20.06%. The average value of K/H from 2004 to 2019 is 38.41%, which means that the average ratio of potential HIV-infected individuals to HIV-infected individuals is 38.41%. Where the greater the proportion of susceptible individuals who changed their sexual habits per unit time, the greater the decrease in the number of potential HIV-infected individuals. Increasing the proportion of susceptible individuals who changed their sexual habits per unit time α by 0.5% reduces potential HIV-infected individuals K by an average of 2,061. As time increases, the proportion of susceptible individuals who changed their sexual habits per unit time α has a greater and greater impact on the change in the number of potential HIV-infected individuals. So to strengthen the publicity of AIDS, increase the proportion of susceptible individuals who changed their sexual habits per unit time α, So that taking more protective measures, practicing safe sex and maintaining good personal hygiene can greatly reduce the number of new HIV cases.

4.2. Proportion of susceptible individuals changed their sexual habits per unit time α on HIV-infected individuals H

The proportion of susceptible individuals who changed their sexual habits per unit time α can have an impact on the change in the number of HIV-infected individuals H, the results are shown in Figure 7. The black curve represents our initial fitted curve, the number of HIV-infected individuals H will still be increasing by 2034 and even if the proportion of susceptible individuals changed their sexual habits per unit time α is increased, the number of HIV-infected individuals H will decrease but will still be on an increasing trend. With the number of HIV patients decreasing as the proportion of susceptible individuals changed their sexual habits per unit time α increases, when the proportion of susceptible individuals changed their sexual habits per unit time α increased by 2%, the average reduction in HIV patients is 29,123. The analysis shows that the greater the proportion of susceptible individuals who changed their sexual habits per unit time, the lower the number of HIV patients, so it is necessary to increase public awareness of AIDS to keep HIV infections to a

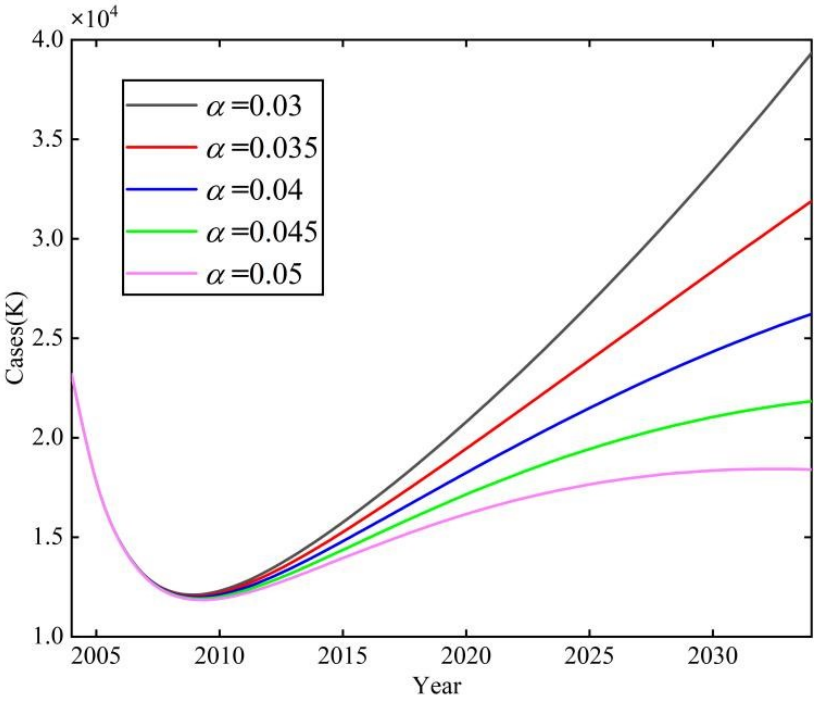

Figure 6. Proportion of susceptible individuals who changed their sexual habits per unit time α on potential HIV-infected individuals K.

low minimum. People learn more about AIDS and change their own sexual habits to keep away from AIDS.

