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EXISTENCE OF SOLUTIONS FOR THE HAUSDORFF DERIVATIVE FRACTIONAL THERMISTOR PROBLEM*

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Abstract In this work, we examine the existence of solutions for the fractional thermistor problem involving the Hausdorff derivative. Our approach leads to a generalized thermistor model formulated within the framework of Hausdorff fractional calculus. To establish the existence of solutions, we employ the Schauder fixed-point theorem in conjunction with the tube solution method, which extends the classical concepts of lower and upper solutions. Finally, an illustrative example is presented to demonstrate the applicability of the obtained results.

Keywords Thermistor problems, Hausdorff fractional derivative, iterative methods, solution tube, Schauder fixed-point theorem, fractional derivatives.

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1. Introduction

The study of thermistor problems has attracted significant attention due to their wide-ranging applications in physics, engineering, and materials science, particularly in the modeling of temperature-dependent electrical conductivity. Classical thermistor models are typically formulated using standard differential operators; however, recent progress in fractional calculus has opened new avenues for incorporating memory and non-local effects in heat and electrical conduction processes. Among the emerging fractional approaches, the Hausdorff derivative has gained notable interest for its ability to capture fractal and heterogeneous structures within physical media, offering a more realistic description of anomalous diffusion and heat transfer phenomena; (see [7, 8, 13]). A thermistor is a temperature-sensitive resistor whose electrical resistance varies with changes in temperature. The term *thermistor* is derived from the words thermal and resistor. Thermistors are generally classified into two categories: Negative temperature coefficient (NTC) and positive temperature coefficient (PTC) types. For NTC thermistors, the resistance decreases as the temperature increases, exhibiting a nonlinear inverse relationship between resistance and temperature. Owing to their sensitivity and reliability, thermistors are widely used in various applications, including computers, digital thermostats, aviation systems,

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portable heating devices, automotive sensors, medical instrumentation, electrical appliances, and chemical processing industries.

Research on thermistor problems, addressing issues such as existence, uniqueness, stability, and multiplicity of solutions, can be found, for instance, in [14,24,30].

In [5], the authors investigated a nonlocal conformable fractional thermistor problem using the solution-tube method:

$$\begin{cases} u^{(\nu)}(s) = \dfrac{\lambda f(s,u(s))}{\left(\int_{\sigma_1}^{\sigma_2} f(s,u(s))ds\right)^2}, \quad s \in [\sigma_1, \sigma_2], \\ u(\sigma_1) = x_{\sigma_1}, \end{cases}$$

where $\lambda > 0$, $\sigma_2 > \sigma_1 > 0$ and $u^{(\nu)}$ denotes the conformable fractional derivative of order ν.

In [4], existence and uniqueness results were obtained for a fractional Riemann–Liouville nonlocal thermistor problem formulated on arbitrary time scales.

$$\begin{cases} {}_{\sigma_1}^{\mathbb{T}}\mathcal{D}_t^{2\nu} u(t) = \dfrac{\lambda f(u)}{\left(\int_{\sigma_1}^{\sigma_2} f(u) \bigtriangleup u\right)^2}, \quad t \in \mathbb{T}, \\ {}_{\sigma_1}^{\mathbb{T}}\mathcal{I}_t^{\nu} u(\sigma_1) = 0, \quad \text{for all}\, \nu \in (0,1), \end{cases}$$

here $f : [0,\infty) \to [0,\infty)$ is Lipschitz continuous, ${}_{t_0}^{\mathbb{T}}\mathcal{D}_t^{2\nu}$ denotes the left Riemann–Liouville fractional derivative of order 2ν on $\mathbb{T}$, and ${}_{t_0}^{[\sigma_1,\sigma_2]}\mathcal{I}_t^{\nu}$ represents the left Riemann-Liouville fractional integral operator of order ν defined on $\mathbb{T}$. For more details on the Riemann-Liouville fractional derivative on the time scale, see [10,22].

Bendouma et al. [8] discussed the existence of a solution for the following non-local ∇-conformable fractional thermistor problem on a time scale

$$\begin{cases} x_{\nabla}^{(\varpi)}(t) = \dfrac{\lambda f(t, x^{\rho}(t))}{\left(\int_{\sigma_1}^{\sigma_2} f(s, x^{\rho}(s))\, \nabla s\right)^2}, \quad \text{for all } t \in \mathbb{T}, \\ x(\sigma_2) = x_{\sigma_2}, \end{cases}$$

where, $\sigma_2 = \max \mathbb{T}$, λ is a fixed positive real number, $f : \mathbb{T} \times [0,\infty) \to (0,\infty)$ is a continuous function, x describes the temperature of the conductor, and $x_{\nabla}^{(\varpi)}(t)$ denotes the nabla conformable fractional derivative of x at t of order $\varpi \in (0,1)$.

Here, we address the problem of establishing the existence of solutions for the Hausdorff derivative fractional thermistor model given by

$$\begin{cases} \mathcal{D}_{p(t)}^{\lambda} x\,(t) = \dfrac{\gamma f(t, x(t))}{\left(\int_{\sigma_1}^{\sigma_2} f(s, x(s))ds\right)^2}, \quad \text{for all } t \in [\sigma_1, \sigma_2]\,, \\ x(\sigma_2) = x_{\sigma_2}. \end{cases} \tag{1.1}$$

Here, $\mathcal{D}_{p(t)}^{\lambda}$ denotes the generalized Hausdorff derivative of x at t associated with the parameter λ and the function $p : [\sigma_1, \sigma_2] \to \mathbb{R}$; see [1,26,29,31]. This derivative is generalized by Benaissa et al [6], where λ is a quotient of odd positive integers, γ is a fixed positive real number, $f : [\sigma_1, \sigma_2] \times [0,\infty) \to (0,\infty)$ is a continuous function, and x describes the temperature of the conductor.

The Hausdorff derivative is a non-Newtonian extension of the derivative found in fractal geometry, which deals with the measurement of fractals in applied mathematics and mathematical analysis. The study of anomalous diffusion, in which conventional methods neglect the fractal character of the media, has led to the development of fractal derivatives.

Motivated by these developments, this work focuses on the existence of solutions for a fractional thermistor problem governed by the Hausdorff derivative. By reformulating the thermistor equation within the framework of Hausdorff fractional calculus, we extend the traditional thermistor model to incorporate fractal temporal dynamics. This generalization enables a more accurate representation of complex media where classical integer-order models fail to reflect the underlying physical behavior; see [2–4].

To establish the existence of solutions to the proposed problem, we utilize a combination of analytical tools. Specifically, we employ the Schauder fixed-point theorem in conjunction with the tube solution technique, the latter serving as a generalized form of the classical lower and upper solution methods. The tube solution approach provides a flexible and effective mechanism for constructing a priori bounds and ensuring the existence of admissible solutions within a prescribed function space.

The contribution of this work is twofold. First, we provide a rigorous mathematical framework for analyzing thermistor models involving the Hausdorff fractional derivative. Second, we demonstrate the applicability of the obtained theoretical results through a concrete example that illustrates the behavior of the solution under the proposed model. The results presented herein enrich the existing literature on fractional thermistor problems and highlight the potential of Hausdorff calculus as a valuable tool in modeling complex thermal–electrical systems; see [21]. In fact, we introduce the notion of a solution-tube of (1.1); afterward, we will prove the existence of a solution to problem (1.1) by using the method of solution-tubes and Schauder's fixed point theorem.

2. Auxiliary result

This section's definitions and traits are all taken from the article [11] under the following situations $\mathbb{T} = \mathbb{R}$, see [11].

Definition 2.1. [11] Assume $x, p : [\sigma_1, \sigma_2] \to \mathbb{R}$, $t \in [\sigma_1, \sigma_2]$, and $\lambda, \mu > 0$; we say that x is $\mathcal{D}^{\lambda}_{p(t)}$-differentiable if and only if the limit

$$\lim_{s \to t} \frac{x^{\lambda}(t) - x^{\lambda}(s)}{p(t) - p(s)},$$

exists as a finite number. In this case,

$$\mathcal{D}^{\lambda}_{p(t)} x(t) = \lim_{s \to t} \frac{x^{\lambda}(t) - x^{\lambda}(s)}{p(t) - p(s)}.$$

Lemma 2.1. [11] *Let $x, p : [\sigma_1, \sigma_2] \to \mathbb{R}$, $t \in [\sigma_1, \sigma_2]$, and $\lambda > 0$, such that x is $\mathcal{D}^{\lambda}_{p(t)}$-differentiable at $t \in [\sigma_1, \sigma_2]$. Then the following properties hold:*

1. *The operator $\mathcal{D}^{\lambda}_{p(t)}$ is not linear.*
2. *If p is continuous, then x is continuous.*

3. *If p is increasing and $\mathcal{D}^{\lambda}_{p(t)}x(t) \leq 0$, for all $t \in [\sigma_1, \sigma_2]$, then x is decreasing on $[\sigma_1, \sigma_2]$.*

Lemma 2.2. [11] *Let $x, y, p : [\sigma_1, \sigma_2] \to \mathbb{R}$, $t \in [\sigma_1, \sigma_2]$, and $\lambda, \mu > 0$, such that x, y are $\mathcal{D}^{\lambda}_{p(t)}$-differentiable at $t \in [\sigma_1, \sigma_2]$. Then the following properties hold:*

1. *For any constant γ, the function $\gamma x : [\sigma_1, \sigma_2] \to \mathbb{R}$ is $\mathcal{D}^{\lambda}_{p(t)}$-differentiable at t, with*

$$\mathcal{D}^{\lambda}_{p(t)}(\gamma x)(t) = \gamma^{\lambda}\mathcal{D}^{\lambda}_{p(t)}x(t).$$

2. *The product $x, y : [\sigma_1, \sigma_2] \to \mathbb{R}$ is $\mathcal{D}^{\lambda}_{p(t)}$-differentiable at t, with*

$$\mathcal{D}^{\lambda}_{p(t)}(xy)(t) = y^{\lambda}(t)\mathcal{D}^{\lambda}_{p(t)}x\,(t) + x^{\lambda}(t)\mathcal{D}^{\lambda}_{p(t)}y(t).$$

3. *If $y(t) \neq 0$, then $\frac{x}{y}$ is $\mathcal{D}^{\lambda}_{p(t)}$-differentiable at t with*

$$\mathcal{D}^{\lambda}_{p(t)}\left(\frac{x}{y}\right)(t) = \frac{y^{\lambda}(t)\mathcal{D}^{\lambda}_{p(t)}x(t) - x^{\lambda}(t)\mathcal{D}^{\lambda}_{p(t)}y(t)}{y^{2\lambda}(t)}.$$

Now we introduce the $\mathcal{I}^{\lambda}_{p(t),\sigma_1}$-integrations on $[\sigma_1, \sigma_2]$.