4.3. Proportion of susceptible individuals who changed their sexual habits per unit time α on HIV carriers (K+H)

The simulation obtained that the proportion of susceptible individuals who changed their sexual habits per unit time α has an effect on the change in the number of HIV carriers (K+H), as is shown in Figure 8. As the proportion of susceptible individuals who changed their sexual habits per unit time α increases, the sum of the number of potential HIV-infected individuals K and the number of people with HIV H decreases, see Figure 8(A). When α increases by an average of 10%, the sum of the number of potential HIV-infected individuals K and the number of diagnosed infected individuals H decreases by an average of 12,779. When α is 3%, the sum of the number of potential HIV-infected individuals K and HIV-infected individuals H is on average 144,870. When α is 43%, the sum of the number of potential HIV-infected individuals K and the number of HIV-infected individuals H is on average 37,126. As we increase the proportion of susceptible individuals who changed their sexual habits per unit time α, the sum of the number of potential HIV-infected individuals K and HIV-infected individuals H peaks first, followed by a decrease in the sum of the number of cases, see Figure 8(B). When α is 53%, starting in 2021, the sum of the number of potential HIV-infected individuals K and HIV-infected individuals H gradually begins to fall below the sum of the number

Table 2. Ratio of potential HIV-infected individuals K to HIV-infected individuals H.

Year	K	H	Value(K/H)	Cumulative percentage
2004	23187	13001	1.7835	1.7835
2005	17795	24741	0.7193	1.0859
2006	14695	31065	0.4730	0.8092
2007	13034	32241	0.4043	0.6800
2008	12280	40497	0.3032	0.5722
2009	12104	42958	0.2818	0.5046
2010	12301	44265	0.2779	0.4607
2011	12744	49672	0.2566	0.4243
2012	13352	56615	0.2358	0.3924
2013	14073	61549	0.2286	0.3670
2014	14881	71864	0.2071	0.3425
2015	15752	79216	0.1989	0.3217
2016	16679	84864	0.1965	0.3049
2017	17649	91992	0.1919	0.2906
2018	18662	99561	0.1874	0.2781
2019	19712	98278	0.2006	0.2698
Average value			0.3841	

Note: The cumulative percentage is calculated by dividing the cumulative number of cases for K by the cumulative number of cases for H.

of cases in 2004. Therefore, we can effectively reduce the number of HIV carriers (K+H) by stepping up our awareness-raising efforts to make more and more people aware of AIDS and have good sexual habits, and by increasing the proportion of susceptible individuals who changed their sexual habits per unit time. The proportion of susceptible individuals who changed their sexual habits per unit time α affects the proportion of potential HIV-infected individuals K in the number of potential HIV-infected individuals K and HIV-infected individuals H. The proportion of potential HIV-infected individuals K varies with the proportion of susceptible individuals who changed their sexual habits per unit time α, as shown in Table 3. The simulation results are shown in Figure 9, it can be seen that the greater the proportion of susceptible individuals who changed their sexual habits per unit time α, the smaller the proportion of potential HIV-infected individuals K. α increases by 10%, the percentage of potential HIV-infected individuals K decreased by an average of 3.62%, with a consistent trend across all curves. The percentage of potential HIV-infected individuals K is decreasing over time. When α is 3%, the average percentage of potential HIV-infected individuals K is 20.21%. When α is 43%, the average percentage of potential HIV-infected individuals K is 14.31%. The proportion of susceptible individuals who changed their sexual habits per unit time α increases, which not only leads to a reduction in the number of potential HIV-infected individuals K and HIV-infected individuals H, but also effectively reduces the proportion of potential HIV-infected individuals K. Therefore, it is necessary to