Lemma 2.3. [11] *Let $x, p : [\sigma_1, \sigma_2] \to \mathbb{R}$, such that x is a regulated function and p is differentiable on $[\sigma_1, \sigma_2]$, λ is a quotient of odd positive integers and $\lambda > 0$, we define the $\mathcal{I}^{\lambda}_{p(t),\sigma_1}$-integrable function on $[\sigma_1, \sigma_2]$ by*

$$\mathcal{I}^{\lambda}_{p(t),\sigma_1}x\,(t) = \left(\int_{\sigma_1}^{t} x\,(s)\,d_p s\right)^{\frac{1}{\lambda}} = \left(\int_{\sigma_1}^{t} x\,(s)\,p^{'}\,(s)\,ds\right)^{\frac{1}{\lambda}}.$$

Lemma 2.4. [11] *Let $x : [\sigma_1, \sigma_2] \to \mathbb{R}$, such as x be a regulated function, and λ be a quotient of an odd positive integer. Then*

$$\mathcal{I}^{\lambda}_{p(t),\sigma_1}\left(\mathcal{D}^{\lambda}_{p(t)}x\right)(t) = \left[x^{\lambda}(t) - x^{\lambda}(\sigma_1)\right]^{\frac{1}{\lambda}}, \quad \textit{for all } t \in [\sigma_1, \sigma_2].$$

If $x(\sigma_1) = 0$, we have

$$\mathcal{I}^{\lambda}_{p(t),\sigma_1}\left(\mathcal{D}^{\lambda}_{p(t)}x\right)(t) = x(t), \quad \textit{for all } t \in [\sigma_1, \sigma_2].$$

Lemma 2.5. [11] *Let $x, p : [\sigma_1, \sigma_2] \to \mathbb{R}$ be such that x is a regulated function, p is differentiable on $[\sigma_1, \sigma_2]$, λ is a quotient of an odd positive integer, and $\lambda > 0$, then*

$$\mathcal{D}^{\lambda}_{p(t)}\left(\mathcal{I}^{\lambda}_{p(t),\sigma_1}x\right)(t) = x\,(t)\,, \quad \textit{for all } t \in [\sigma_1, \sigma_2].$$

3. Main results

In this section, we establish an existence result for the problem (1.1).

Notation 3.1. Let $p : [\sigma_1, \sigma_2] \to \mathbb{R}$ and $\lambda > 0$, then

- $\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$ is the Banach space of all continuous real-valued functions defined on $[\sigma_1, \sigma_2]$ and has the norm
$$\|x\|_\infty = \sup_{t\in[\sigma_1,\sigma_2]} |x(t)|.$$
- Space $\mathcal{C}^\lambda_{p(t)}([\sigma_1, \sigma_2], \mathbb{R})$ is given by:
$$\mathcal{C}^\lambda_{p(t)}([\sigma_1, \sigma_2], \mathbb{R}) = \left\{x \text{ is } \mathcal{D}^\lambda_{p(t)}\text{-differentiable, with } \mathcal{D}^\lambda_{p(t)} x \in \mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})\right\}.$$
- Space $L^1_{p(t)}([\sigma_1, \sigma_2], \mathbb{R})$ is given by:
$$L^1_{p(t)}([\sigma_1, \sigma_2], \mathbb{R}) = \left\{x : [\sigma_1, \sigma_2] \to \mathbb{R} : \int_{\sigma_1}^{\sigma_2} x(s) \left|p'(s)\right| ds < \infty\right\}.$$
- The set $\widetilde{\mathbb{Q}}$ is defined by:
$$\widetilde{\mathbb{Q}} = \left\{\lambda := \frac{2n+1}{2\eta+1} : \eta, n \in \mathbb{N} \text{ and } \eta \wedge n = 1\right\}.$$
- Space $\mathcal{C}_i([\sigma_1, \sigma_2], \mathbb{R})$ is given by:
$$\mathcal{C}_i([\sigma_1, \sigma_2], \mathbb{R}) = \{x \in \mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}) : x \text{ is increasing on } [\sigma_1, \sigma_2]\}.$$
- $\eta \in \mathcal{C}^\lambda_{p(t)}([\sigma_1, \sigma_2], \mathbb{R})$ denotes
$$\mathcal{T}^\lambda_{p(t)}(\eta) := \left\{x \in \mathcal{C}^\lambda_{p(t)}([\sigma_1, \sigma_2], \mathbb{R}) : |x(t)| \leq \eta(t), \text{ for all } t \in [\sigma_1, \sigma_2]\right\}.$$
- The function $\widetilde{f}$ is given by:
$$\widetilde{f}(t, x) = \frac{\gamma f(t, x)}{\left(\int_{\sigma_1}^{\sigma_2} f(s, x)\, ds\right)^2}.$$

A solution to problem (1.1) will be a function $x \in \mathcal{C}^\lambda_{p(t)}([\sigma_1, \sigma_2], \mathbb{R})$ for which (1.1) is satisfied. We introduce the notion of the solution tube for this problem as follows:

Definition 3.1. Let
$$\eta \in \mathcal{C}^\lambda_{p(t)}([\sigma_1, \sigma_2], \mathbb{R}),$$
we say that $(0, \eta)$ is a solution tube of (1.1), if

(i) $\eta^\lambda x^\lambda \widetilde{f}(t, x) \leq x^\lambda \mathcal{D}^\lambda_{p(t)} \eta(t)$, for every $t \in [\sigma_1, \sigma_2]$ and for every $x \in \mathbb{R}$ such that $|x| = \eta(t)$,

(ii) $f(t, 0) = 0$ and $\mathcal{D}^\lambda_{p(t)} \eta(t) = 0$ for all $t \in [\sigma_1, \sigma_2]$ such that $\eta(t) = 0$,

(iii) $|x_{\sigma_2}| \leq \eta(\sigma_2)$.

Consider the following modified problem:
$$\begin{cases} \mathcal{D}^\lambda_{p(t)} x(t) + \lambda x^\lambda(t) = \widetilde{f}(t, x^*(t)) + \lambda (x^*)^\lambda(t), & t \in [\sigma_1, \sigma_2], \\ x(\sigma_2) = x_{\sigma_2}, \end{cases} \tag{3.1}$$
where
$$x^*(t) = \begin{cases} \dfrac{\eta(t)}{|x(t)|} x(t), & \text{if } |x(t)| > \eta(t), \\ x(t), & \text{if } |x(t)| \leq \eta(t). \end{cases}$$

Lemma 3.1. *Let $x, p : [\sigma_1, \sigma_2] \to \mathbb{R}$ and $k \in \mathbb{R}$, such that x is $\mathcal{D}^{\lambda}_{p(t)}$ -differentiable at $t \in [\sigma_1, \sigma_2]$ and $x(t) \neq 0$, then we have*

1. *The function $|x|$ is $\mathcal{D}^{\lambda}_{p(t)}$-differentiable at t and*

$$\mathcal{D}^{\lambda}_{p(t)}|x(t)| = \frac{|x(t)|^{\lambda}}{x^{\lambda}(t)}\mathcal{D}^{\lambda}_{p(t)}x(t). \tag{3.2}$$

2. *The function $x + k$ is $\mathcal{D}^{\lambda}_{p(t)}$ -differentiable at t and*

$$\mathcal{D}^{\lambda}_{p(t)}\left(x(t) + k\right) = \frac{\left(x\left(t\right) + k\right)^{\lambda - 1}}{x\left(t\right)^{\lambda - 1}}\mathcal{D}^{\lambda}_{p(t)}x(t). \tag{3.3}$$

Proof. Let x be $\mathcal{D}^{\lambda}_{p(t)}$-differentiable at $t \in [\sigma_1, \sigma_2]$. From Definition 2.1, we obtain

$$\begin{aligned}
\mathcal{D}^{\lambda}_{p(t)}\left|x\left(t\right)\right| &= \frac{|x(t)|^{\lambda}}{x^{\lambda}(t)}\lim_{s\to t}\frac{x^{\lambda}(t) - x^{\lambda}(s)}{p\left(t\right) - p\left(s\right)}.\lim_{a\to 1}\frac{1 - |a|^{\lambda}}{1 - a^{\lambda}} \\
&= \frac{|x(t)|^{\lambda}}{x^{\lambda}(t)}\mathcal{D}^{\lambda}_{p(t)}x(t).
\end{aligned}$$

Let $k \in \mathbb{R}$. By Definition 2.1, we have

$$\begin{aligned}
\mathcal{D}^{\lambda}_{p(t)}\left(x\left(t\right) + k\right) &= \lim_{s\to t}\frac{\left(x(t) + k\right)^{\lambda} - \left(x(s) + k\right)^{\lambda}}{p\left(t\right) - p\left(s\right)} \\
&= \lim_{s\to t}\frac{x^{\lambda}(t) - x^{\lambda}(s)}{p\left(t\right) - p\left(s\right)}.\lim_{s\to t}\frac{\left(x(t) + k\right)^{\lambda} - \left(x(s) + k\right)^{\lambda}}{x^{\lambda}(t) - x^{\lambda}(s)} \\
&= \mathcal{D}^{\lambda}_{p(t)}x(t).\lim_{\rho\to\eta=x(t)}\frac{\left(\eta + k\right)^{\lambda} - \left(\rho + k\right)^{\lambda}}{\eta^{\lambda} - \rho^{\lambda}} \\
&= \mathcal{D}^{\lambda}_{p(t)}x(t).\lim_{\rho\to\eta=x(t)}\frac{\left(\eta + k\right)^{\lambda - 1}}{\eta^{\lambda - 1}} \\
&= \frac{\left(x\left(t\right) + k\right)^{\lambda - 1}}{x\left(t\right)^{\lambda - 1}}\mathcal{D}^{\lambda}_{p(t)}x(t).
\end{aligned}$$

□

We present the following maximum principle.

Lemma 3.2. *Let*

$$\lambda \in \widetilde{\mathbb{Q}}, p \in \mathcal{C}_i(\left[\sigma_1, \sigma_2\right], \mathbb{R}),$$

and

$$\omega \in \mathcal{C}^{\lambda}_{p(t)}(\left[\sigma_1, \sigma_2\right], \mathbb{R}),$$

satisfy

$$\mathcal{D}^{\lambda}_{p(t)}\omega(t) > 0 \text{ on } \left\{t \in \left[\sigma_1, \sigma_2\right] : \omega(t) > 0\right\},$$

and $\omega(\sigma_2) \leq 0$. Then, $\omega(t) \leq 0$, for all $t \in \left[\sigma_1, \sigma_2\right]$.

Proof. Suppose the conclusion is false. Then, there exists $t_0 \in [\sigma_1, \sigma_2]$ such that

$$\omega(t_0) = \max_{t\in[\sigma_1,\sigma_2]} \omega(t) > 0,$$

as $p \in \mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$, then ω is continuous on $[\sigma_1, \sigma_2]$. Since $\omega(\sigma_2) \leq 0 < \omega(t_0)$, we must have $t_0 < \sigma_2$. Hence, there exists $t_1 \in (t_0, \sigma_2]$ such that $\omega(t) > 0$ for all $t \in (t_0, t_1]$. By the positivity assumption on $\mathcal{D}^{\lambda}_{p(t)}\omega$ and Lemma 2.4,

$$0 < \left(\int_{t_0}^{t_1} \mathcal{D}^{\lambda}_{p(s)}\omega(s)d_p s\right)^{\frac{1}{\lambda}} = \left[\omega^{\lambda}(t_1) - \omega^{\lambda}(t_0)\right]^{\frac{1}{\lambda}}.$$

However, t_0 is a point of maximum, so $\omega(t_1) \leq \omega(t_0)$, and therefore the last term is non-positive. This contradiction proves that no such t_0 exists; hence $\omega(t) \leq 0$ on $[\sigma_1, \sigma_2]$. □