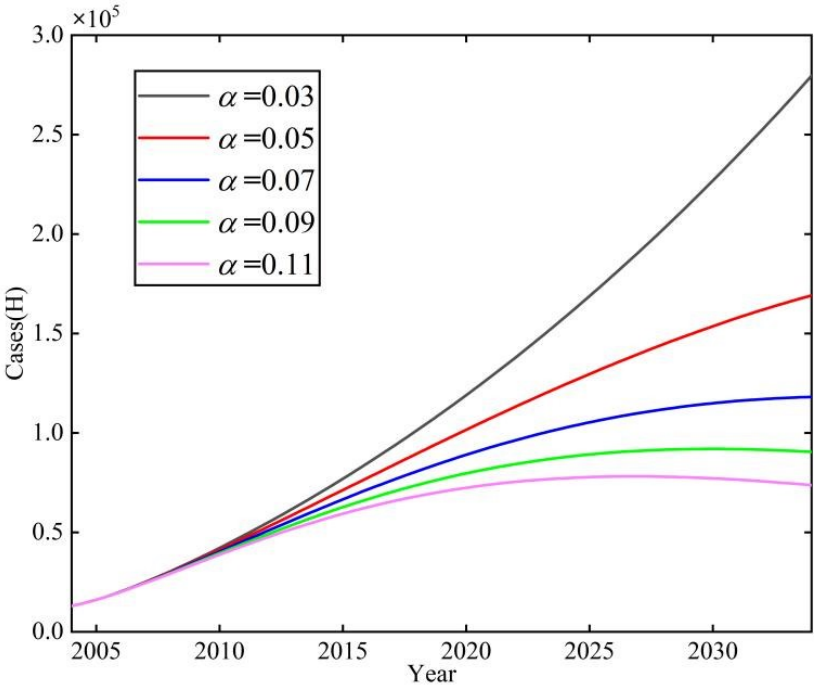

Figure 7. Proportion of susceptible individuals who changed their sexual habits per unit time on HIV-infected individuals H.

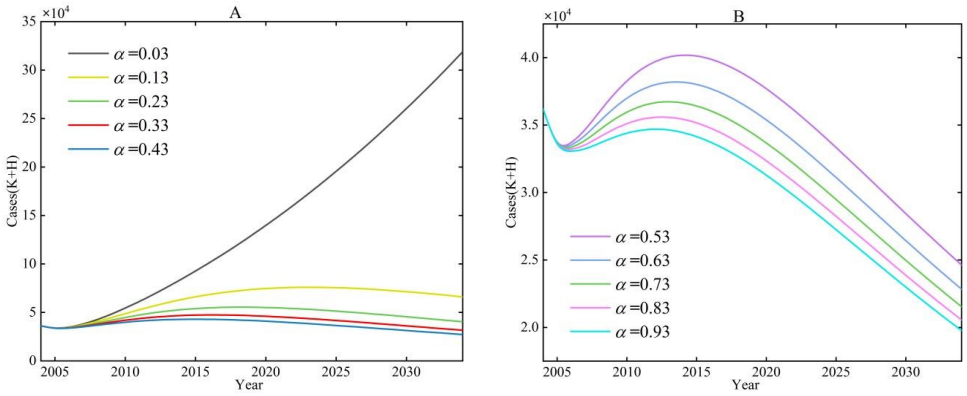

Figure 8. Impact of the proportion of susceptible individuals changed their sexual habits per unit time α on the number of HIV carriers (K+H).

increase public awareness regarding AIDS prevention and treatment to advocate a healthy lifestyle in helping to reduce the number of infected persons.