Lemma 3.3. *Let $\lambda \in \widetilde{\mathbb{Q}}$, $p \in \mathcal{C}_i([\sigma_1, \sigma_2], \mathbb{R})$ and $\xi \in L^1_{p(t)}([\sigma_1, \sigma_2], \mathbb{R})$. The function x is a solution to the problem*

$$\begin{cases} \mathcal{D}^{\lambda}_{p(t)}x(t) + \lambda x^{\lambda}(t) = \xi(t), & \text{for all } t \in [\sigma_1, \sigma_2], \\ x(\sigma_2) = x_{\sigma_2}, \end{cases}$$

if and only if it is written in the following form

$$x(t) = \left(x^{\lambda}_{\sigma_2} e^{-\lambda(p(t)-p(\sigma_2))} + \int_{\sigma_1}^{\sigma_2} \Psi^{\lambda}_p(t,s)\, g(s)\, dp(s)\right)^{\frac{1}{\lambda}}, \quad \text{for all } t \in [\sigma_1, \sigma_2], \tag{3.4}$$

where

$$\Psi^{\lambda}_p(t,s) = e^{\lambda(p(s)-p(t))} \begin{cases} 1, & \sigma_1 < s < t < \sigma_2, \\ \left(1 - x^{\lambda}_{\sigma_2}\right), & \sigma_1 < t < s < \sigma_2. \end{cases}$$

Proof. By Lemma 2.2, we get

$$\begin{aligned} \mathcal{D}^{\lambda}_{p(t)}\left(x(t)e^{p(t)}\right) &= e^{\lambda p(t)}\mathcal{D}^{\lambda}_{p(t)}(x(t)) + x^{\lambda}(t)\,\mathcal{D}^{\lambda}_{p(t)}e^{p(t)} \\ &= e^{\lambda p(t)}\left(\mathcal{D}^{\lambda}_{p(t)}(x(t)) + \lambda x^{\lambda}(t)\right) \\ &= \xi(t)\, e^{\lambda p(t)}. \end{aligned}$$

Since,

$$\mathcal{D}^{\lambda}_{p(t)}e^{p(t)} = \lambda e^{\lambda p(t)}, \quad \text{for all } t \in [\sigma_1, \sigma_2].$$

By the last two equalities and Lemma 2.2, we find

$$\left(x^{\lambda}(t)e^{\lambda p(t)} - \underbrace{x^{\lambda}(\sigma_1)e^{\lambda p(\sigma_1)}}_{c}\right)^{\frac{1}{\lambda}} = \mathcal{I}^{\lambda}_{p(t),\sigma_1}\xi(t)\, e^{\lambda p(t)} = \left(\int_{\sigma_1}^{t} \xi(s)\, e^{\lambda p(s)} dp(s)\right)^{\frac{1}{\lambda}}.$$

Here,

$$x(t) = \left(ce^{-\lambda p(t)} + \int_{\sigma_1}^{t} \xi(s)\, e^{\lambda(p(s)-p(t))} dp(s)\right)^{\frac{1}{\lambda}}, \quad \text{for all } t \in [\sigma_1, \sigma_2].$$

As $x(\sigma_2) = x_{\sigma_2}$, then

$$c = x_{\sigma_2}^{\lambda} e^{\lambda p(\sigma_2)} - \int_{\sigma_1}^{\sigma_2} \xi\left(s\right) e^{\lambda p(s)} dp\left(s\right),$$

which means

$$x(t) = \left(x_{\sigma_2}^{\lambda} e^{-\lambda(p(t)-p(b))} - \int_{\sigma_1}^{\sigma_2} x_{\sigma_2}^{\lambda} \xi\left(s\right) e^{\lambda(p(s)-p(t))} dp\left(s\right) + \int_{\sigma_1}^{t} \xi\left(s\right) e^{\lambda(p(s)-p(t))} dp\left(s\right) \right)^{\frac{1}{\lambda}}.$$

Splitting the last expression at $s = t$ and using the definition of Ψ_p^{λ} gives (3.4). Conversely, if x is given by (3.4), then applying $\mathcal{D}_{p(t)}^{\lambda}$ and Lemma 2.5 to the integral term yields

$$\mathcal{D}_{p(t)}^{\lambda} x(t) + \lambda x^{\lambda}(t) = \xi(t),$$

and substituting $t = \sigma_2$ in (3.4) gives $x(\sigma_2) = x_{\sigma_2}$. Thus the integral representation is equivalent to the boundary-value problem. $\square$

Definition 3.2. We define the operators $\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}) \to \mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$ by

$$\begin{aligned} P_{\varpi}\left(x\right) &:= x^{\varpi}, \\ E_{\eta}\left(x\right) &:= x^{*}, \\ I_{y}\left(x\right) &:= \int_{\sigma_1}^{\sigma_2} y\left(t, s\right) x\left(s\right) dp\left(s\right), \\ M_{\pi}\left(x\right) &= \pi\left(s, x\right), \end{aligned}$$

where

$$\begin{aligned} &\varpi \in \widetilde{\mathbb{Q}}, \eta \in \mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}), \\ &y \in \mathcal{C}([\sigma_1, \sigma_2] \times [\sigma_1, \sigma_2], \mathbb{R}), \end{aligned}$$

and

$$\pi \in \mathcal{C}([\sigma_1, \sigma_2] \times \mathbb{R}, \mathbb{R}).$$

Lemma 3.4. *Let*

$$\begin{aligned} &\varpi \in \widetilde{\mathbb{Q}}, \eta \in \mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}), \\ &y \in \mathcal{C}([\sigma_1, \sigma_2] \times [\sigma_1, \sigma_2], \mathbb{R}), \end{aligned}$$

and

$$\pi \in \mathcal{C}([\sigma_1, \sigma_2] \times \mathbb{R}, \mathbb{R}),$$

then the operators P_{ϖ}, E_{η}, I_y and M_{π} are continuous.

Proof. Let $\varpi \in \widetilde{\mathbb{Q}}$. if $(x_n)_n$ is a sequence of $\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$ converging to $x \in \mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$, then the sequence $(x_n^{\varpi})_n$ converges towards x^{ϖ}, which means that P_{ϖ} is continuous.

Let $\eta \in \mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$. By the definition of x^*, we have

$$x_n^{*}(t) = \begin{cases} \dfrac{\eta(t)}{|x_n\left(t\right)|} x_n\left(t\right), \text{if } |x_n\left(t\right)| > \eta(t), \\ x_n(t), \qquad\qquad \text{if } |x_n\left(t\right)| \leq \eta(t), \end{cases} \to x^{*}\left(t\right),$$

consequently, E_{η} is continuous.

Let

$$y \in \mathcal{C}([\sigma_1, \sigma_2] \times [\sigma_1, \sigma_2], \mathbb{R}).$$

We obtain

$$\begin{aligned}|I_y(x_n)(t) - I_y(x)(t)| &\leq \int_{\sigma_1}^{\sigma_2} |y(t,s)| \, |x_n(s) - x(s)| \, |dp(s)| \\ &\leq |p(\sigma_2) - p(\sigma_1)| \sup_{t,s \in [\sigma_1, \sigma_2]} |y(t,s)| \, \|x_n - x\|,\end{aligned}$$

which means that $I_y(x_n)$ has converged to $I_y(x)$ in $\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$. Hence the continuity of I_y.

Let $\pi \in \mathcal{C}([\sigma_1, \sigma_2] \times \mathbb{R}, \mathbb{R})$. since $(x_n)_n$ is converged, there exist $n_0 \in \mathbb{N}$ and $T > 0$ such that

$$\|x_n\| \leq T, \text{ for all } n \geq n_0.$$

By the uniform continuity of π on $[\sigma_1, \sigma_2] \times [-T, T]$, for $\varepsilon > 0$, there exists $\delta > 0$ for $x_1, x_2 \in [-T, T]$. if $|x_1 - x_2| \leq \delta$, we have

$$|\pi(s, x_1) - \pi(s, x_2)| \leq \varepsilon,$$

on the other hand, we have $x_n \to x$ in $\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$. there exists $n_1 \in \mathbb{N}$ for $n \geq n_1$, we get $\|x_n - x\| \leq \delta$. what this implies is that we have

$$|\pi(s, x_n) - \pi(s, x_n)| \leq \varepsilon,$$

which means that M_{π} is continuous. □

Lemma 3.5. *Let*

$$\eta \in \mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}),$$

and

$$y \in \mathcal{C}([\sigma_1, \sigma_2] \times [\sigma_1, \sigma_2], \mathbb{R}),$$

be given

1. *The set* $\eta(\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}))$ *is bounded in* $\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$*,*
2. *The set* $I_y(\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}))$ *is equicontinuous.*

Proof. Let $\eta \in \mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$ be such that we have

$$\|E_{\eta}(x)\| = \|x^*\| \leq \|\eta\|,$$

that is to say,

$$\eta(\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})) \subseteq B(0, \|\eta\|),$$

hence

$$\eta(\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})),$$

is bounded in $\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$.

Let $t_1, t_2 \in [\sigma_1, \sigma_2]$, then

$$|I_y(x)(t_1) - I_y(x)(t_2)| \leq \int_{\sigma_1}^{\sigma_2} |y(t_1, s) - y(t_2, s)| \, |x(s)| \, |dp(s)|$$

$$\leq \|x\| \left|p(\sigma_2) - p(\sigma_1)\right| \sup_{s\in[\sigma_1,\sigma_2]} \left|y(t_1, s) - y(t_2, s)\right|.$$

As $t_1 \to t_2$, the RHS of the above inequality tends to zero; consequently, $I_y\left(\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})\right)$ is equi-continuous. □

Lemma 3.6. *Let*

$$f \in \mathcal{C}\left([\sigma_1, \sigma_2] \times [0, \infty), (0, \infty)\right),$$

and

$$\lambda \in \widetilde{\mathbb{Q}}, p \in \mathcal{C}_i([\sigma_1, \sigma_2], \mathbb{R}),$$
$$\eta \in \mathcal{C}^{\lambda}_{p(t)}([\sigma_1, \sigma_2], \mathbb{R}).$$

If $(0, \eta)$ *is a solution tube of* (1.1)*, then there is at least one solution to problem* (3.1)*.*

Proof. We will convert problem (3.1) into a fixed-point problem. Let $A : \mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}) \to \mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$ be

$$Ax(t) := \left(\int_{\sigma_1}^{\sigma_2} \Psi_p^{\lambda}(t, s)\left[\widetilde{f}(s, x^*(s)) + \lambda\left(x^*\right)^{\lambda}(s)\right] dp(s) + x_{\sigma_2}^{\lambda} e^{-\lambda(p(t)-p(\sigma_2))}\right)^{\frac{1}{\lambda}}.$$

We prove the existence of a fixed point of A by applying Schauder's fixed-point theorem [18]. The operator A can be written in the following form

$$Ax(t) = P_{\frac{1}{\lambda}}\left[\left(I_{\Psi_p^{\lambda}} \circ M_{\widetilde{f}} \circ E_{\eta}\right)(x) + \left(I_{\lambda\Psi_p^{\lambda}} \circ P_{\lambda} \circ E_{\eta}\right)(x) + x_{\sigma_2}^{\lambda} e^{-\lambda(p(t)-p(\sigma_2))}\right].$$

The proof proceeds in three steps:

Step 1. By Lemma 3.4, the maps P_{λ}, $P_{\frac{1}{\lambda}}$, E_{η}, $I_{\lambda\Psi_p^{\lambda}}$, $I_{\Psi_p^{\lambda}}$, and $M_{\widetilde{f}}$ are continuous since A is the sum and composition of continuous maps; therefore, A is continuous.

Step 2. The set $A(\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}))$ is uniformly bounded. By Lemma 3.5, we have $E_{\eta}\left(\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})\right)$ is bounded in $\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$, as $P_{\frac{1}{\lambda}}$, $I_{\Psi_p^{\lambda}}$, $M_{\widetilde{f}}$ and $I_{\lambda\Psi_p^{\lambda}}$ are continuous, we will deduce that $A(\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}))$ is bounded in $\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R})$.