4.4. Impact of the mortality rate due to disease d on potential HIV-infected individuals K, HIV-infected individuals H

The mortality rate due to disease d can have an impact on changes in the number of potential HIV-infected individuals K, the results are shown in Figure 10. The black

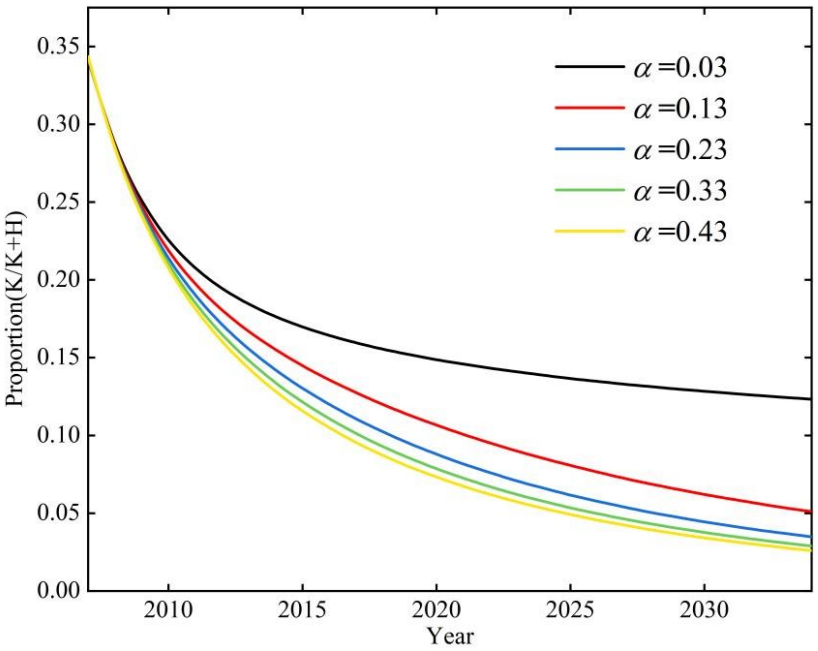

Figure 9. Effect of the proportion of susceptible individuals changed their sexual habits per unit time α on the proportion of potential HIV-infected individuals K.

curve is our original fitted curve, the number of potential HIV-infected individuals K is decreasing as the mortality rate due to disease d increases. When d increases by 3%, the number of potential HIV-infected individuals K decreases by an average of 3017. The above analysis reveals that the mortality rate due to disease d has an increasing impact on changes in the number of potential HIV-infected individuals K over time.

The mortality rate due to disease d has an impact on the number of HIV-infected individuals H, the results are shown in Figure 11. The black curve is our initial fitting curve, the number of people living with HIV is decreasing as the mortality rate due to disease d increases, with the number of people living with HIV decreasing by an average of 23,749 when the mortality rate due to disease d increases by 10% between 2004 and 2034.

5. Discussion and conclusions

This study established a dynamic model that includes a compartment for potential HIV-infected individuals in the 15-74 age group, the average number of potential HIV-infected individuals is 16,847, the average ratio of potential HIV-infected individuals to HIV-infected individuals from 2004 to 2019 is 38.41%. With the basic reproduction number R_0=1.514 for HIV virus transmission in China, it indicates that the spread of HIV in China is still in a severe situation, the number of new AIDS patients continues to rise at a high level within the population. Therefore, there is a need to enhance control measures to slow down the spread of the virus.

This study considers the impact of different parameters on the transmission of

Table 3. Percentage of potential HIV-infected individuals K in HIV carrier compartments (K+H).