Step 3. The set $A(\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}))$ is equicontinuous. By Lemma 3.5, we have that

$$I_{\Psi_p^{\lambda}}\left(E_{\eta}(\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}))\right),$$

and

$$I_{\lambda\Psi_p^{\lambda}}\left(E_{\eta}(\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}))\right),$$

are equicontinuous.

Consequently, $A(\mathcal{C}([\sigma_1, \sigma_2], \mathbb{R}))$ is equicontinuous.

By Steps 1 through 3, and owing to the Arzelà–Ascoli theorem, we find that A is completely continuous. Using Schauder's fixed point theorem, we find that A has a fixed point, that is a solution to (3.1). □

Theorem 3.1. *Let*

$$f \in \mathcal{C}\left([\sigma_1, \sigma_2] \times [0, \infty), (0, \infty)\right),$$

and

$$\lambda \in \widetilde{\mathbb{Q}},\ p \in \mathcal{C}_i([\sigma_1, \sigma_2], \mathbb{R}),$$

$$\eta \in \mathcal{C}^{\lambda}_{p(t)}([\sigma_1, \sigma_2], \mathbb{R}),$$

if $(0, \eta)$ *a solution tube of* (1.1)*. Then, problem* (1.1) *has a solution* $x \in \mathcal{T}^{\lambda}_{p(t)}(\eta)$*.*

Proof. By Lemma 2.4, the operator A is compact. It has a fixed point by the Schauder fixed point theorem. we show that each solution x of the problem verifies $x \in \mathcal{T}^{\lambda}_{p(t)}(\eta)$; that is to say, we demonstrate that

$$\mathcal{B} := \{t \in [\sigma_1, \sigma_2] : |x(t)| > \eta(t)\} = \emptyset.$$

Assume, for contradiction, that $\mathcal{B} \neq \emptyset$ and let $t \in \mathcal{B}$. On $\mathcal{B}$ we have $x^*(t) = \eta(t)x(t)/|x(t)|$, so $|x^*(t)| = \eta(t)$ and $x^*(t)$ lie on the boundary of the tube. By Lemmas 2.1 and 3.1, we get

$$\begin{aligned}
\mathcal{D}^{\lambda}_{p(t)}\left(\frac{|x(t)|}{\eta(t)}\right) &= \frac{1}{\eta^2(t)}\left(\eta^{\lambda}(t)\mathcal{D}^{\lambda}_{p(t)}|x(t)| - x^{\lambda}(t)\,\mathcal{D}^{\lambda}_{p(t)}\eta(t)\right) \\
&= \frac{|x(t)|^{\lambda}}{x^{\lambda}(t)\eta^{\lambda}(t)}\mathcal{D}^{\lambda}_{p(t)}x(t) - \frac{x^{\lambda}(t)}{\eta^{2\lambda}(t)}\frac{\mathcal{D}^{\lambda}_{p(t)}\eta(t)}{|x(t)|^{\lambda}} \\
&= \frac{|x(t)|^{\lambda}}{x^{\lambda}(t)\eta^{\lambda}(t)}\widetilde{f}(t, x^*(t)) + \frac{\lambda\,|x(t)|^{\lambda}}{x^{\lambda}(t)\eta^{\lambda}(t)}\left(x^*\right)^{\lambda}(t) \\
&\quad - \frac{\lambda\,|x(t)|^{\lambda}}{x^{\lambda}(t)\eta^{\lambda}(t)}x^{\lambda}(t) - \frac{x^{\lambda}(t)}{\eta^{2\lambda}(t)}\frac{\mathcal{D}^{\lambda}_{p(t)}\eta(t)}{|x(t)|^{\lambda}} \\
&= \frac{\widetilde{f}(t, x^*(t))}{\left(x^*\right)^{\lambda}(t)} + \lambda\left(1 - \frac{|x(t)|^{\lambda}}{\eta^{\lambda}(t)}\right) - \frac{\mathcal{D}^{\lambda}_{p(t)}\eta(t)}{\eta^{\lambda}(t)\left(x^*\right)^{\lambda}(t)} \\
&\leq \frac{\widetilde{f}(t, x^*(t))}{\left(x^*\right)^{\lambda}(t)} - \frac{\mathcal{D}^{\lambda}_{p(t)}\eta(t)}{\eta^{\lambda}(t)\left(x^*\right)^{\lambda}(t)}.
\end{aligned}$$

Since $(0, \eta)$ is a solution to the problem (1.1), we have

$$\mathcal{D}^{\lambda}_{p(t)}\left(\frac{|x(t)|}{\eta(t)}\right) \leq \frac{\eta^{\lambda}(t)\left(x^*\right)^{\lambda}(t)\,\widetilde{f}(t, x^*(t)) - \left(x^*\right)^{\lambda}(t)\,\mathcal{D}^{\lambda}_{p(t)}\eta(t)}{\left(x^*\right)^{2\lambda}(t)\,\eta^{\lambda}(t)} \leq 0,$$

for all $t \in \mathcal{B}$. Let

$$\theta(t) = \frac{|x(t)|}{\eta(t)} - 1,$$

for all $t \in \mathcal{B}$, then $\theta(t) \geq 0$, for all $t \in \mathcal{B}$, by Lemma 2.1, we get

$$\mathcal{D}^{\lambda}_{p(t)}\theta(t) = \frac{\eta^{\lambda-1}(t)}{|x(t)|^{\lambda-1}}\theta^{\lambda-1}(t)\,\mathcal{D}^{\lambda}_{p(t)}\frac{|x(t)|}{\eta(t)} \leq 0, \text{ for all } t \in \mathcal{B},$$

so $\mathcal{D}^{\lambda}_{p(t)}\theta(t) < 0$ on $\mathcal{B} := \{t \in [\sigma_1, \sigma_2] : \theta(t) > 0\}$. Moreover, since $(0, \eta)$ is a solution to problems (1.1) and $x(\sigma_2) = x_{\sigma_2}$, then $\theta(\sigma_2) < 0$, and as a consequence, Lemma 3.2 implies that $\mathcal{B} = \emptyset$. Thus, $x \in \mathcal{T}^{\lambda}_{p(t)}(\eta)$, which proves the theorem. □

Theorem 3.2. *Let* $\varrho \in \mathcal{C}^{\lambda,\mu}([\sigma_1, \sigma_2], [0, \infty))$*, such that*

1. $\begin{cases} \varrho^{\lambda}(t)\,\widetilde{f}(t, \varrho(t)) \leq \mathcal{D}^{\lambda}_{p(t)}\varrho(t), \\ -\varrho^{\lambda}\widetilde{f}(t, -\varrho(t)) \leq \mathcal{D}^{\lambda}_{p(t)}\varrho(t), \end{cases}$ *for all* $t \in [\sigma_1, \sigma_2]$*,*

2. $f(t,0)=0$ *and* $\mathcal{D}^{\lambda}_{p(t)}\varrho(t)=0$ *for all* $t\in[\sigma_1,\sigma_2]$ *such that* $\varrho(t)=0$,
3. $|x_{\sigma_2}|\leq\varrho(\sigma_2)$.

Then there is at least one solution x *to problem* (1.1) *verifying* $x\in\mathcal{T}^{\lambda}_{p(t)}(\varrho)$.

Proof. Let us show that $(0,\varrho)$ is a tube solution to problem (1.1). If $|x|=\varrho(t)$, then $x=\varrho(t)$ or $x=-\varrho(t)$, so

$$\varrho^{\lambda}(t)\,x^{\lambda}\widetilde{f}(t,x)=\begin{cases}\varrho^{2\lambda}(t)\,\widetilde{f}(t,\varrho(t)), & \text{if } x=\varrho(t)\\ -\varrho^{2\lambda}(t)\,\widetilde{f}(t,-\varrho(t)), & \text{if } x=-\varrho(t)\end{cases}\leq x^{\lambda}(t)\,\mathcal{D}^{\lambda}_{p(t)}\varrho(t),$$

conditions (2) and (3) are conditions (ii) and (iii) of tube-solutions in Definition 3.1, respectively. By Theorem 3.1, it is implied that problem (1.1) possesses at least one solution $x\in\mathcal{T}^{\lambda}_{p(t)}(\varrho)$.

4. Illustrative example

To demonstrate the applicability of the obtained existence result, we consider the following Hausdorff fractional thermistor problem:

$$\begin{cases}\mathcal{D}^{\lambda}_{p(t)}x(t)=\dfrac{A}{(1+x(t))^2}+h(t), & t\in[0,1],\\ x(0)=x(1),\end{cases}\tag{4.1}$$

where $0<\lambda<1$, $A>0$ are real parameters, $h(t)$ is a continuous function on $[0,1]$, and $\mathcal{D}^{\lambda}_{p(t)}$ denotes the Hausdorff fractional derivative of order λ. For illustrative purposes, we choose

$$h(t)=t^2+1.\tag{4.2}$$

Define the operator

$$\mathcal{T}:C([0,1],\mathbb{R})\to C([0,1],\mathbb{R}),$$

by

$$(\mathcal{T}x)(t)=x(0)+\int_0^t\left(\frac{A}{(1+x(s))^2}+h(s)\right)\frac{ds}{s^{1-\lambda}}.\tag{4.3}$$

Let us consider the tube

$$T=\{x\in C([0,1]):m(t)\leq x(t)\leq M(t)\},\tag{4.4}$$

where we take

$$m(t)=0,\qquad M(t)=2+t.\tag{4.5}$$

For every $x\in T$, we have

$$\frac{A}{(1+x(t))^2}+h(t)\leq\frac{A}{1}+(t^2+1)\leq A+2,\tag{4.6}$$

which is bounded on $[0,1]$. Hence, $\mathcal{T}$ maps T into itself. Moreover, $\mathcal{T}$ is continuous and compact due to the Arzelà–Ascoli theorem.

Thus, by Schauder's fixed point theorem [17,25], the operator $\mathcal{T}$ has at least one fixed point in T, which corresponds to a solution of the Hausdorff fractional thermistor problem (4.1).

This example confirms that the hypotheses of the main existence theorem are satisfied for a nontrivial choice of data and demonstrates the feasibility and effectiveness of the proposed approach. □

5. Conclusion and perspectives

This article investigates the existence of solutions for a thermistor problem involving the Hausdorff fractional derivative. The proposed framework extends the classical thermistor equation by replacing the standard derivative with the Hausdorff derivative. In the special case $\lambda = 1$ and $p(t) = t$, the model reduces to the classical formulation. The existence analysis is based on Schauder's fixed-point theorem and the solution-tube method.

The results are consistent with several known existence theorems in the literature. They may also be extended to the structurally differentiable nabla thermistor problem on time scales,

$$\begin{cases} x^{\nabla^{\lambda}_{p(t)}}(t) = \dfrac{\gamma f(t, x(t))}{\left(\int_{\sigma_1}^{\sigma_2} f(s, x^{\rho}(s))\, \nabla s\right)^2}, \quad \text{for all } [\sigma_1, \sigma_2] \subset \mathbb{T}_k, \\ \varpi x(\sigma_1) - \nu x(\sigma_2) = \mu, \end{cases} \tag{5.1}$$

here $x^{\nabla^{\lambda}_{p(t)}}$ denotes the structural ∇-derivative on a time scale. For more details on this derivative, see [11].

Existence conditions for (5.1) can be obtained by adapting the methodology developed in this work. In particular, Theorems 3.1 and 3.2 suggest corresponding extensions to this time-scale setting; the detailed development will be considered in future work.

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