Year	$\alpha = 0.03$ $\frac{K}{K+H}$	$\alpha = 0.13$ $\frac{K}{K+H}$	$\alpha = 0.23$ $\frac{K}{K+H}$	$\alpha = 0.33$ $\frac{K}{K+H}$	$\alpha = 0.43$ $\frac{K}{K+H}$
2004	0.6407	0.6407	0.6407	0.6407	0.6407
2005	0.5242	0.5244	0.5246	0.5248	0.5250
2006	0.4202	0.4209	0.4215	0.4221	0.4227
2007	0.3419	0.3423	0.3427	0.3431	0.3435
2008	0.2876	0.2865	0.2857	0.2852	0.2848
2009	0.2509	0.2473	0.2446	0.2425	0.2411
2010	0.2256	0.2189	0.2140	0.2104	0.2077
2011	0.2077	0.1975	0.1903	0.1852	0.1815
2012	0.1945	0.1806	0.1713	0.1649	0.1604
2013	0.1844	0.1668	0.1555	0.1481	0.1430
2014	0.1764	0.1551	0.1421	0.1338	0.1283
2015	0.1698	0.1448	0.1303	0.1215	0.1158
2016	0.1643	0.1357	0.1200	0.1108	0.1050
2017	0.1596	0.1275	0.1107	0.1013	0.0955
2018	0.1555	0.1199	0.1024	0.0929	0.0872
2019	0.1519	0.1130	0.0948	0.0854	0.0798
2020	0.1487	0.1066	0.0880	0.0786	0.0733
2021	0.1459	0.1007	0.0817	0.0726	0.0674
2022	0.1432	0.0952	0.0761	0.0671	0.0621
2023	0.1409	0.0900	0.0708	0.0621	0.0574
2024	0.1387	0.0852	0.0661	0.0576	0.0531
2025	0.1366	0.0807	0.0617	0.0535	0.0492
2026	0.1348	0.0765	0.0577	0.0498	0.0456
2027	0.1330	0.0725	0.0540	0.0464	0.0424
2028	0.1314	0.0688	0.0505	0.0432	0.0394
2029	0.1298	0.0654	0.0474	0.0403	0.0367
2030	0.1284	0.0621	0.0445	0.0377	0.0342
2031	0.1270	0.0591	0.0418	0.0352	0.0319
2032	0.1257	0.0562	0.0393	0.0330	0.0298
2033	0.1245	0.0535	0.0370	0.0309	0.0278
2034	0.1233	0.0510	0.0348	0.0290	0.0260

the HIV virus. We find that the proportion of susceptible individuals changed their sexual habits per unit time α influences the spread of the HIV virus. Additionally, changes in the mortality rate due to disease d have an impact on the numbers of HIV-infected individuals (H) and potential HIV-infected individuals (K). In our research, we delve into the impact of these parameters on model predictions, especially the

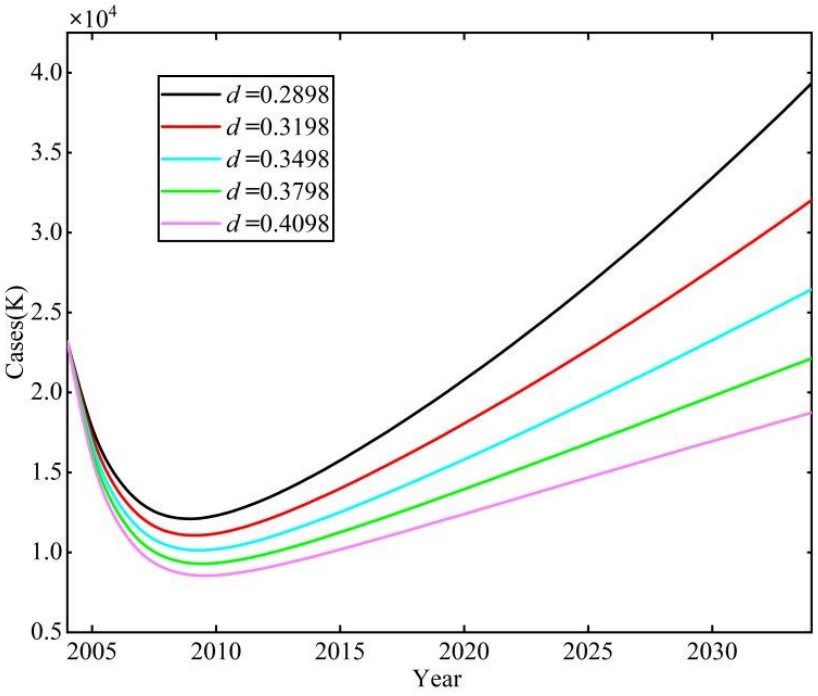

Figure 10. Impact of the mortality rate due to disease d on potential HIV-infected individuals K.

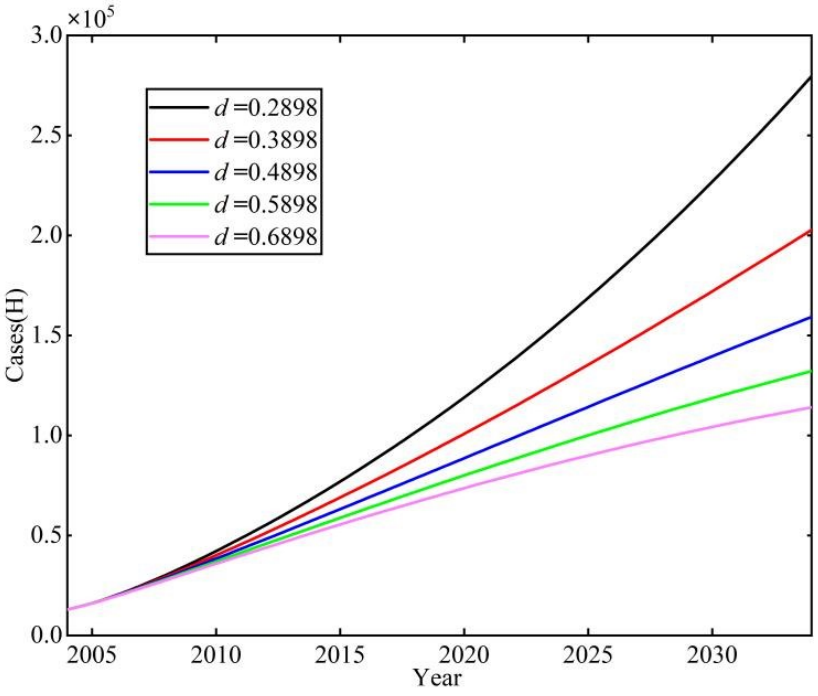

Figure 11. The effect of the mortality rate due to disease d on HIV-infected individuals H.

proportion of individuals in the potential infection compartment.We observe that the number of potential HIV-infected individuals constitutes a significant portion of the overall infected population. This finding is of great importance in devising effective prevention and control strategies because interventions targeting potential infection cases may have a significant impact on disease transmission. Furthermore, we analyze the basic reproduction number, a critical indicator for assessing disease transmission potential. Our results indicate that the basic reproduction number is greater than 1, implying that the disease may continue to spread. Therefore, to control the disease's spread, proactive prevention and control measures, such as enhanced education and awareness campaigns are necessary. In summary, our study reveals the important role of potential HIV-infected individuals in the spread of the disease and provides an important basis for the development of targeted prevention and control strategies.

This research obtains there is a high number of potentially infectious HIV-infected individuals in China. Currently, there is no effective vaccine for preventing AIDS [28]. In order to better control the spread of the HIV virus, the most effective measures to prevent and control AIDS is to increase publicity, adopt health education and behavioral intervention strategies, increasing awareness of AIDS transmission and to focus on personal protection and cleanliness. The elimination of drug use, the avoidance of contaminated blood, semen and breast milk of AIDS patients will also likely decrease the channels. China promotes early diagnosis, early treatment and early detection of acute HIV infections. Strengthening integrated interventions for key populations (gay and lesbian populations, drug users). The use of new media and new technologies to publicize and educate high-risk groups and college students, to innovate social management and to expand the scope of HIV testing [32].

This study provides important clues for understanding the key role of potential HIV-infected individuals in transmission and provides useful insights for developing targeted control strategies. However, there is certain limitations in this research, such as the complete reliability of the data used, future studies can further consider these factors and incorporate more real-world data to refine our model and conclusions. Since the incidence of HIV varies across different age groups, our subsequent research will explore the impact of heterogeneity on HIV virus transmission and seek effective strategies for controlling HIV incidence.

Acknowledgements

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